A TEXT-BOOK

ON

SURVEYING AND LEVELLING

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PREFACE TO THIRD EDITION.

THE call for a further edition of this work is gratifying to both the Author and the Publishers, and in meeting this call the opportunity has been taken to revise the work thoroughly. This had become imperative owing to alterations in the *Nautical Almanac* in connection with the change in zero hour from 0h. noon to 0h. midnight G.M.T. and the change in the method of interpolation, and also on account of the advent of wireless rhythmic time signals, which renders the task of obtaining G.M.T. at any place so simple. The alterations in method brought about by these changes have been fully explained and illustrated by a large number of examples in Chapters XVI. and XVII. New matter has been inserted on (*a*) traversing ; (*b*) the mass diagram , (*c*) heights of survey scaffolds ; (*d*) interpolation ; and (*e*) determination of longitude.

I am under a great obligation to Mr. R. J. Cornish, M. Sc. (Manchester), A. M. Inst. C. E., A. M. I. Mech. E., M.R.San.I., Head of the Municipal Engineering Department, College of Technology, Manchester, for his kindness in reading through Chapters XVI. and XVII. and for many useful suggestions ; also to the Controller of H.M. Stationery Office for permission to reproduce the table of $h_2 = \dfrac{2 \sin^2 (\frac{1}{2}\text{H} \times 15)}{\sin 1''}$ from a " Text-book of Topographical Surveying," by Colonel C. F. Close, C.M.G., R.E. and Captain E. W. Cox, R.E., and given here in Appendix C.

H. T.

PREFACE TO FIRST EDITION.

PROFESSIONAL men who require a knowledge of Surveying and Levelling in the exercise of their profession, may be divided into two main groups, viz.:—(*a*) those for whom the preparation of land plans is only the preliminary to further operations, and (*b*) those for whom it is the sole object of their work. The former, which is nowadays by far the larger group, comprises Architects and Civil Engineers, while the latter comprises Surveyors whose operations vary from the preparation of plans of small estates, to those engaged in the preparation of maps covering a greater or less extent of country, or of a route passed through.

The knowledge of the subject required by Architects and Estate Surveyors is, as a rule, not very extensive; a good working knowledge of Chain Surveying, Traversing, and Levelling will, in the main, satisfy their needs. The Civil Engineer is required, in addition, to know the principles of Setting Out whether on the straight or the curve, on the surface or underground, while he may occasionally be called upon to determine Azimuth, Latitude, and Time from astronomical observations, hence he must be thoroughly conversant with the construction and manipulation of the theodolite and spirit level, and ·be able at any time to test and adjust both types of instrument.

It is with the object of endeavouring to meet the needs of both groups of men, and of students of the subject generally, that this book has been written and divided into two parts. It is the outcome of the Author's practical experience both in the field, and as the Lecturer in the subject for the past twenty years in the Municipal School (now College) of Technology, Manchester.

Part I. is divided into the five main sections of Chain Surveying, Traversing, the Computation and Setting Out of Cuttings and Embankments, and Office Work, and should be sufficient to meet the needs of those whose operations are of a simple character, not requiring the use of the theodolite. In this part the effect of the spherical form of the earth on surveying operations has been but briefly mentioned.

The matter in Part II. covers the ground required by the Civil Engineer engaged in making surveys for (or in setting out the sites of) proposed works, and also that required by the Surveyor engaged on Geodetic Surveys. The influence of the sphericity of the earth both on final delineation of extensive surveys and on the field operations is here more fully dealt with.

It is hoped that the arrangement of the chief problems which arise in survey practice in one chapter, as is done in Chapter XI., will be found convenient for reference.

The chapter on Tacheometry and also the brief description of the methods applied and principles involved in Photographic Surveying have been introduced to meet the needs of Surveyors in the Colonies, where both systems of Surveying are now largely employed.

As Spherical Trigonometry is not taught in many Secondary Schools and Colleges, it has been thought desirable to introduce a brief description of the subject and to give the demonstrations of the more important

formulæ in order to render the matter on Practical Astronomy more intelligible to the student.

Wherever it has been thought desirable, full demonstrations of the formulæ required in the solutions of the various problems dealt with have been given, and the methods of applying the formulæ to particular cases are copiously illustrated by examples throughout the text. This should be of great assistance to students generally and particularly to those who are preparing for the examination of Associateship of the Institution of Civil Engineers, or for the Final Examination in the Faculty of Engineering in most Universities. The examples given at the end of each chapter will enable the student to test his reading at the different stages of his studies.

Although every care has been taken both in working out and checking the results of the numerical examples in the text, and at the end of each chapter, yet in spite of these efforts some errors may remain, and I shall be obliged for any intimation of these which readers may discover.

My thanks are due, and gratefully tendered, to Mr. W. C. Jenkins, F.R.A.S., the Curator of the Godlee Observatory at the Municipal School (now College) of Technology, Manchester, for his assistance in reading and checking the manuscript of the section on Practical Astronomy; to Mr. G. S. Coleman, D.Sc. (Eng. London), A.M.Inst.C.E., formerly Trench Warfare Engineer under the Ministry of Munitions, Newcastle, for reading through the manuscript and for many valuable suggestions which have greatly enhanced the value of the book; to the Controller of H.M. Stationery Office for permission to reproduce the tables of Astronomical Refraction given in Appendix A; to Messrs. A. G. Thornton, Ltd., Manchester; Messrs. J. Halden & Co., Manchester; Carl Zeiss, Jena; and to

Messrs. W. F. Stanley & Co., Ltd., London, for their kind-
ness in allowing me the use of their blocks of surveying
instruments, used throughout the text; also to Messrs.
Charles Griffin & Co., Ltd., for permission to make use of
several diagrams from two of their publications.

<div style="text-align: right">H. THRELFALL.</div>

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TEXT-BOOK ON

SURVEYING AND LEVELLING

PART I.

SURVEYING.

PRELIMINARY REMARKS.

FORMULÆ, ETC.

SURVEYING may be defined as "the art of ascertaining and representing the shape and extent of any portion of the earth's surface and of the objects upon it, as projected on a horizontal surface." In this restricted sense, surveying does not include levelling, but the latter is quite as necessary to the engineer as the former, hence in its wider sense surveying is understood to include levelling.

We may define the term levelling as "the art of ascertaining and representing the relative elevations of different points on the earth's surface and of the objects upon it."

In surveys covering a large extent of country, many refinements, both instrumental and mathematical, have to be introduced in order to allow for the curvature of the earth's surface. To the subject dealing with this—the most accurate mode of surveying—the term Geodesy is applied, and the surveys are spoken of as Geodetic Surveys, or as Great Trigonometrical Surveys.

The refinements introduced in geodetic surveying are not necessary in surveys for ordinary engineering purposes, as the errors introduced by neglecting the sphericity of the earth are negligible in work of this character, and consequently for all ordinary purposes the operations of the surveyor are carried out as though the earth's surface were plane and not spheroidal.

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The most imperfect type of surveying is that carried out for military purposes, and is known as Military Surveying.

Military Surveying must not, however, be confounded with the work done by the Ordnance Surveyors, who are drawn very largely from the officers and men of the Royal Engineers. The surveying work done by the Ordnance Department is of the highest kind, both in this Country and in the Crown Colonies, and comprises the delimitation of boundaries as well as topographical surveys. Much of this work is done in wild and unexplored countries devoid of artificial landmarks, and involves a considerable amount of astronomical work.

Military surveying is carried out on the principles of ordinary practice, but in a very rough manner, rapidity of execution being of paramount importance, the permissible error is large, and the relative positions of the chief surface features are obtained by aid of pocket instruments, the smaller detail being filled in by guesswork and sketching.

All ordinary survey work—from that embracing a large district to the measurement of plots of land for building purposes —will lie between these two extreme types.

Surveying may be looked upon as a branch of practical mathematics, as it is based on the principles of geometry and trigonometry ; hence it is necessary that a surveyor should have a fair general knowledge, not only of these subjects, but of all the subjects comprised by the term mathematics. In addition to this, he should know something of the principles of magnetism and optics, and of the arts of drawing and colouring.

The knowledge of mathematics required in ordinary chain surveying and levelling is not very extensive, but in geodetical work the highest mathematical ability and great organising power are required for a proper concep ion and supervision of the operations.

In the field considerable powers of judgment, observation, and organisation are required. These can only be acquired by practice in the field, and the student must guard himself against thinking that the mere reading of this or any other book on the subject will give him that technical knowledge which field practice alone can give. The marking out of base lines for a survey is a case in point. Every surveyor knows that the best lay-out of base lines on paper may be the worst, or even an impossible one, on the site. The actual lay-out may appear very unsatisfactory when plotted, but may be the best for the ground surveyed, as unforeseen difficulties and obstacles crop up as soon

as the work commences, and it is in overcoming these difficulties that the greatest calls are made on the practical experience and judgment of the surveyor.

FORMULÆ.

The following formulæ in plane trigonometry are given on account of their general utility to the surveyor, but no attempt is made to prove them, as it is assumed that this has already been done in the student's general education, otherwise any good text-book on plane trigonometry must be referred to.

Other formulæ will be given as the need arises.

Let A, B, and C be the angles of a right-angled triangle, right-angled at C, and let a, b, and c be the sides of the triangle opposite to the angles A, B, and C respectively.

By taking the ratios of the sides of the triangle we obtain the six fundamental equations, thus :—

$$\text{Sin A} = \frac{a}{c}. \qquad \bullet \quad \bullet \quad \bullet \quad \bullet \quad (1)$$

$$\text{Cos A} = \frac{b}{c}. \qquad \bullet \quad \bullet \quad \bullet \quad \bullet \quad (2)$$

$$\text{Tan A} = \frac{a}{b}. \qquad \bullet \quad \bullet \quad \bullet \quad \bullet \quad (3)$$

$$\text{Cosec A} = \frac{c}{a}. \qquad \bullet \quad \bullet \quad \bullet \quad \bullet \quad (4)$$

$$\text{Sec A} = \frac{c}{b}. \qquad \bullet \quad \bullet \quad \bullet \quad \bullet \quad (5)$$

$$\text{Cot A} = \frac{b}{a}. \qquad \bullet \quad \bullet \quad \bullet \quad \bullet \quad (6)$$

Hence,
$$\text{Sin A} = \frac{1}{\text{cosec A}}. \qquad \bullet \quad \bullet \quad \bullet \quad (7)$$

$$\text{Cosec A} = \frac{1}{\text{sin A}}. \qquad \bullet \quad \bullet \quad \bullet \quad (8)$$

$$\text{Cos A} = \frac{1}{\text{sec A}}. \qquad \bullet \quad \bullet \quad \bullet \quad (9)$$

$$\text{Sec A} = \frac{1}{\text{cos A}}. \qquad \bullet \quad \bullet \quad \bullet \quad (10)$$

$$\text{Tan A} = \frac{1}{\text{cot A}}. \qquad \bullet \quad \bullet \quad \bullet \quad (11)$$

$$\text{Cot A} = \frac{1}{\text{tan A}}. \qquad \bullet \quad \bullet \quad \bullet \quad (12)$$

Functions of the Complements of Angles, etc.

$$\text{Sin A} = \cos (90° - \text{A}). \quad . \quad . \quad (13)$$
$$\text{Cos A} = \sin (90° - \text{A}). \quad . \quad . \quad (14)$$
$$\text{Cosec A} = \sec (90° - \text{A}). \quad . \quad . \quad (15)$$
$$\text{Sec A} = \text{cosec} (90° - \text{A}). \quad . \quad (16)$$
$$\text{Tan A} = \cot (90° - \text{A}). \quad . \quad . \quad (17)$$
$$\text{Cot A} = \tan (90° - \text{A}). \quad . \quad . \quad (18)$$
$$\text{Sin}^2 \text{A} + \cos^2 \text{A} = 1. \quad . \quad . \quad . \quad (19)$$
$$\text{Sec}^2 \text{A} = \tan^2 \text{A} + 1. \quad . \quad . \quad (20)$$
$$\text{Cot}^2 \text{A} = \text{cosec}^2 \text{A} - 1. \quad . \quad . \quad (21)$$

Angles Greater than a Right Angle.

$$\text{Sin} (n\,360° + \text{A}) = \sin \text{A}. \quad . \quad . \quad (22)$$
$$\text{Cos} (n\,360° + \text{A}) = \cos \text{A}. \quad . \quad . \quad (23)$$
$$\text{Sin} (180° + \text{A}) = -\sin \text{A}. \quad . \quad . \quad (24)$$
$$\text{Cos} (180° + \text{A}) = -\cos \text{A}. \quad . \quad . \quad (25)$$
$$\text{Sin} (180° - \text{A}) = \sin \text{A}. \quad . \quad . \quad (26)$$
$$\text{Cos} (180° - \text{A}) = -\cos \text{A}. \quad . \quad . \quad (27)$$
$$\text{Sin} (360° - \text{A}) = -\sin \text{A}. \quad . \quad . \quad (28)$$
$$\text{Cos} (360° - \text{A}) = \cos \text{A}. \quad . \quad . \quad (29)$$
$$\text{Sin} (90° + \text{A}) = \cos \text{A}. \quad . \quad . \quad (30)$$
$$\text{Cos} (90° + \text{A}) = -\sin \text{A}. \quad . \quad . \quad (31)$$
$$\text{Sin} (-\text{A}) = -\sin \text{A}. \quad . \quad . \quad (32)$$
$$\text{Cos} (-\text{A}) = \cos \text{A}. \quad . \quad . \quad (33)$$
$$\text{Cot} (180° - \text{A}) = -\cot \text{A}. \quad . \quad . \quad (34)$$
$$\text{Tan} (180° - \text{A}) = -\tan \text{A}. \quad . \quad . \quad (35)$$

Sums, Differences, etc., of Angles.

$$\text{Sin} (\text{A} + \text{B}) = \sin \text{A} \cos \text{B} + \cos \text{A} \sin \text{B}. \quad . \quad . \quad (37)$$
$$\text{Sin} (\text{A} - \text{B}) = \sin \text{A} \cos \text{B} - \cos \text{A} \sin \text{B}. \quad . \quad . \quad (38)$$
$$\text{Cos} (\text{A} + \text{B}) = \cos \text{A} \cos \text{B} - \sin \text{A} \sin \text{B}. \quad . \quad . \quad (39)$$
$$\text{Cos} (\text{A} - \text{B}) = \cos \text{A} \cos \text{B} + \sin \text{A} \sin \text{B}. \quad . \quad . \quad (40)$$

$$\text{Tan} (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}. \qquad \text{. . . (41)}$$

$$\text{Tan} (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}. \qquad \text{. . . (42)}$$

$$\text{Cot} (A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}. \qquad \text{. . . (43)}$$

$$\text{Cot} (A - B) = \frac{\cot A \cot B + 1}{\cot A - \cot B}. \qquad \text{. . . (44)}$$

Hence, when $A = B$, we have :—

$$\text{Sin } 2 A = 2 \sin A \cos A. \qquad \text{. . . . (45)}$$

$$\text{Cos } 2 A = \cos^2 A - \sin^2 A. \qquad \text{. . . (46)}$$

$$= \cos^2 A - (1 - \cos^2 A).$$

$$= 2 \cos^2 A - 1. \qquad \text{. . . . (47)}$$

$$\text{Tan } 2 A = \frac{2 \tan A}{1 - \tan^2 A}. \qquad \text{. . . . (48)}$$

$$\text{Cot } 2 A = \frac{\cot^2 A - 1}{2 \cot A}. \qquad \text{. . . . (49)}$$

Sums and Differences of Functions.

$$\text{Sin } A + \sin B = 2 \sin \left(\frac{A + B}{2} \right) \cos \left(\frac{A - B}{2} \right). \text{. . (50)}$$

$$\text{Sin } A - \sin B = 2 \sin \left(\frac{A - B}{2} \right) \cos \left(\frac{A + B}{2} \right). \text{. . (51)}$$

$$\text{Cos } A + \cos B = 2 \cos \left(\frac{A + B}{2} \right) \cos \left(\frac{A - B}{2} \right). \text{. . (52)}$$

$$\text{Cos } A - \cos B = - 2 \sin \left(\frac{A + B}{2} \right) \sin \left(\frac{A - B}{2} \right). \text{. (53)}$$

$$\text{Tan } A + \tan B = \frac{\sin (A + B)}{\cos A \cos B}. \text{ . . . . (54)}$$

$$\text{Tan } A - \tan B = \frac{\sin (A - B)}{\cos A \cos B}. \text{ . . . . (55)}$$

$$\text{Cot } A + \cot B = \frac{\sin (A + B)}{\sin A \sin B}. \text{ . . . . (56)}$$

$$\text{Cot } A - \cot B = \frac{\sin (A - B)}{\sin A \sin B}. \text{ . . . . (57)}$$

Sides and Angles.

$$\text{Sin A} = \sin(B + C). \qquad \bullet \qquad \bullet \qquad \bullet \quad (58)$$

$$\text{Cos A} = -\cos(B + C). \quad \bullet \qquad \bullet \qquad \bullet \quad (59)$$

$$\text{Tan}\, \frac{A}{2} = \cot\left(\frac{B + C}{2}\right). \quad \bullet \qquad \bullet \qquad \bullet \quad (60)$$

$$\text{Tan}\, \frac{A - B}{2} = \frac{a - b}{a + b} \cot\frac{C}{2}. \qquad \bullet \qquad \bullet \qquad \bullet \quad (61)$$

$$\frac{\text{Sin A}}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}. \qquad \bullet \qquad \bullet \qquad \bullet \quad (62)$$

$$a^2 = b^2 + c^2 - 2bc \cos A. \qquad \bullet \qquad \bullet \quad (63)$$

$$\text{Cos A} = \frac{b^2 + c^2 - a^2}{2bc}.$$

$$\text{Cos}\, \frac{A}{2} = \sqrt{\frac{s\,(s - a)}{bc}}. \qquad \bullet \qquad \bullet \qquad \bullet \quad (64)$$

$$\text{Sin}\, \frac{A}{2} = \sqrt{\frac{(s - b)\,(s - c)}{bc}}. \qquad \bullet \qquad \bullet \quad (65)$$

Where
$$s = \frac{a + b + c}{2}.$$

CHAPTER I.

INSTRUMENTS FOR MEASURING DISTANCE.

Rods, Poles, etc.

THE instruments in most general use for the direct measurement of distance consist of the following :—(1) Chains, (2) steel bands, (3) tapes, (4) offset and butt rods, (5) wheel pedometers, and (6) pacing with or without passometers.

Chains.—The chain is constructed of long and short links made of stout steel wire. Generally the long links are connected by three short oval links. This construction is the best, since by using long links the number of joints is reduced, and by connecting the long links with short ones the pliability of the chain is not interfered with. Further, the chain may be wrapped up into a compact bundle, as shown in Fig. 1, hence is extremely portable, and, lastly, this construction enables the surveyor to obtain odd lengths on the chain more easily. In the best chains the joints in the wire forming the links are brazed. The ends of the chain are provided with brass swivel handles. There are

Fig. 1.

two chains in common use in this country—viz., the 66-feet or " Gunter " chain, and the 100-feet chain. Both chains are divided into 100 equal parts, which are spoken of as links. Evidently the length of the Gunter link = 66 × 12/100 = 7·92 inches, and that of the 100-feet chain equals 1 foot. The length of the chain is the distance between the outer edges of the

handles, when the chain is stretched tight and laid on a flat surface; consequently, the first and last links include the handles. On the remainder of the chain, a link is denoted by the distance between the centres of two consecutive middle small links.

Tallies.—In order that distances may be more easily read off, the chain is divided into ten equal parts by means of small pointed brass tallies. The number of points on each tally corresponds to the number of tens of links that it is from the nearest end of the chain. Thus the 10th and 90th links are each marked by a tally having one point; the 20th and 80th links by a tally having two points, and so on, the centre or 50th link being marked by a round tally. The forms of the tallies are shown in Fig. 1.

The Gunter chain is generally used when the object of the survey is to determine the acreage of the ground surveyed, as the length of this chain was chosen by its inventor, the Rev. Edmund Gunter, in 1620, in order to facilitate the computation of land areas, 10 square chains being equal to 1 acre. For a like reason, and on account of its handiness, it is much used in colliery surveying, for which purpose it is sometimes made with the first ten links at each end in brass.

The 100-feet chain is used almost exclusively on surveys for engineering purposes. It is preferred on account of its unit being 1 foot, and owing to its greater length it is more accurate when used for the measurement of long lines.

In the surveys of metalliferous mines, the chain is generally 60 feet or 10 fathoms in length. The links are each 6 inches long, a brass link being inserted to mark the fathoms. In some cases the chain is made entirely in brass and is provided with suitable tallies.

Arrows.—Accompanying each chain are ten arrows or pins, which are made of stout steel wire, sharpened at one end and bent into a ring at the other (see Fig. 2); sometimes an eleventh arrow made of brass is added. The arrows are used for the purpose of marking the end of each chain length, and the brass arrow for marking the end of each tenth chain. Arrows have usually a piece of red or white tape tied on at the ring, to make them more easily visible at a distance.

The chain cannot be considered an accurate measuring instrument, as it is liable to have its length increased by stretching when in continued use, or when working over muddy ground to be shortened by mud clogging the links.

Chain Standard.—The chain should be frequently tested either by applying it to the standard, a copy of which is laid down on some public building in most large towns, or to a steel measuring band, which should be kept in the office for this sole purpose. If the surveying operations about to be carried out are of an extended character, a chain standard should be made by driving two pegs into the ground at the requisite distance apart, and the chain should be applied to this standard before commencing each day's work. The length of the chain may be adjusted by bending one of the long links or by removing one or more of the small oval links, if it is too long, or if too short, by straightening any bent links and flattening out a few of the small links.

The change of length due to alteration of temperature is so small that it may be neglected in all ordinary surveying operations.
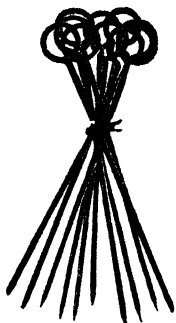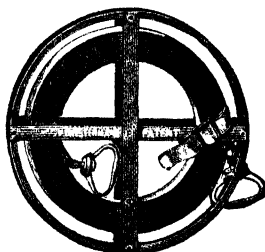
Fig. 2.   Fig. 3.

Steel Bands.—For the accurate measurement of distance steel bands are now largely used. These bands may be purchased of almost any length up to 500 feet, and in width from $\frac{1}{16}$ inch to $\frac{3}{4}$ inch. They are provided with swivel handles at each end, and when not in use are wound on a steel cross, or on a reel (see Fig. 3). The divisions are etched in feet, inches, and $\frac{1}{8}$ths on one side, and links on the other in the broader and shorter bands. In the narrower and longer bands the first and last chains are divided into links, the remainder by a short numbered brass sleeve at every chain.

The steel band is used in precisely the same way as the chain, and affords the same facilities for the rapid measurement of distance, but it is more accurate, since it is not so liable to kink

or stretch when in use. Its chief disadvantage is that it is easily broken and difficult to repair.

For accurate work the steel band should always be used in preference to the chain, but it should only be placed in the hands of careful chainmen.

The steel tape is much lighter than the chain, and consequently it can be more easily kept horizontal when working over broken ground. The chain is rarely used in the United States of America or in the Colonies, as it is not so handy in use as the steel tape.

Wooden Rods.—When more accurate measurements are required than it is possible to obtain with a steel band, rods made of deal or lance wood are occasionally used. The rods are rarely placed directly on the ground, which is seldom even enough for the purpose, but are supported on stakes or trestles. When the ground is sufficiently level, the rods (supported on the stakes) are placed end to end. On inclined ground the ends of the rods are brought into the same vertical plane by means of a plumb-bob, or an accurate set square placed upright on the lower rod, the rods themselves being set horizontal by a hand level. Great care is necessary to keep the rods in line during the measurements, and for this purpose a cord stretched accurately in the line is sometimes made use of. Wooden measuring rods are usually made 5 or 10 feet long, of well-seasoned straight wood, and to prevent changes of length from dryness or humidity, they should be saturated with boiling oil and heavily varnished.
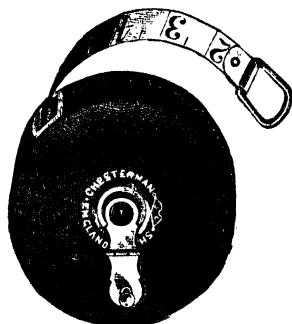
Fig. 4.

Field Compasses.—On the Continent rods three to four yards long are employed. These rods are terminated by two points, and are provided with a handle and a spirit level in the middle of their length. This apparatus is called the field-compasses, and is often used for filling in the details of a survey.

The Linen Tape.—This is probably the most unsatisfactory instrument used by the surveyor for the measurement of distance; its use should be restricted to the measurement of offsets. It consists of a painted linen band or tape about ⅝ inch wide and 66 or 100 feet long. One one side it is divided in feet, inches and half-inches, and in links and poles on the other. The end

of the tape is provided with a brass ring, the length of which is included in the first inch or link. For convenience in use, the tape is wound on a brass arbor provided with a small folding handle. The arbor is fitted to the centre of a circular leather case, as shown in Fig. 4. The linen tape is very liable to stretch when in use ; to shorten when wet, and to blow about when used in windy weather. On no account should it be used for measuring main lines, or for measuring an offset of a greater length than two chains.

Steel band tapes, constructed and divided like linen tapes, may be obtained, and should be used on all work where great accuracy is required.

Offset Rods. — These are round wooden rods, shod with a pointed iron shoe at one end, and provided with a notch or a hook at the other. The rod is marked out in links or feet by alternate bands of red, white, and black paint (see Fig. 5). The rod is shod with iron in order that it may be used as a ranging rod, while the hook at the opposite end is often useful for drawing the handle of the chain through the roots of hedges and brushwood. As its name implies, the rod is intended for the measurement of offsets—*i.e.*, short distances at right angles to the chain.

The Butt Rod, shown in Fig. 6, is also used for measuring offsets, but is more often used by the building surveyor or by architects than by the land surveyor. It consists of two laths, each one yard in length, loosely riveted together at one end, the joint being provided with a spring

Fig. 5.   Fig. 6.

catch to keep the rod extended. The rod is painted black, and
the divisions (feet and inches) are marked out with white and
red paint.

The Wheel Pedometer or Viameter.—This instrument (see
Fig. 7) is sometimes used for the rapid determination of the
distance between two points. The wheel is constructed so that
its circumference is a multiple of a yard, and its axis is fixed to
the prongs of a forked handle, the wheel turning between
them. The wheel is rolled along the line whose length is
required, its motion being communicated to a cyclometer
arrangement, the dials on which register the distance
traversed to the nearest yard.
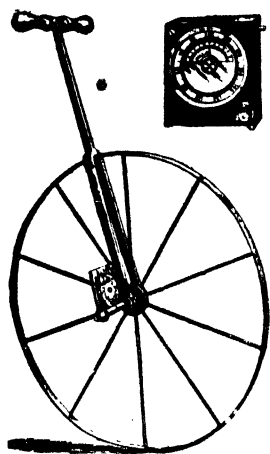
Fig. 7.

Since the instrument registers the
length of the surface actually passed
over, it is obvious that the readings
obtained on undulating ground are
too inaccurate to be of much use to
the surveyor. On even ground of
uniform surface the readings given
by this instrument are fairly satis-
factory.

This instrument is chiefly used by
road authorities for fixing the posi-
tions of mile stones.

Pacing. — The measurement of
distance by pacing is chiefly con-
fined to the preparation of military
plans, explorations, preliminary sur-
veys, or to rough levelling with the
aneroid barometer. To obtain satis-
factory results, the surveyor must
know the average length of his
step, which he should obtain by pacing a measured distance
over ground of varying character. The average length of the
step varies with (*a*) the age of the person making the
measurement; the length diminishes rapidly with increasing
age after the age of 25 to 30; (*b*) the gradient of the
line to be measured, being shorter both on going uphill and
downhill than the normal; (*c*) the height of the individual,
varying from 29·5 inches in a person 5 feet high to 32·6 inches
when his height is 6 feet; (*d*) the physical condition of the
observer; and (*e*) the direction of the wind.

It is obvious that a unit of measurement, which depends for

its length on so many varying factors, cannot be used for any but the most approximate work.

Passometers.—To relieve the observer from the monotony of counting his steps in pacing long lines, a pocket instrument, called a passometer, may be used. This instrument automatically records the number of steps taken in pacing a given distance.

Pedometers are used for the same purpose, but they may be adjusted to the length of the step of the person using them, and record the distance traversed in miles and yards.

INSTRUMENTS FOR MARKING STATIONS.

Pegs.—These are used for marking the positions of stations. They are made of any fairly hard timber, a convenient size being 10 inches long by $1\frac{1}{2}$ inches square. One end is sharpened, and the other should be painted a bright scarlet, so that the peg may be more easily found. Pegs when driven into the ground should not project more than $1\frac{1}{2}$ inches, otherwise they may injure cattle grazing on the land. It is a good plan to have a fine saw cut made in the top of each peg, so that a slip of paper may be inserted giving the peg number or other data.

Ranging Rods.—These are used for the purpose of marking out the positions of stations and lines, which are necessary for the survey. They are constructed of straight-grained spruce or pitch pine rods, 6 to 9 feet long, generally circular in section, tapering from about $1\frac{1}{4}$ inches in diameter at the bottom to $\frac{3}{4}$ inch at the top. The bottom of the rod is provided with a pointed iron shoe, so that it may be driven firmly into the ground. In order to distinguish the rods against backgrounds of different colour, they are painted in alternate bands of black, white, and red. Unless seen against the sky these rods are almost invisible at a distance of about 700 feet, hence when used on long lines each rod should have a red, white, or yellow flag, about 18 inches square, tied on near its top. White and yellow flags are visible over much greater distances than red flags.

Cases often occur in which a clear sight of the rod is prevented by the intervention of some obstacle, such as a depression in the ground, a high wall, or a hedge. When this happens two or more rods should be lashed together until a rod of the requisite length is obtained, care being taken that the rod stands quite vertical, and that it is firmly stayed with any handy material.

In town work, where it is difficult to drive the point of the rod into the ground, it is usual to hold the rod in position by

clamping it in a wood or metal tripod, the actual station being indicated either by a cross cut into the pavement or by an iron spike driven between the setts.

Poles.—In the case of very long lines, long fir poles are used, varying in length from 12 to 30 feet, and from $2\frac{1}{2}$ to 4 inches diameter at the base. These poles must be quite straight, but need not be painted; they should in all cases be provided with a large flag. The foot of each pole is sunk about 2 feet into the ground, the pole being set quite vertical by aid of a plumb-bob.

Permanent stations are distinguished from temporary stations by using a pole which is stayed at its base by cross struts, and by having a cross batten nailed to it near its top.

Plasterer's Laths.—A bundle of plasterer's laths will often be found of great service in a country survey. The laths are light both in colour and weight; they are easily carried about and easily sharpened with a knife when required. They are chiefly used for marking out a line when crossing a depression from which the forward rod is invisible, or when it is hidden by obstacles, such as hedges, walls, buildings, etc.

Whites.—These are used for the same purpose as the laths above referred to, but are not so satisfactory in use. They are simply sharpened thin sticks cut from the nearest hedge. The sticks are split with the knife at the top, and pieces of white paper are inserted in the clefts in order to make them more visible when stuck up in the grass.

CHAPTER II.

FUNDAMENTAL OPERATIONS—INSTRUMENTS.

To Set up a Rod.—It may appear trivial to introduce a description of such a simple operation in a book of this kind, but it is the author's experience that all beginners are notoriously careless about the way in which they set a rod in the ground, and it does not improve one's temper to see your forward rod—maybe a quarter of a mile away—suddenly disappear as it is blown over by the wind, or to have to wait—when about to read an angle with an instrument—until your chainman goes to set a rod upright. With men new to the work, the time spent in explaining this simple operation is well repaid at later stages.

To place the rod in position, hold it firmly in the hand with its point about a foot above the ground, then drive it into its place by a quick, downward blow. If not driven far enough, pull the rod out of the ground and enlarge the hole by using the rod as a reamer, then drive the rod in a second time. When driven home, the shoe should be almost buried and the rod stand quite firm and upright. If a rod has to be held up by a chainman, as sometimes occurs in a town, make him stand *sideways* to the rod, so as to keep it upright from the side while you keep it upright on the chain line.

Ranging Rods in Line.—To range a rod in line with two others which are already in position, send an assistant forward with the rod to the place at which it is to be set up, and standing about two yards away from the near rod, place the eye in line with the two rods which are in position. Now signal your assistant to move his rod to the right or left as required until the three rods are exactly in the same straight line, then signal him to plant the rod, and finally check its position before allowing him to leave it. In the same way laths and whites are ranged in line where required.

Method of Approximations.—It frequently happens that a line is placed in such a position that on standing at either end

of it the rod at the other is invisible owing to a rise in the ground. In such a case, an intermediate rod must be ranged in line near the top of the rise, in order to guide the chainmen.

To do this, let each of your assistants take a rod and place himself in such a position that he can see the forward or backward rod, as the case may be. The positions taken up by the chainmen are indicated in Figs. 8 and 9 by the letters C and D, A and B being the forward and backward rods respectively. The rods having been placed approximately in the line, the assistant at C (Fig. 9) plants his rod and ranges the rod at D_1 exactly in line with his rod and the rod at B. The rod at C is then moved into line with D_1 and A, when it will occupy the position C_1. D_1 is then moved to D_2, in line with C_1 and B, and so on, until, finally, D is in line with C and B, and at the same time C is in line with A and D. The four rods will now be in the same straight line, and this state of affairs will be reached
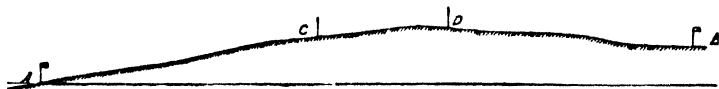
Fig. 8.
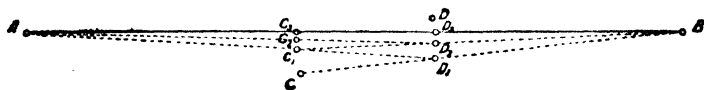
Fig. 9.

when the assistants are no longer able to alter each other's position by sighting along their respective lines. If required, a rod may be ranged in between C and D in the usual way, and the rods at these points removed.

Field Glasses.—A good pair of field glasses should always be considered as part of the equipment of a surveyor, as they are often of great service, and are especially useful in ranging work on long lines.

Signals.—The surveyor should always arrange a code of signals with his assistants before commencing a survey.

On short lines the required instructions may be signalled by motions of the arms, shouting instructions should always be avoided. If it is necessary for your assistant to move his rod towards your right, signal him to this effect by putting up your right arm horizontally, if he must move to the left put up the

left arm, and when his rod is in its correct position give both arms a downward motion in front of the body. Signalling with the arms alone is not feasible at greater distances than about 800 feet, under ordinary conditions. In such cases flag signalling should be resorted to. For simple right and left instructions it is sufficient to have two spare rods provided with flags of different colours. On signalling with (say) a yellow flag the assistant moves his rod to the right, if a red flag is exhibited he moves to the left, while if both flags are shown at the same time his rod is in its correct position.

Chaining a Line.—In all chaining operations two assistants are necessary. The duty of the man at the forward end of the chain—who is known as the leader—is to drag the chain forward and to obey the instructions of the second assistant—known as the follower—at the rear end of the chain. In selecting the men for their respective duties, always choose the man who appears to be the most intelligent of the two to act as the follower, since the accuracy with which the chain measurements are made will depend to a great extent on his care and judgment.

In commencing to chain a line, the follower places the rear handle of the chain on the ground in contact with the rod from which the measurement is to commence, then standing with his back to the rod—which he should feel between his shoulders —he places both heels firmly on the handle of the chain. Meanwhile the leader—who must be provided with all the arrows— takes the other handle and stretches the chain out, in the direction of the forward station. The leader, crouching down to the ground, takes an arrow in his right hand, holds it in contact with the outside of the chain handle, the two being held at arm's length so that the arrow is vertical, faces the follower ready to carry out his instructions. The follower now sights over the arrow to the forward rod, and signals the leader to move the arrow to the right or left as required, until it is accurately in line, when—after noting that the chain lies straight and tight —he signals the leader to mark the end of the chain. The leader does this by driving the arrow upright into the ground, or, if on a hard surface, by scratching a cross (or marking with chalk) on the ground at the end of the chain and laying an arrow beside it. The leader now drags the chain forward another chain's length, while the follower places the rear handle over the arrow, or to the scratched mark on the ground, and directs the chain into line by sighting over leader's arrow as before. When in

its correct position the leader marks the end of the chain, the follower releases his end of the chain, picks up the arrow at his feet, while the leader again drags the chain forward.

Thus, for each chain length measured, an arrow passes from the leader's into the follower's possession, and the surveyor can at any moment obtain the length which has been measured by counting the arrows in the hands of the follower. The measurement of the line proceeds in the manner indicated until the leader has placed his last arrow (the tenth) in the ground, when he should call out *ten*, and he must then replace the arrow by a brass arrow or by a twig. The follower then passes the ten arrows back to the leader; the surveyor records the fact that ten chains have been measured, and the measurement proceeds as before; the brass arrow, twig, or other convenient token in the hands of the follower reminds him that the tenth chain has been passed. On arriving at the end of the line, the leader drags his end of the chain past the forward rod and, when in line, the odd links are counted on the chain and added to the length already recorded.

Before entering the length of the line in the field book, the number of arrows that the leader has should be checked against the number in the possession of the follower as a safeguard against the loss of arrows and the consequent chaining error. The sum of the arrows in the possession of the two men should, of course, at any moment be ten.

Errors in chaining are largely due to (*a*) faulty alignment on the part of the follower; (*b*) the lack of uniformity in the tension of the chain; and (*c*) the stretching of the chain due to continued use. The first may be reduced when measuring long unbroken lines, by ranging in a few laths or whites before chaining; the second by taking care to observe that the chain lies straight and tight before allowing the leader to move forward; and the third by testing the chain on the standard before commencing each day's work.

It is evident that the error caused by faulty alignment will make the line appear longer than its real length, and in a similar way if the chain is too long, the measured length will be shorter than the real length. The error caused by the variation in tension tends to be of one sign, and to cause the measured length to be greater than the real length.

From some 6,000 measurements of a line by six different methods, Professor F. Lorber, of the Leoben School of Mines, deduced the results given in the following table :—

Method.	Probable Mean Error.
Two rods along a stretched cord, . . .	$0\cdot000535 \ \sqrt{L}$
Two rods without cord, . . .	$0\cdot000927 \ \sqrt{L}$
Chain, ...	$0\cdot003000 \ \sqrt{L}$
Steel band, ...	$0\cdot002160 \ \sqrt{L}$
Field compasses, ...	$0\cdot002120 \ \sqrt{L}$
Measuring wheel, ...	$0\cdot003600 \ \sqrt{L}$

The quantity L being the measured length.

Defects in the measuring instrument and errors in alignment cause an error which is directly proportional to the length measured, and, according to Prof. Lorber, these give rise to the following errors :—

Method.	Error.
Two rods without cord, ...	$- 0\cdot00008 \ L$
Chain, ...	$+ 0\cdot00046 \ L$
Steel band, ...	$- 0\cdot00032 \ L$
Field compasses, ...	$- 0\cdot00079 \ L$

In the chain alone is the error positive—that is to say, the measured length is greater than the real length.

Chaining on Inclined Ground.—Since a plan is the horizontal projection of the ground it represents, all measurements of distance must be made horizontally, or be reduced to the horizontal. Thus, if Fig. 10 represents the section of the ground between two stations A and B, the distance required is not the distance A B, but its horizontal projection A C. Obviously A C is always less than A B, and it is, therefore, necessary when chaining a line on sloping ground to make an allowance for the slope. This may be done in two ways, either by the process known as " stepping " or by calculating A C from the known length of A B and its inclination.

In the first method the allowance is made automatically in the following way as the measurement proceeds :—On reaching an ascending gradient the follower elevates his end of the chain until it is stretched horizontally and the rear handle is vertically over the arrow left in the ground by the leader. At the same

time the follower directs the leader into line with the forward
rod and the leader marks the position of the end of the chain
in the usual manner. This process is repeated for each chain
until the top of the rise is reached.

The chief difficulty in using this method is to ensure that the
rear handle is held vertically over the marked point on the
ground. This is best done by aid of a plummet, but failing this
useful instrument, the follower should support his handle of the
chain by grasping it against a spare rod, the foot of the rod being
placed at the mark on the ground and the rod sighted vertical
by the surveyor, who stands for this purpose at some distance
to one side of the rod and chain.

In chaining down-hill, the follower stands on the rear handle
of the chain and directs the leader into line. The leader lifts up
the chain until it is horizontal, then either by using a plummet,
a spare rod, or a drop arrow he finds the point on the ground
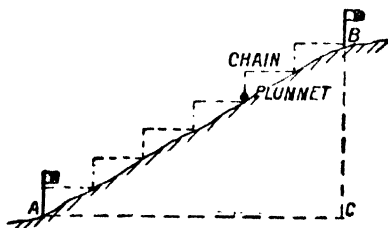
Fig. 10.

which is vertically under the end of the chain. This point found,
he marks it with an arrow, drags forward the chain for the next
length, and repeats the process as often as necessary.

If the ground is steep, the steps must be made in half or quarter-
chain lengths, and great care is necessary to see that the follower
only picks up the arrows at the end of each chain.

The steel tape is much handier than the chain for " stepping
up or down inclines as it is much lighter in weight.

Drop Arrow.—The drop arrow, already referred to, consists
of an arrow which is loaded near its point with a boss of lead.
The arrow is held on the end of the leader's thumb in contact
with the handle of the chain, and when the latter is in its correct
position the arrow is allowed to slip off the thumb. In falling
the lead boss keeps the arrow upright, and the point of the arrow
penetrates the ground at the point vertically under the end of

the chain. The plummet is, however, by far the most accurate instrument for this purpose.

Allowance for Slope.—Of the two methods of making allowance for the slope on inclined ground, the method of stepping is to be preferred, if the ground is undulating and the gradients short, but when the gradients are long and regular, the method of making the required allowance by means of a calculation based on the inclination of the ground gives the best result.

In the second method, the inclination of the slope is first measured with a clinometer; from this we may either calculate the allowance which must be made for each chain measured on the slope, or the horizontal projection of the whole line. In the former case, as each chain length is measured on the slope, the arrow is set forward by an amount equal to the calculated correction, and thus the slope is allowed for as the work proceeds. This should always be done on long lines from which offsets
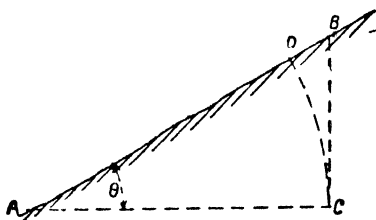
Fig. 11.

are taken. In the latter case the whole line is chained on the slope and afterwards reduced to the horizontal.

To determine the amount the arrow must be set forward, proceed as follows :—

Referring to Fig. 11, let θ be the inclination of the ground, A C the horizontal projection of A B, and let the length of A C equal 1 chain.

With A as centre, A C as radius, strike an arc cutting A B in the point D. Then the error due to chaining A B on the slope is D B,

and $\qquad$ D B = A B — A D

$\qquad\qquad$ = A B — A C,

but $\qquad$ A B = A C sec θ,

∴ $\qquad$ D B = A C sec θ — A C

$\qquad\qquad$ = A C (sec θ — 1).

Consequently the correction in links per chain = 100 (sec $\theta - 1$), and this is the amount that the arrow must be set forward at each chain.

For example, let $\theta = 10°$, then the arrow must be set forward by an amount

$$= 100 \ (\sec 10° - 1)$$
$$= 100 \ (1{\cdot}0154 - 1)$$
$$= 1{\cdot}54 \text{ links.}$$

The amount which must be deducted from each chain measured on the slope is calculated as follows :—

Let A B (Fig. 11) equal one chain.

Now $\qquad\qquad \dfrac{\text{A C}}{\text{A B}} = \cos \theta \text{ ;}$

∴   A C = A B $\cos \theta$.

As before, the correction to be applied is equal to D B, and

$$\text{D B} = \text{A B} - \text{A C}$$
$$= \text{A B} \ (1 - \cos \theta).$$

Therefore, the correction in links per chain = 100 $(1 - \cos \theta)$.

When $\qquad\qquad \theta = 10°$, the required correction
is $\qquad\qquad\quad = 100 \ (1 - \cos 10°)$
$$= 100 \ (1 - {\cdot}9848)$$
$$= 1{\cdot}52 \text{ links.}$$

To use this formula in the field, it is necessary to be always provided with a table of natural cosines, but this may be avoided by changing the formula into the approximate form—

$$\text{C} = k \ \theta^a, \text{ where } k \text{ and } a \text{ are constants.}$$

When $\qquad\qquad \theta = 10°$, C = $1{\cdot}52$ links,

and when $\qquad\quad \theta = \ 5°$, C = $0{\cdot}38$ links,

∴ $\qquad\qquad 1{\cdot}52 \ = k \ 10^a,$.   .   .   .   .   (1)

and $\qquad\qquad {\cdot}38 \ = k \ 5^a.$.   .   .   .   .   (2)

Dividing equation (1) by equation (2) we get

$$\frac{1{\cdot}52}{{\cdot}38} = \frac{10^a}{5^a} \cdot$$

$$\therefore \quad \log 1{\cdot}52 - \log {\cdot}38 = a \, (\log 10 - \log 5),$$

or

$$a = \frac{\log 1{\cdot}52 - \log {\cdot}38}{\log 10 - \log 5}$$

$$= \frac{0{\cdot}1818 - \bar{1}{\cdot}5798}{1 - {\cdot}6990}$$

$$= \frac{{\cdot}602}{{\cdot}301} = 2.$$

Substituting this value of a in equation (1), we have :—

$$1{\cdot}52 = k \times 10^2$$

$$\therefore \quad k = \frac{1{\cdot}52}{100},$$

or

$$k = {\cdot}0152.$$

$$\therefore \quad C = {\cdot}0152 \, \theta^2 \text{ links per chain.}$$

The values of k and a are, however, not quite constant for all values of θ, but if we make k equal to ·015 instead of ·0152 we shall find the results obtained by using the approximate formula to agree very closely with those obtained by using the more accurate one.

The close agreement of the results given by the two formulæ is shown in the following table :—

θ.	$C = 100 \, (1 - \cos \theta)$.	$C = {\cdot}015 \, \theta^2$.
5	0·38	0·37
10	1·52	1·50
15	3·41	3·37
20	6·03	6·00
25	9·37	9·37
30	13·40	13·50
35	18·08	18·37
40	23·40	24·00
45	29·29	30·37

When the vertical fall of the ground in links per chain is known, the correction for the slope is obtained as follows :—Let n be the vertical fall in links per chain. Referring to Fig. 11, the correction $C = B \, D$,

$$= A \, B - A \, C$$

$$= A \, B - \sqrt{A \, B^2 - B \, C^2}.$$

When A B = 1 chain, the correction in links per chain

$$= 100 - \sqrt{10,000 - n^2}.$$

When the fall is gentle the following approximate formula may be used :—

$$C = \frac{n^2}{200}.$$

The value of n is easily found by pinning down one handle of the chain, which is then stretched out horizontally. The height of the other handle above the ground gives the value of n for that particular slope.

Clinometers.—These are small hand instruments for measuring angles of elevation or depression. They do not as a rule read finer than 1°, as this degree of accuracy is fine enough for the purpose for which they are intended. Although the various forms of the instrument are constructed on different principles, yet they are all used in the same way. To read an angle of elevation or depression, the surveyor marks the height of his eye upon a spare rod, to which his assistant fixes a piece of straight stick at right angles to the rod at the point indicated by the mark. The assistant now takes the rod in the direction of the line and holds it upright. The observer stations himself in the line and sights the stick through his instrument; the angle recorded by the instrument gives the inclination of the ground.

Rule Clinometer.—The clinometer illustrated in Fig. 12 consists of two pieces of boxwood, 6 inches long, jointed together at one end like a two-foot rule. Both parts of the instrument are fitted with small spirit levels shown at C and D, the level D giving the horizontal line of reference when the instrument is being used to determine angles of elevation or depression of survey lines, and the level C gives the corresponding line of reference when the instrument is being used to determine the inclination of any sloping surface on which it is placed. The upper part of the instrument carries a pair of folding brass plates A and B, in each of which is a sight hole about $\frac{1}{16}$ inch diameter, and a hole about $\frac{1}{4}$ inch diameter, in which is fitted a piece of plate glass. On the surface of the plate glass a cross is scratched, the line of sight being that formed by joining the centre of the sight hole to the centre of the corresponding cross. The sights are placed alternate, the upper pair being used in reading an angle of elevation, and the lower pair when obtaining an angle of depression. The angles are read on the quadrant F, which

is divided in degrees from 0° to 90°. A small compass (E) is recessed into the lower part of the instrument, and can be used for the rough determination of magnetic bearings. On most instruments of this type tables (G) are engraved, which give the hypotenusal allowance in inches per yard, and angles corresponding to given gradients.

In using the instrument to determine an angle of elevation or depression, the sights are turned up into position and the sight vane on the assistant's rod is sighted through the instrument. The lower part of the instrument is then depressed until the bubble in the spirit level D is in the centre. The angle required will now be obtained by noting the reading on the quadrant.

As it is impossible to sight the vane on the assistant's rod and to see the bubble in the lower spirit level at the same time, a much better plan of using this instrument is to place it on
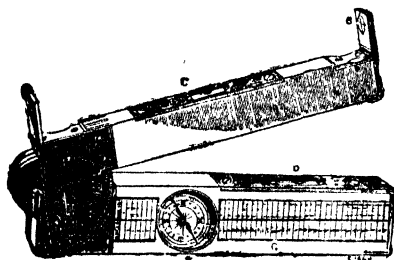
Fig. 12.

the top of a wall or post, and pack up the lower part until it is horizontal, then open the instrument and sight in the required direction. A more accurate reading will be obtained in this way than by holding the instrument in the hand.

Abney Level.—The difficulty of observing the bubble of the spirit level and the distant object at the same time has been overcome by Captain Abney in the reflecting level which bears his name.

The principle underlying the construction of this instrument is as follows :—If a ray of light impinges on a plane mirror at an angle of 45°, then the ray will be reflected at right angles to its original direction. This principle is often made use of in surveying instruments.

Abney's level consists of a square tube about 6 inches long

and $\frac{5}{8}$ inch square in cross-section. One end of the tube is pro-
vided with an eyepiece, which is perforated with a small hole—
the sight hole—and the other end is quite open. Crossing the
bore of the tube and inclined at an angle of 45° to its axis is a
small mirror, which half-closes the opening of the tube. A thin
horizontal wire is placed in the axis of the tube a short distance
behind the mirror. The wire and mirror are carried on a short
internal square tube, which may be withdrawn when the mirror
requires cleaning. Immediately above the centre of the mirror
a rectangular slot is cut in the upper surface of the tube in order
that the reflection of the bubble of a small spirit level—the lower
portion of the level case being cut away for this purpose—may
be seen in the mirror. When the spirit level is quite horizontal,
on looking through the instrument the reflection of the bubble
is seen as though bisected by the wire. The spirit level is fixed
to a short horizontal axis, to which is also attached a vernier
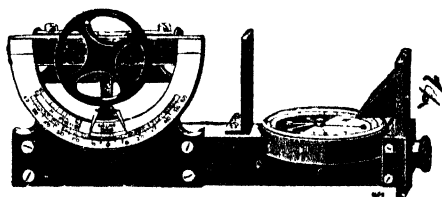
Fig. 13.

arm and a small wheel. The vernier (this is a fine reading
accessory, and is explained in detail on p. 222), on the extremity
of the arm, moves over a graduated arc engraved on the plate,
which supports the axis of the level. This plate is fixed to the
side of the tube by screws, as shown in the figure. In order to
read angles of elevation and depression, the scale reads right
and left to 60° on each side of zero, which is so placed that when
the axis of the instrument is horizontal the zeros of the vernier
and scale coincide. The vernier is also graduated right and left
hand, so that it may be used on either scale. A small compass,
provided with a pair of sights, is shown in Fig. 13 fitted to the
top of the instrument.

In using the instrument, the observer sights the distant mark,
which he covers with the cross-wire. The spirit level is then
rotated by turning the small wheel until the image of the bubble
appears to be bisected by the cross-wire, the reading at the

vernier will now give the required angle of elevation or depression.

In the more expensive types of this instrument the tube is formed into a telescope, the vernier is fixed, and the scale is given a slow motion by a rack and pinion arrangement.

Barker's Combined Clinometer and Prismatic Compass.—The clinometer portion of this instrument consists of a thin segmental brass plate carrying a circular right- and left-hand scale divided in degrees. The plate is axially supported on two fine conical centres, on which it turns with very little friction. One part of the plate is weighted, so that, when the plane of the plate is vertical, it comes to rest with the 90° mark on the scale vertically under the axis of suspension. The clinometer plate and the compass are enclosed in a cylindrical brass box about 3¼ inches
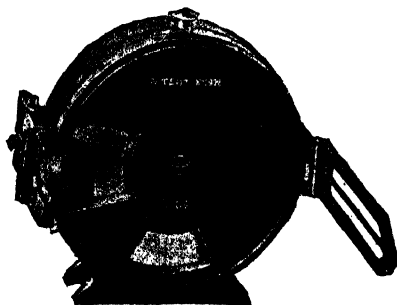
Fig. 14.

in diameter and 1 inch deep. Two glazed window openings are provided in the cover of the box, as shown in Fig. 14, through which the scales of the clinometer disc and compass ring may be seen. When about to be used as a compass, the box is turned until the opening in the clinometer disc is vertically under the upper window, the disc is then clamped in position by pressing in the clamp shown at the top of the figure. The compass ring will now be seen at a lower level in the box.

Two folding sights are attached to the box in diametrically opposite positions. One sight is a simple window opening crossed by a horse hair or a fine wire at right angles to the plane of the box. The other sight is a reflecting glass prism, the lower surface of which is formed into a lens, in order to magnify the scale divisions. The prism is enclosed in a brass case, which

may be adjusted in position for focussing the lens, by means of a slide. The prism case is perforated by a sight opening above which a narrow sighting slit is cut ; a second opening admits the rays of light from the scale, which are bent at right angles by the reflecting surface, and emerge from the prism at the circular sight opening. Thus, on looking through the prism and sighting slit, we are able to see at the same time the distant object, the sighting wire, and the scale.

In using the instrument as a clinometer, it is either held in the hand or supported on a tripod with its plane vertical. The case of the instrument carrying the sights is turned until the hair sight is in line with the distant vane. The angle of elevation (or depression) of the line of sight is now given by the apparent intersection of the hair sight and scale as seen through the slit and circular sight hole.

The instrument—mounted on a tripod—can be used for rough levelling work, since by turning the sights until the reading is zero, the line of sight will be horizontal.

Fixing the Position of a Point relative to a Line.—The position of the point P (Fig. 15) relative to station A, and the line A B may be fixed by measuring (1) A P and the angle P A B ; (2) A C and P C ; (3) A D, D P, P E, and A E.

The first method is not often used in chain surveying, but forms the basis of surveys carried out by means of telemeters. The distance A P is determined by instrumental readings on a graduated staff held at P, and the angle that A P deviates from some standard direction is also determined by the instrument. The second and third methods are the most frequently recurring operations in chain surveying, as it is by measuring distances along a line and at right angles or oblique thereto that the outlines of the surface features of the ground are fixed relative to the surveyor's system of main lines

Offsets.—The distances, such as P C (Fig. 15), which are measured at right angles to the main line A B are called *offsets*, those, such as D P and P E, which are not at right angles to the main line, are spoken of as *oblique offsets*.

When the surveyor decides to take an offset to some point, he signifies his intention to the chainmen, who leave the chain pinned down on the ground. The leader then takes the end of the tape to the point indicated by the surveyor, the follower takes the tape box and moves along the chain until he finds the point at which the tape is at right angles to it, the position of the point sought on the chain being judged by the eye only.

After a little practice the position of the foot of the perpendicular on the chain can readily be judged by the eye to within half a link of its true position. Having found this point, the follower calls out the length of the offset and the corresponding distance
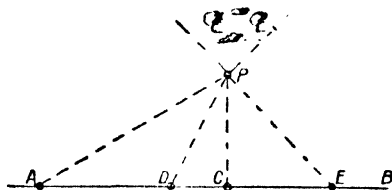
Fig. 15.

on the chain, which—after checking—the surveyor enters in his field notes.

If the offset to be measured is fairly short, the distance may be determined by the offset rod ; long offsets (above 20 links) are more quickly and accurately determined by means of the tape, but in no case should offsets of a greater length than two chains be measured with this instrument. Should an offset of this length be necessary, the direction of the offset line should be set out accurately at right angles to the chain.

Shortening Offsets.—Long offsets should in all cases be avoided. They may be largely obviated by judiciously placing the main lines of the survey, or by making use of subsidiary figures. For example, in the case of a deep bend in the outline of a fence, as shown in Fig. 16, a subsidiary triangle *a b c* should be formed
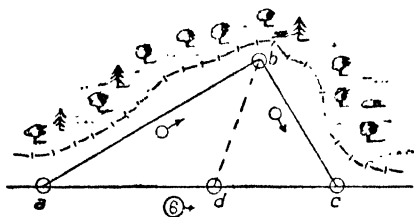
Fig. 16.

having its base on the main line, the offsets to the fence being taken from the sides *a b* and *b c*, as indicated in the figure. The proof line *b d* should also be measured to check the accuracy of the work.

Oblique offsets are only used when the point to be fixed is an important one, such as the corner of a building or the end of a straight fence. The method is illustrated by Fig. 17. In the case of the fence $c\,m$, it may happen that only one other point on it may be obtainable, hence it is necessary to fix the two points on the fence with great care. The oblique offsets $a\,c$ and $c\,b$ are measured, and the point d, at which the prolongation of the fence $m\,c$ cuts the main line is found and recorded with the other measurements in the field notes. The same mode of procedure is adopted in the case of the building, also shown in the figure, the point of intersection of the two wall faces at g and l with the main line at h and j being found and noted. It is desirable that the lines $c\,d$, $g\,h$, and $l\,j$ should be measured in order to fully check the other measurements, for it must be clearly borne in mind that, although it is sufficient to measure the three sides of a triangle in order to plot it, the lengths of the three sides
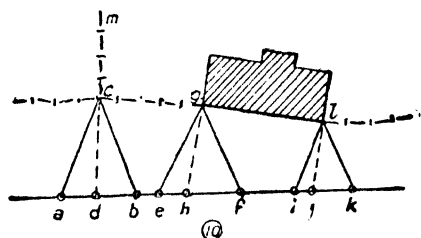
Fig 17.

alone will not give us sufficient data to prove the figure. Hence it is necessary to measure a fourth quantity, and in chain surveying this is done by measuring a fourth line.

Setting out Right Angles.—For many purposes the surveyor finds it necessary to set out a line at right angles to another, or to find the foot of the perpendicular from a point on to a given line. In either case the problem may be effected by aid of the chain and tape, or by means of a simple instrument designed for this purpose.

We will consider the chain and tape methods first.

Problem I.—To set out a right angle at a point A in a given line A B.

First Method.—By Euclid I. 47, we know that in any right-angled triangle A B C, right-angled at C,

$$A\,B^2 = A\,C^2 + B\,C^2,$$

hence, in order to set out the required right angle, it is only necessary to select three lengths which satisfy the above equation, and to use these lengths as the sides of a triangle, having the right angle at the given point. The most easily remembered numbers which will answer our purpose are 3, 4, and 5, and convenient multiples of these numbers are 30, 40, and 50, or 60, 80, and 100.

To set out the right angle, stretch the chain from A (to the right or left as most convenient) in the given line, and mark the position of the 40th link (C, Fig. 18) by an arrow. Now pin down the handle of the chain at C and the 80th link (50 + 30) at A, take hold of the 50 tally and pull the two parts of the chain out tight. The 50 tally will now occupy the position of the point D in Fig. 18, and A D will be at right angles to the line
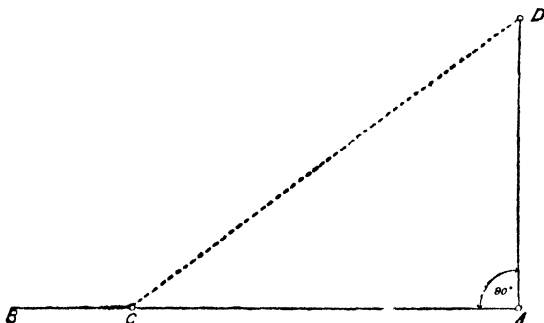
Fig. 18.

B A. The line A D may be prolonged as far as required by ranging a rod in line with rods placed at the points A and D.

Second Method.—Another simple method is illustrated in Fig. 19. In this case stretch the chain in the given line B C, with the 50 tally at A, and mark the points E and D with arrows such that A E = A D (say 60 links). Pin down the handle of the chain at E and the end of the tape at D ; pull out both chain and tape in the direction of the point F, and bring the handle of the chain to meet the corresponding length on the tape. When both chain and tape lie straight and horizontal, their point of intersection will give the position of the point F. The geometry of the construction is obvious.

Problem II.—To drop a perpendicular from a point on to a given line.

This is the converse of the preceding problem, as in this case the point F is fixed, and we wish to find the point A.

First Method.—Lay the chain on the ground in the given line, so that the centre of the chain is roughly in the position of the point A (Fig. 19). Pin down the tape at the point F, and pull it out tight, so as to cut the chain obliquely, at the point E. Note the position of the point E. Without altering

Fig. 19.

the length on the tape, swing it round the pin at F, until it again cuts the chain at some point D, whose position must be noted. The point midway between E and D is the position of the desired point. For example, if the reading on the chain at E is 20 links and at D 84 links, then A occurs at (20 + 84)/2 = 52 links from the left-hand end of the chain.

Second Method.—Stretch the chain in line as before, and pin down the end of the tape at C (Fig. 20) near the left-hand end

Fig. 20.

of the chain. Measure the length C F, and make C D equal to C F by swinging the tape round the arrow at C ; measure also the distance F D. Then the point A is found by laying off a length D A from D, such that $D A = \dfrac{F D^2}{2 C D}$.

The proof of this is as follows :—With C as centre, radius C F, describe the semicircle D F B. Join F A and F D.

Since F A is perpendicular to B D,

$$B A . A D = A F^2 \text{ (Euc. III. 35).}$$

$$(2 C D - D A) A D = A F^2$$

$$\therefore D A = \frac{A F^2 + A D^2}{2 C D}$$

$$= \frac{F D^2}{2 C D}.$$

Instruments for Setting-out Right Angles.

Cross Staff.—This instrument, in its simplest form, consists of a block of wood in which two fine saw cuts, about $1\frac{1}{2}$ inches deep, are made accurately at right angles to each other in the upper part of the block. A hole is bored in the lower part of the block, so that it may be placed on the top of a staff, which is about 4 feet long.

In setting out a right angle at a given point with this instrument, the staff is set up at the point, and the block is turned until, on sighting through one of the saw cuts, the forward rod is bisected by the line of sight. A second rod may now be set in a line at right angles to the first by sighting through the second saw cut.

The foot of the perpendicular from a point to a line is readily found by setting up the staff in the given line, in the neighbourhood of the required point, and sighting the forward rod ; on looking through the second saw cut it will be at once seen whether the instrument requires to be moved to the right or left in order to place it at the desired point. A few trials made in this way will enable the observer to determine accurately the position of the point sought.

In its more modern form the instrument is constructed as a thin brass box (see Fig. 21), either octagonal or circular in cross-section. The sights consist of a narrow vertical slit, and a vertical window bisected by a vertical horse-hair placed opposite each other. The sights are arranged in pairs for left and right-hand sighting. The base of the instrument is provided with a socket, so that it may be placed on a staff when in use.

, The form of cross staff shown in Fig. 22 is fitted with a compass, and the upper portion of the instrument is divided from, and may be moved relative to, the lower portion, the motion being governed by a circular rack and pinion. The angle turned through by the upper part of the staff may be read on a divided scale, a vernier being fitted as shown in the figure. Angles read with this instrument, however, can only be considered approximate, as the sights on the instrument are too crude to give an accurate result. This remark applies to all forms of cross staff, but so long as the use of this type of instrument is restricted to short lines it is a useful addition to the surveyor's field equipment.
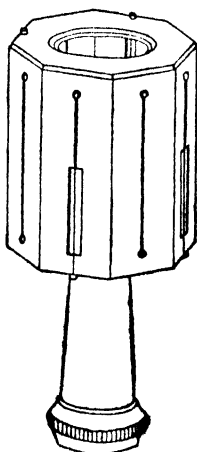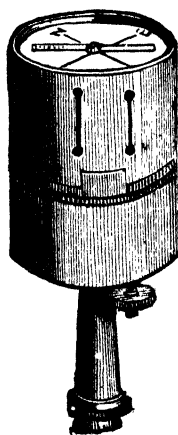
Fig. 21.   Fig. 22.

The Optical Square.—There are several forms of this accurate little instrument, but the form in most general use is that shown in Fig. 23.

It is constructed in the form of a circular brass box from 2 to $2\frac{1}{2}$ inches in diameter, made of two closed tubes closely fitting the one within the other. Both tubes are perforated on their curved surfaces by (1) a small sight hole about $\frac{1}{16}$ inch diameter; (2) a small rectangular slot or window placed diametrically opposite to the sight hole; (3) a large window placed at right angles to the line joining the sight hole and small window; and (4) a small circular hole, through which a key may be passed in order to adjust the working parts of the instrument. When

the instrument is not in use a slight rotation of the outer tube relative to the inner one closes all the apertures and excludes dust. Within the box, and fixed to the flat end of one of the tubes, are two mirrors. One of these mirrors—the half-mirror
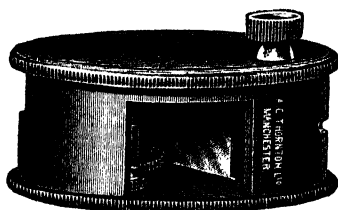
Fig. 23.

(A, Fig. 24)—cuts the line joining the centre of the sight hole and small window at an angle of about 120°; the lower half of this mirror is silvered, the upper half being of plain glass, so that an object in the direct line of sight may be seen through it. The second mirror B is placed so that its plane is inclined at an angle of 45° with the plane of the mirror A. The half-mirror is fixed in position, but the second mirror may have its position adjusted by applying the key K to the screw, shown at P.

The principle underlying the construction of the instrument
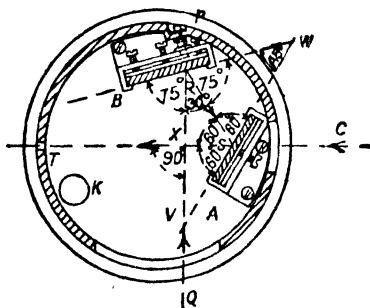
Fig. 24.

is as follows :—If a ray of light undergoes two successive reflections in a plane at right angles to each of two plane mirrors, the angle between the first incident ray and the last reflected ray is double the angle between the mirrors.

The first incident ray (Q R, Fig. 24) strikes the mirror **B at R**, and is reflected to S in the second mirror. From this point it is thrown off in the direction S T, and enters the eye at the same time as the direct ray C T. By a well-known law in optics, the angle B R V is equal to the angle S R W, and the angle R S W is equal to the angle T S V. Also the

$$\angle \text{ B R V} + \angle \text{ V R S} = \text{the exterior angle of the}$$
$$\text{triangle R S W}$$

$$= \angle \text{ R S W} + 45° \text{ (Euc. I. 32).}$$

Similarly $\angle$ T S V + $\angle$ T S R = $\angle$ S R W + 45°, by addition we get,

$$\angle \text{ B R V} + \angle \text{ V R S} + \angle \text{ T S V} + \angle \text{ T S R} = \angle \text{ R S W}$$
$$+ \angle \text{ S R W} + 90°.$$

But $\angle$ B R V = $\angle$ S R W, and $\angle$ R S W = $\angle$ T S V,

∴ $\angle$ V R S + $\angle$ T S R = 90°

$$= \text{sum of the interior angles}$$
$$\text{X R S and X S R of the}$$
$$\text{triangle X R S.}$$

Therefore, the third angle R X S = 90°

$$= \angle \dot{\text{Q}} \text{ X T (Euc. I. 15),}$$

or when the angle between the mirrors is 45°, the angle between the first incident ray (Q R) and the last reflected one (S T) is 90°.

In using the optical square, the operator holds the instrument in his hand, vertically over the point at which the right angle is to be set out, and sights the forward rod. An assistant, provided with a second rod—having placed himself approximately in the direction of the required perpendicular—will be seen by reflection in the two mirrors. The operator now signals his assistant to move to the right or left as required until his rod as seen by reflection appears in line with the forward rod seen directly. The appearance of the two rods when this occurs, as seen in the half-mirror, is shown in Fig. 25.

In order to establish the position of the point on the chain line, which is the foot of the perpendicular, from some object on the right or left, all that is necessary is to sight the forward rod through the instrument and walk in the line—forward or backward as the case may be—until the forward rod is seen exactly in line with the image of the given object. The point sought is then vertically under the instrument.

It is a good plan when using this instrument to place a second forward rod in the given line, as by this means the observer can easily range himself accurately in the line.

As this instrument marks out a right angle in its own plane, care must be taken to hold the instrument quite horizontal. If the right angle has to be set out on side-long ground, a rod should be placed in the chain line 50 or 100 feet from the observer, and the perpendicular set out should be a short one, so that the three points marked by the forward rod, the position of the instrument, and the rod on the perpendicular are approximately in the same horizontal plane. The perpendicular may then be prolonged by ranging in the usual way.

Testing Optical Square.—To test the accuracy of the optical square, the instrument is taken into the field and is placed on the flat top of a stake about 5 feet long and 2 inches square,

PLAIN
GLASS

SILVERED
GLASS
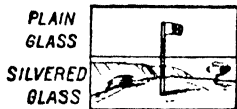
Fig. 25.
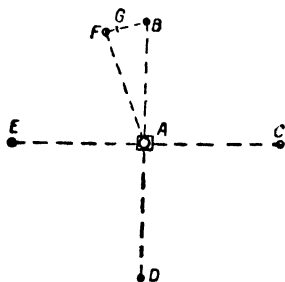
Fig. 26.

which has been driven firmly and vertically into the ground. The ground selected should be fairly level and free from obstacles. The position of the instrument is shown at A in Fig. 26. A rod B is set up about 300 feet from A, and by aid of the instrument the (presumed) right angle B A C is set out, the point C being marked with a rod. The instrument is then turned through a right angle, C being sighted as the forward station, and C A D is set out. Similarly, the right angle D A E is marked out. Lastly, if the instrument is in adjustment and the work has been correctly carried out, on sighting E as the forward station, the rod at B will appear to coincide with the rod at E. If the instrument is in error or the work has been carelessly done, the rod at B will appear out of its true position, in which case set in a rod at F, so placed that the angle F A E—according to the

instrument—is a right angle. Measure the distance F B, and
shift the rod at F to G, F G being one-quarter of F B. Now
apply the key K (Fig. 24) to the adjusting screw P, and move
the mirror B until the rods at E and G appear to coincide when
viewed through the instrument. If the work has been accurately
carried out, the instrument should now be in perfect adjustment.
As a check on the result, the four right angles should be again
set out and the error (if any) corrected as before.

Chaining Past Obstacles.—It occasionally happens that a main
line on a survey passes over, or through, some object which
prevents the direct measurement of that part of the line which
the object intersects. The interfering object, in a case of this
kind, is spoken of as an *obstacle*

The method of dealing with an obstacle depends both on its
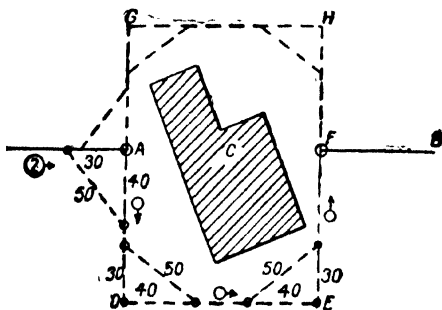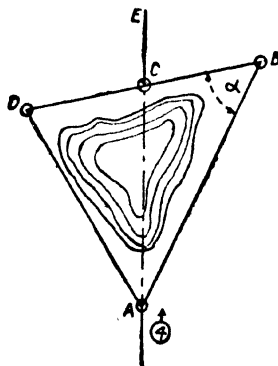
Fig. 27.   Fig. 28.

form and the nature of the surrounding objects. A method
which would apply in one case may be quite inapplicable in
another; in all cases, however, the greatest care is needed in
taking the necessary measurements and in setting out the re-
quired figures, in order to obtain an accurate result.

In passing an obstacle, it is usual to throw around (or across)
it a geometrical figure, from which the required distance may be
obtained either by direct measurement or by calculation. This
may be best illustrated by assuming a series of cases.

Case I.—When the obstacle can be seen over and chained
round—*i.e.*, a low building or a pond.

In this case the main line may be continued beyond the ob-
stacle by ranging in the usual way.

First Method.—Referring to Fig. 27, A B is the main line, and C the obstacle to be passed. At A set out the perpendicular A D, and prolong it (if necessary) until a second line D E, perpendicular to A D, runs clear of the obstacle. Measure the length of A D, and at E set out the right angle D E F, making E F equal to A D. Then, as may be seen from the figure, the point F should be in the line A B, and the accuracy of the work is checked by observing if the point F is accurately in the line. The break in the chainage A F may be filled in by measuring the equal line D E, since the figure A D E F is a rectangle.

If the line A B is an important one, a second figure A G H F should be set out, by means of which the length of D E may be checked. The outlines of the obstacle can be readily determined by taking offsets from the sides of the figures.

Second Method.—In this case (Fig. 28) a triangle A B D is thrown around the obstacle, the point of intersection C of the main line A E and the line B D being found by ranging. The sides of the triangle and the distances B C and C D are measured on the ground, from which the distance A C may be obtained either by plotting or by calculation.

To calculate the distance A C, we proceed as follows :—By reference to Fig. 28, we see that

$$\text{Cos } \alpha = \frac{A B^2 + B D^2 - A D^2}{2 A B . B D},$$

also,
$$\text{Cos } \alpha = \frac{A B^2 + B C^2 - A C^2}{2 A B . B C},$$

∴
$$\frac{A B^2 + B D^2 - A D^2}{2 A B . B D} = \frac{A B^2 + B C^2 - A C^2}{2 A B . B C},$$

and
$$\frac{A C^2}{B C} = \frac{A B^2 + B C^2}{B C} - \frac{A B^2 + B D^2 - A D^2}{B D},$$

Solving for A C, we get

$$A C = \sqrt{\frac{A B^2 . D C + A D^2 . B C}{B D} - B C . D C}.$$

This formula is of considerable service, inasmuch as it gives the length of the proof line, drawn from one angular point to cut the opposite side of any triangle.

Offsets to the outline of the obstacle from the sides of the triangle A B D will determine its form.

Case II.—The obstacle cannot be seen over but can be chained around—*i.e.*, a tall building or a thick wood.

In this case the main line A B (Fig. 29) cannot be continued past the obstacle by ranging in the usual way ; it is necessary to determine two points such as I and H on A B produced, in order to continue the line.

First Method.—Select two points C and B on the line A B about two chains apart, and at these points set out equal perpendiculars C E and B D. Prolong these lines until the line D E joining their extremities will pass the obstacle. On the line D E produced, select two points F and G, and at these points set out the perpendiculars F I and G H, which must be made equal in length to B D and C E. The points I and H— as may be seen from the figure—are in A B produced, and the line I H may be prolonged as far as required. The break in the chainage C I is equal to E F.
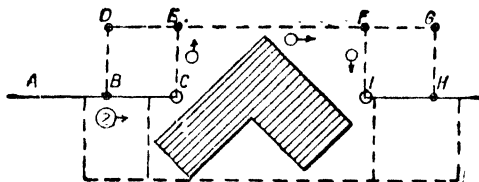
Fig. 29.

A second figure should be placed around the obstacle, in order to check the positions of the points I and H, and the measured distance E F.

Second Method.—Near the extremity of the line A B mark two points B and C (Fig. 30), 50 links apart. Pin down one handle of the chain at B and the other at C ; take hold of the 50 tally and pull the chain out tight. The two parts of the chain will now form the sides of an equilateral triangle having its vertex at D, the point occupied by the 50 tally. Set up rods at the points B and D, and prolong the line to the point F, the point F being so chosen that the side F J of the equilateral triangle B F J will clear the obstacle. Chain B F, and on E F as base set out the equilateral triangle E F G. Prolong F G to J, making F J equal in length to B F, and on H J as base set out the equilateral triangle H I J. If the work has been accurately carried out, the line joining the points I and J will be in

A B produced, and the break in the chainage B J will be equal to B F or F J.

As in the other methods, a check figure should be set out, otherwise we may have no guarantee that the work is correct.
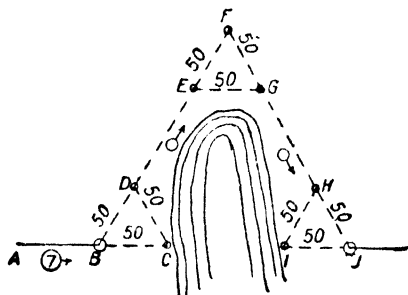
Fig. 30.

Case III.—When the obstacle can be seen over but not chained around—*i.e.*, a river or a long arm of the sea.

First Method.—Range a rod C (Fig. 31) on the far bank of the river in line with the rods A and B on the main line. At
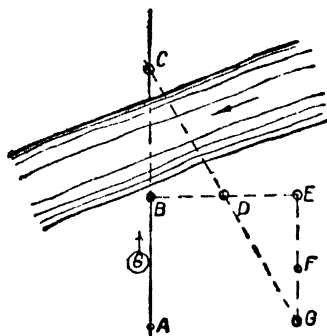
Fig. 31.

any convenient point B on the line, set out a perpendicular B E and mark its middle point D. At E set out the perpendicular E F. Now take a spare rod and range it exactly in line with F and E, and D and C, thus obtaining the point G, then, as

may be seen by inspection, the triangles B C D and D E G are similar.

$$\therefore \qquad \frac{B\,C}{B\,D} = \frac{E\,G}{D\,E} \text{ (Euc. VI. 4),}$$

but B D is equal to D E,

$$\therefore \qquad\qquad B\,C = E\,G.$$

Chain the line E G and record the measurement as the equivalent of B C, the break in the chainage.

Second Method.—Range the point H on the far bank in the
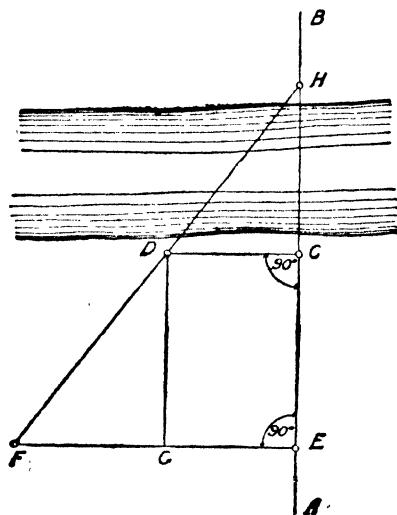
Fig. 32.

given line A B (Fig. 32). At any convenient points E and C in the main line set out the perpendiculars C D and E F. Set up a rod at F and range a rod D on the perpendicular C D exactly in line with H and F. Chain the lengths C D, C E, and E F, then the required distance

$$C\,H = \frac{C\,E \,.\, C\,D}{E\,F - C\,D}.$$

We may prove this equation in the following way:—Draw through D a perpendicular to the line E F, meeting it in the

point G. Then, as may be seen by inspection, the triangles H C D and D G F are similar.

Hence,
$$\frac{C H}{C D} = \frac{D G}{G F} \text{ (Euc. VI. 4),}$$

or
$$C H = \frac{C D . D G}{G F},$$

but D G is equal to E C, since the figure E G D C is a rectangle, and G F = E F — C D for the same reason, ∴ C H = $\dfrac{C E . C D}{E F - C D}$.

Third Method.—As before, range the point E on the far bank
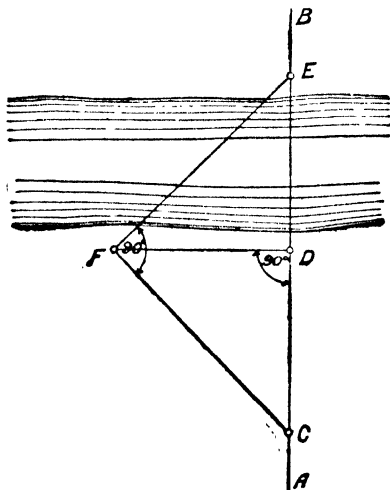
Fig. 33.

in line with A and B. At any convenient point D in the main line set out the perpendicular D F. Make the length of D F (Fig. 33) any convenient number of whole chains. With the cross staff, or optical square, set out the right angle E F C, and find by ranging the lines F C and A B, their point of intersection C. Then, since the angles C D F and E F C are right angles, we have
$$C D . D E = D F^2 \text{ (Euc. III. 35),}$$

or
$$D E = \frac{D F^2}{D C}.$$

From this equation, the break in the chainage D E is easily found after the distance D C has been chained.

Magnetic Bearings.—In making a survey, it is necessary to obtain either the magnetic bearing of any main line, or the deviation of a main line from the true meridian, in order to show the cardinal directions on the plan of the ground surveyed. In the former case, it is also necessary to know the declination of the compass needle (*i.e.*, the angle between the magnetic and true meridians) for the year and place in which the survey is made.

The method of finding the direction of the true or geographic meridian at a station is dealt with in Chap. XVII., Part II.
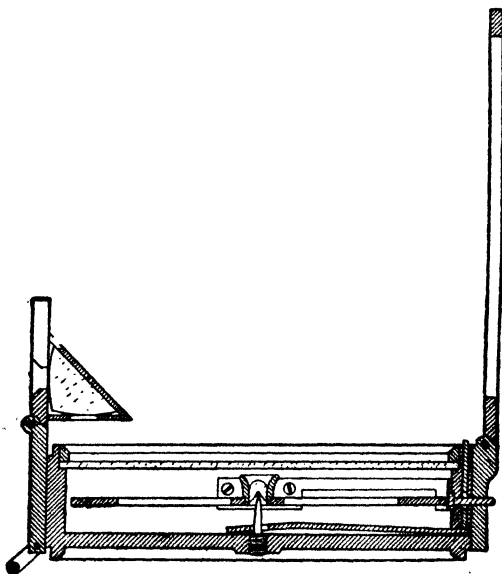
Fig. 34.

Magnetic bearings are also required in that form of surveying, which is known as compass traversing, a method which is largely used in filling in the detail of surface surveys, and also in mine surveys.

Prismatic Compass.—The simplest form of instrument for the purpose of taking magnetic bearings is the prismatic compass. This instrument consists of a light magnetic needle carrying either a thin graduated cardboard disc or a light graduated metal

ring. The centre of the graduations on the ring (Fig. 34) coincides with the centre of the magnetic needle, at which is placed a brass cell having an agate cap. This cap is supported on the point of a hard steel needle fixed to the centre of the base of the compass box. The magnetic needle thus supported can turn on its centre with a very small amount of friction.

The compass box is a circular brass box about 3 inches in diameter, having a plane glass lid, or a metal lid provided with a glazed window, through which the graduations on the ring may be seen. The sights consist of a totally reflecting prism with a sighting slit, and a hair sight fixed at diametrically opposite points on the sides of the box. These sights may be folded close to the box when the instrument is not in use. The hair sight,
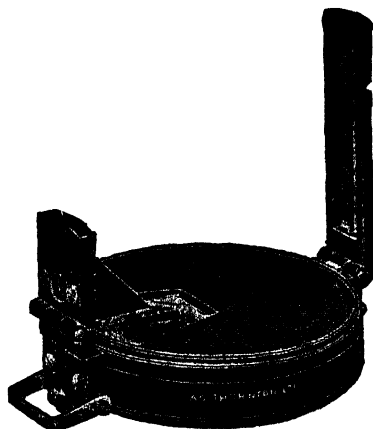
Fig. 35.

when folded down, depresses a trigger, which, acting on a bent lever, raises the needle off its centre and holds it firmly against the glass lid of the box, thus preventing undue wear on the point of the supporting pivot. In order to still the more violent vibrations of the needle when about to take a reading, a damping spring may be brought into contact with the edge of the compass ring by depressing the stud shown at the base of the hair sight.

The ring is generally divided to read degrees and half-degrees, the graduations have the zero at the south end of the compass needle, and are numbered clockwise. The general appearance of the instrument is shown in Fig. 35.

The beginner, in using this instrument, has always some difficulty in deciding the cardinal direction in which a line is proceeding. This is due, on the one hand, to overlooking the fact that the sights rotate round the needle, the scale remaining stationary, and on the other, to the absence of any direct indication of the cardinal direction on the graduated ring. A sketch will, however, enable him to easily overcome this difficulty. Thus, if the line is going in the direction A B (Fig. 36), the bearing as given by the instrument will be the acute angle O F A, and the line is going N.E. The bearing of C D is the angle O F C, and the line is going S.E. Similarly, the bearings of the lines B A and D C—read clockwise—are given on the instrument by the angles O F B and O F D, and the lines are going S.W. and N.W. respectively.
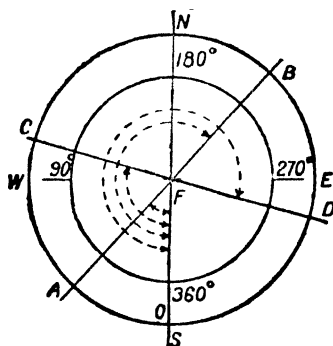
Fig. 36.

The cardinal directions of lines in the four quadrants are :

When the bearing lies between	The line is going
0° and 90°	N.E.
90° and 180°	S.E.
180° and 270°	S.W.
270° and 360°	N.W.

Reduced Bearings.—The actual deviation of the line from the magnetic meridian—*i.e.*, the acute angle the line makes with the magnetic meridian—is called the *reduced bearing* of the line.

The method of obtaining the reduced bearings of lines in the four quadrants is shown in the following table :—

When the bearing lies between	The reduced bearing is equal to
0° and 90°	The given bearing.
90° and 180°	180° — given bearing.
180° and 270°	Given bearing — 180°.
270° and 360°	360° — given bearing.

For example, the reduced bearings of the lines having bearings of 75° 30′, 124° 15′, 245° 30′, and 344° 20′ are N. 75° 30′ E., S. 55° 45′ E., S. 65° 30′ W., and N. 15° 40′ W. respectively. The reduced bearing of a line is its *nautical bearing*.

Declination of the Compass.—The declination of the compass needle varies from year to year, and varies also with the place of observation. The (Westerly) declination for the week ending 26th Oct., 1935, at the Abinger Magnetic Observatory was 11° 27′ W. The declination at Manchester on 26th Oct., 1935, was 12° 36′ W. Annual decrease 10·8′ (not constant). " In the British Isles the value of the (Westerly) declination increases by 1° for about every 75 miles travelled in a W.N.W. direction, while there is approximately zero change between places on lines running from N.N.E. to S.S.W." * The average rate of diminution of the Westerly declination in Great Britain is approximately 12′ per annum, but the change is neither permanent nor regularly distributed. Hence in using the needle for the purpose of approximately determining the direction of the true meridian at any place, it is necessary to know the declination at that place.

Variation in the Declination.—The declination of the needle is also subject to slight diurnal changes, which vary with the season of the year and the time of the day. These changes are much the same over the Northern Hemisphere, though they differ in amount. The needle occupies its mean position at about 10 a.m. and 6 p.m., reaches its most easterly position at about 8 a.m., and at 1 p.m. has its greatest westerly variation. These diurnal variations are, however, limited to a few minutes of arc—about 12 minutes in summer and 7 in winter—and when using the prismatic compass they may be disregarded.

Reading Bearings.—To obtain the bearing of a line, the instrument is either held in the hand or supported on a tripod over the rear station, and the sights are turned in the direction of the forward station. When the needle comes to rest, the reading at which the hair sight appears to cut the scale gives the required

* Report on the Results of the Magnetic Observations made by the Ordnance Survey, 1927.

fore-bearing. As a check on the accuracy of the reading, the bearing of the line should be taken in the reverse or backward direction from the forward station. This second reading is spoken of as the back-bearing of the line. The two bearings should differ by 180°, a variation of ¼° may be permitted in using this instrument, but this amount should not be exceeded.

Both fore- and back-bearings should be entered in the field notes.

Disturbance of the Needle.—The divergence in the fore- and back-bearings of a line may be caused either by defects in the construction of the instrument or by the presence of some magnetic substance in the neighbourhood of one or both stations. The former may generally be detected by the sluggish action of the needle, which may be caused by (*a*) the loss of magnetism in the needle, or (*b*) by the bluntness of the supporting pivot.

Fig. 37.

The first defect is remedied by remagnetising the needle, and the second by regrinding the point of the pivot, which should always be sharp enough to scratch the thumb nail under a moderate pressure.

If the instrument is known to be in good working order, the divergence in the bearings must be due to the presence of some magnetic substance in the neighbourhood, or on the person of the observer. A bunch of keys, or a knife, in the pocket of the observer, the steel frames of spectacles or eyeglasses, the stiffening wire which is put in the brims of some makes of felt hats, a chain or arrows lying nearby on the ground, may be the cause of the trouble, or, failing these, the presence of magnetic rocks or ores in the neighbourhood may be suspected.
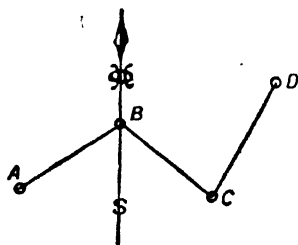
When a disturbance of this kind arises at one of the stations (B, Fig. 37), set out a line B C and determine its fore- and back-bearings. If these agree, then there is no magnetic disturbance at B, and hence the bearing of A B obtained at B is the correct one. If, however, the fore- and back-bearings of the line B C do not agree, set out a third line C D, and obtain its fore- and back-bearings. We will suppose that these bearings agree, then at station C the needle is free from disturbance, and the bearing of C B read at C is the correct bearing.

Let the bearing of C B be 290° 30′, then the bearing of B C

should be 110° 30′. Suppose, however, that the bearing of B C as read at B is 112° 15′, then the needle must be deflected from the meridian an amount equal to 1° 45′. In reading the bearings of B A and B C at B the instrument occupies the same position relative to surrounding objects, hence both bearings will be in error by the same amount. If the bearing of B A as obtained at B is 210° 30′, its correct bearing will be 210° 30′ − 1° 45′, or 208° 45′. If correct, the bearing of A B should be 28° 45′. Suppose, however, that the bearing as obtained at A is 26° 15′, then the difference of 28° 45′ and 26° 15′ or 2° 30′ must be added to all bearings read at A. Thus, if the correct bearing of any one line in a series linked together as in Fig. 37 be known, the correct bearings of all the others may be calculated from their observed bearings.

EXAMPLES.

1. Assume that you are about to make a survey in the country, and you have obtained the assistance of two farm labourers—new to the work—to act as chainmen. Which of the two men would you select to lead the chain ? What instructions would you give these men in chaining on (a) flat ground, (b) hilly ground ? In taking offsets, which of the men would hold the tape-box, the offside man or the man on the chain ?

2. Standing at one end of a line it is impossible to see the rod at the other owing to an intervening hill. Explain how you would place a rod on the top of the hill accurately in the given line.

3. In correcting the lengths of lines measured on sloping ground the following formula is sometimes used :—$c = 100$ $(1 - \cos \theta)$, c being the correction in links per chain to be applied to the measured length, and θ the angular inclination of the ground. Prove the formula, and show how the length of the line may be corrected as the measurement proceeds, without making use of a formula.

4. Show how to calculate the correction in links per chain when the inclination of the ground is θ degrees. There is an approximate formula—giving the necessary correction (c)—of the form $c = K \theta^a$. Write down the value of the constants K and a.

5. Prove the formula $C = 100 - \sqrt{10,000 - n^2}$, where $C =$ the correction in links per chain, and $n =$ the vertical fall in links per chain. Calculate the correction per chain for a case where the gradient is 1 in $5\frac{1}{2}$.   (*Ans.* 1·62 links.)

4

6. A line was measured on a rising gradient of 1 in 10 (1 vertical, 10 horizontal), and found to be 10·45 chains. It was afterwards found that the chain was 2 inches too long. What length should the line scale on a plan ?   (*Ans.* 10·66 chains.)

7. Sketch and describe the construction of any form of clino meter. Explain how you would use this instrument.

8. Sketch and describe the cross staff. How would you alter this instrument so that it may be used for measuring horizontal angles of any magnitude ?

9. Sketch and describe the prismatic compass. What is the value of the declination of the compass needle in your neighbourhood ?

10. To continue a survey line A B past an obstacle, a line B C 250 yards long was set out perpendicular to A B, and from C angles B C D and B C E were set out of 60° and 45° respectively. Determine the lengths which must be chained off along C D and C E in order that E D may be in A B produced.
          (*Ans.* 500 and 353·5 yards.)

11. In passing an obstacle in the form of a pond, stations A and D—on the main line—were taken on the opposite sides of the pond. On the left of A D a line A B, 800 feet long, was laid down, and a second line A C, 1,000 feet long, was ranged on the right of A D, the points B D and C being in the same straight line. B D and D C were then chained and found to be 500 and 600 feet respectively. Find the length of A D.
          (*Ans.* 709·7 feet.)

12. A survey line B A C crosses a river, A and C being on the near and distant banks respectively. Standing at D, a point 200 feet measured perpendicularly to A B from A, the bearings of C and B are 305° 30′ and 215° 30′ respectively, A B being 100 feet, find the width of the river.   (*Ans.* 400 feet.)

13. A main line of a survey crosses a river about 50 yards wide. To find the gap in the line stations A and B are established on the opposite banks of the river, and a perpendicular A C, 120 feet long, is set out at A. If the bearings of A C and C B are 40° 30′ and 280° 30′ respectively, and the chainage at A is 1,536 feet, find the chainage at B.   (*Ans.* 1,743·8 feet.)

14. A survey line A B whose bearing is 90° intersects a tall building. To pass this obstacle, at B a line B C bearing due north and having a length of 300 yards was set out; from C two lines C D and C E, whose bearings were 135° and 120° respectively, were ranged in position. Find the lengths which must be chained off along the lines C D and C E in order that

the stations D and E may be in A B produced. Find also the break in the chainage.   (*Ans.* 424·2 ; 600 ; 300 yards.)

15. The following bearings show a probable magnetic disturbance at the stations A, B, and C. Assuming the disturbance to remain constant while the instrument remains at the stations, find the correct fore- and back-bearings, and the true-bearings of each of the lines. Take the declination of the compass needle N. 16° 20′ W.

Line.	Fore-bearing.	Back-bearing
A B	40° 30′	221° 10′
B C	98° 55′	277° 2′
C D	27° 49′	209° 2′
D E	324° 13′	144° 18′

(*Vict. University, Certificate in Technology,* 1913.)

Ans.

Line.	Fore-bearing.	Back-bearing.	True-bearing.
A B	40° 30′	220° 30′	N. 24° 10′ E.
B C	98° 15′	278° 15′	N. 81° 55′ E.
C D	29° 2′	209° 2′	N. 12° 42′ E.
D E	324° 18′	144° 18′	N. 52° 2′ W

16. A survey line crosses a pond which is too wide to chain across. Give two methods of finding the distance across the pond, and show how you would arrange the lines for the purpose of taking offsets, and to give a check on the result in each case.

17. A survey line A B C cuts the banks of a river at B and C, and to determine the distance B C, a line B E, 3 chains long, was set-out roughly parallel to the river. A point D was then found in C E produced and the middle point F of D B determined. E F was then produced to G, making F G equal to E F, and D G produced to cut the survey line at H. G H and H B were found to be 2 and 4 chains long respectively. Find the distance from B to C.   (*Ans.* 6 chains.)

CHAPTER III.

CHAIN SURVEYING.

Apparatus Required.—Before proceeding to make a survey, whether in the town or country, a list of the apparatus required for the work should be drawn up before leaving the office. This list is useful in checking off the apparatus before leaving the ground, on the completion of the survey. A sample list of apparatus for a survey of about 120 acres would include the following :—

> 1 Field book and 2 or 3 pencils.
> 1 Gunter (or 100 feet) chain and 10 arrows.
> 1 Tape.
> 1 Offset rod.
> 1 or 2 dozen rods with flags.
> 1 Optical square or one cross staff.
> 1 Compass.
> A good field glass.
> About 3 dozen pegs.
> A piece of chalk, a good supply of stout string, and
>> a good pocket knife.

Preliminary Reconnoitre.—On arriving at the ground to be surveyed a preliminary reconnaissance should be made and a fairly correct sketch showing the boundaries, buildings, and other surface features of the ground, drawn in the field book. In the reconnaissance, the main boundaries should be paced, and these lengths used in the preparation of the sketch, so that a fairly accurate knowledge may be obtained both as regards the shape and the extent of the property.

Enquiries should be made as to the owners of the surrounding properties.

Fixing Stations.—After the sketch plan has been completed, the positions of the main survey lines and the stations marking their extremities are next determined. The positions occupied by the stations and lines should be shown on the sketch plan,

the stations being distinguished by capital letters, and the lines numbered by consecutive numbers placed in a circle, to which an arrow is attached showing the direction in which the line is chained. Care and attention spent at this stage in placing the lines in position and setting the order in which they are to be chained, will be well repaid at later stages of the work.

Marking Stations.—The stations are next marked on the ground by driving in a peg at each station and removing the turf around it. The pegs should not project more than 1½ inches above the ground, or they may cause injury to cattle grazing on the land.

It must be understood that no hard and fast rules can be laid down for the guidance of the beginner, in settling the positions of his survey lines. In surveying a plot of land, no two surveyors would use precisely the same system of lines in their work, hence the use of the best system of lines in any particular case depends on the experience and judgment of the surveyor himself.

Conditions to be Fulfilled by Survey Lines.—A good system of lines should, however, fulfil the following conditions :—

(1) One or two lines should form a base line or lines. When two base lines are used they should intersect in the form of the letter X.

(2) The main lines should form triangles of good condition— *i.e.*, should not have angles greater than 90° or less than 30°.

(3) The main lines must not cut through buildings, ponds, etc., necessitating a gap in the chainage.

(4) The rods at the ends of main lines should be mutually visible.

(5) All the figures formed by the main and filling-in lines must contain a sufficient number of tie and check lines.

(6) All the lines from which offsets are taken should be placed close to the corresponding surface features.

(7) To avoid trespassing, the main and filling-in lines should fall within the boundaries of the property to be surveyed.

When the nature of the ground will permit of the use of two base lines running diagonally across the property, they should always be put in, as they simplify considerably the division of the ground into triangles.

Condition (2) is of more importance in trigonometrical than in chain surveying on account of the rapid variation in the value of the sines of angles less than 30°. It must, however, be borne in mind that a triangle which approaches the equilateral in form can be more accurately plotted than an obtuse-angled triangle.

While methods of dealing with obstacles have been given in the preceding chapter, these methods should only be made use of under dire necessity. By shifting a main line a few feet it is often possible to get rid of an awkward chainage gap, and this should be done wherever possible.

Condition (4) is not so important as the others, as it generally is possible to place intermediate rods in position by the method of approximations.

Tie and Check Lines.—The fifth condition is the most important of the series, as without a sufficient number of tie lines the survey could not be plotted, and similarly without a sufficient number of check lines the work would not prove itself.

A good idea of the importance of tie and check lines may be obtained by considering a survey in which the main lines form a four-sided figure A B C D, as in Fig. 38. We could not draw the figure A B C D if we were given the lengths of its sides only
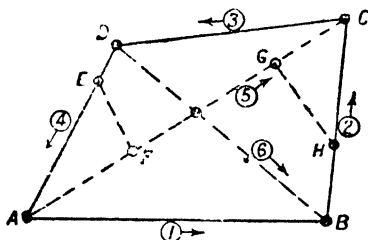
Fig. 38.

since we can construct any number of four-sided figures with four given lengths as sides ; but if we are given also the length of A C, the shape of the figure is fixed, and we could draw only one figure from the given data. Since the line A C is necessary to fix (or tie-in) the shape of the figure, it is called a *tie line*. If one or more of the sides had been inaccurately measured in the field, we should still be able to plot the figure from the given measurements, and nothing on the plan would enable us to detect the inaccuracy ; but if we know the length of the other diagonal B D—or any two lengths such as E F and G H and their respective positions—we have a complete check, not only on all the field measurements, but on the accuracy of the plotting work also. These additional lines are called *check* or *proof lines*. Obviously, either A C or B D may be considered as a tie and the other as a check line.

In most cases the main figures will be crossed by the lines required for filling in detail, and these will serve as tie or check lines.

Short Offsets.—It is obvious that if condition (6) is satisfied much time will be saved in the measurement of the offsets,

since short offsets are more quickly and accurately measured than long ones.

Practice Drawing Survey Lines.—Although the facility and judgment required in deciding on the best positions of survey lines can only be learned as the result of experience in the field, yet much may be learned of the conditions to be satisfied, by drawing survey lines on a plan. The student is recommended to practice drawing systems of survey lines on tracings taken from a local 6-inch or 25-inch ordnance sheet, the stations being lettered and the lines numbered in the order in which he would propose to chain them. The tracings should then be taken to the ground they represent, and from a comparison of the plan and the ground, he will be able to decide whether his system of lines is, or is not, feasible.

The positions of the stations having been finally decided, rods are set up at each peg, and the measurement of the lines may be commenced. Prior to this, however, a chain standard should be made as described on p. 9, so that the chain may be tested daily ; also the fore- and back-bearings of at least one of the main lines should be obtained.

Field Book.—For the purpose of entering field measurements, a special form of note book is used. The form of the book in general use is oblong, about 8 inches by 5 inches, opening lengthwise. The pages are blank, with the exception of either two lines, about half-an-inch apart, or a single line ruled down the centre of the page. The lines down the centre of the page represent the chain, and all numbers booked between them represent chain measurements.

Entering Field Notes.—In entering field notes of distances and offsets, a surveyor always commences his notes on the back page of his field book, and books forward from the back of the book to the front ; also, the entries on any particular page are booked from the bottom of the page upwards.

There are several conventional methods of entering field notes, but they all agree in placing chain measurements up the centre of the page and offsets on the right or left of these, according to the side on which they have been measured. The nature and form of the objects to which offsets are taken should be sketched in the field book as the work proceeds, and marginal notes made wherever such notes are thought to be desirable. In fact, the field book should contain all the information necessary for the preparation of the plan, so that any draughtsman may carry out the plotting work. In order that nothing may be over-

looked, it is absolutely necessary that the whole of the work be carried out in a methodical manner. Hence all lines on the sketch plan must be numbered consecutively in the order in which they are chained, and the numbers of the lines in the field book and sketch plan must agree.

All entries in the field book should be clearly written in pencil, with not more than one survey line to a page ; the figures should not be crowded together, a space of about a quarter of an inch being left between the entries of consecutive chain measurements ; wrong entries should not be erased, but cancelled by drawing a line through them, the correct figures being entered at the side of those cancelled. In long lines an entry should be made at the end of every ten chains. An example of one method of entering field notes which the author has found very satisfactory is shown in Fig. 39.

The chief points to be noticed in this method of booking are :—(1) The numbers giving the chainage at the stations (F, I, and G) are enclosed in an oval ; (2) the directions of joining lines and their numbers are indicated by lines drawn from the oval surrounding the chainage number to the right or left as required, the number of the line being shown by a figure surrounded by a circle.

The student should also notice that (a) the corners of the straight orchard wall being important points, are fixed by oblique off-sets ; (b) the post and rail fence on the left of the line touches the chain line at 70, this is indicated by the word " at " replacing a figured offset ; (c) the points of crossing by the chain of the fences, road, and edges of the stream are shown at 000, 230, 506, etc., on the chainage.

The end of the line is denoted by drawing two lines straight across the page. This should always be done to prevent confusion.

Further examples of this method of entering field notes are given at the end of this book, in the records of the chain survey at Wigglesworth Hall. The latter was carried out by the University students of the Surveying Class of the Municipal School of Technology, Manchester (now the College of Technology, Manchester), during the surveying excursion from July 17th to 26th, 1912, the work being carried out under the personal supervision of the author and his colleague, Mr. H. F. V. Newsome, A.M.S.A.

Instead of showing the direction of joining lines at stations on a main line by straight lines and figures enclosed in circles,

some surveyors make use of fractions written beside the station, the numerator of each fraction denoting the distance of the station from the commencement of the line whose number is given
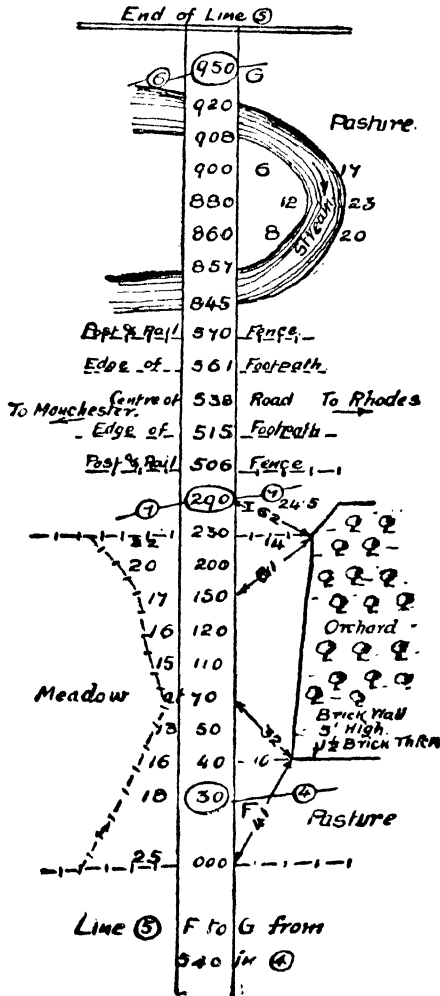
Fig. 39.

by the denominator. Thus, in Fig. 39, at station F, instead of showing line 4 joining line 5 at F as indicated, if F is at 345 links on line 4, the fraction $\frac{345}{5}$ would be written near station F. By this method the joining of the lines 7 and 6 at I and G might be indicated by the fractions $\frac{672}{7}$ and $\frac{080}{6}$ respectively. In the latter, line 6 commences at station G.

This system of booking distances and lines at stations saves time in the drawing office at the expense of the time spent in the field. It adds another to the many difficulties on which the mind of the field surveyor is constantly concentrated, since on reaching any station he has not only to note the numbers of the lines coming in to the station by reference to his sketch of the lay-out, but he must find from his bookings the distance of the station from the commencement of each joining line, or if the joining lines have not been measured, he must make the necessary entries at a later period.

We sometimes come across chain entries given in the form 36 + 45, meaning thereby 36 whole chains + 45 links. If the chain were not decimally divided this method of entering chain measurements might be useful; but since the chain is always divided into 100 links, it is obviously simpler to write the above distance 3645; the number of chains is at once obtained by putting a decimal point to the left of the second figure from the right, thus 3645 links = 36·45 chains = 36 chains 45 links.

Where to Take Offsets.—The beginner is often puzzled in deciding where he should take offsets to surface features, and how many offsets he requires in any particular case.

In deciding where to take offsets, the rule to be followed is, *take an offset wherever the outline of an object changes.* For example, in the case of a straight wall, an offset at each end of the wall is sufficient, while in the case of a crooked fence an offset is required at each change in direction. If the fence dividing two fields is a quick-set hedge, the offset should be measured to the centre of the root of the hedge. Similarly, if the division between two properties is a ditch, a stream, or road, the offset should be taken to the centre in each case, but in the case where the boundary consists of a hedge and ditch, as in Fig. 40, the offset should be taken to the brow of the ditch. Generally, the owner of the hedge owns the ditch also; but this rule is subject to local custom, and it is advisable—in a case of this kind—to make enquiries on the spot. The same remark applies to boundary walls, which are not always subject to joint ownership.

The number of offsets required in a particular case is settled

by the shape of the object and the scale to which the plan is to be plotted. If the rule that an offset must be taken wherever the outline of a surface feature changes is strictly followed, in some cases (*i.e.*, where the scale of the plan is small) more measurements may be taken than can be usefully employed in plotting the plan, yet it is always better to err on the side of taking too many measurements than too few.
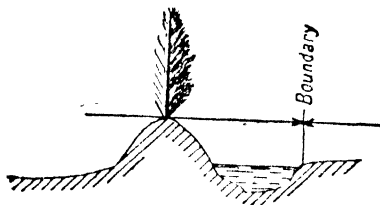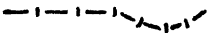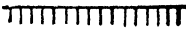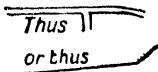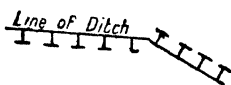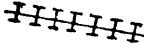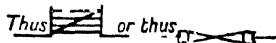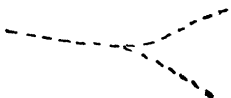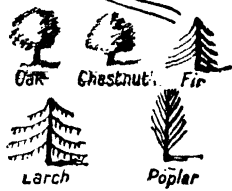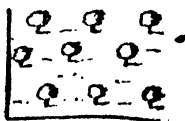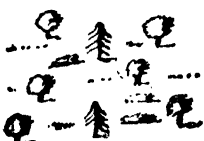
Fig. 40.

Degree of Precision.—In commencing field measurements, the beginner meets with a further difficulty. On taking his first offset the question arises, " To what fraction of a link or a foot ought the distance to be measured ? " This question is settled very largely by the scale of representation, which must be known before the survey is commenced.

A draughtsman with good eyesight can plot a point on paper to within $\frac{1}{150}$ of an inch of its true position ; but this will not be his ordinary degree of precision in plotting. He should, however, be able to lay down his points with a permissible error of $\frac{1}{100}$ of an inch. If we take the former value as the limit of precision in plotting, then, when we know the scale of representation, we can at once settle the degree of precision of the field measurements. For example, take one of the largest scales used in survey work—viz., 10 feet to an inch. The representative fraction of the scale is $\frac{1}{120}$, and hence $\frac{1}{150}$ of an inch on the plan would represent $\frac{120}{150} = \cdot 8$ of an inch on the ground. The offsets should, therefore, be measured to the nearest half-inch. If the scale is 2 chains to 1 inch, measure the offsets to the nearest link $\left(\frac{200}{150} = 1\frac{1}{3}\right)$, if 4 chains to 1 inch it is sufficient to measure the offsets to the nearest 2 links $\left(\frac{400}{150} = 2\frac{2}{3} \text{ links}\right)$.

Attention to this point will often save time in the field.

Conventional Signs.—The chief conventional signs used in this country by surveyors, to indicate special surface features, are as follows :—

1. Post and rail fence,   .   .

2. Close paling.   .   .   ⌐   .

3. Walls, ...

Thus
or thus

4. Hedge and ditch,   .   .   .
The hedge being indicated by
the T and its position shows to
which side it belongs.

Line of Ditch

5. Foot-set fence,   .   .   .

6. Gates,   .   .   ⌐   ⌐   .

Thus   *or thus*

7. Footpaths, ...

8. Cart-track or bridle-path,   .

9. Roads, ...

10. Isolated trees are shown and
described thus :—

Oak   Chestnut   Fir

Larch   Poplar

11. Orchards are indicated by
small round trees regularly spaced
in rows, ...

12. Woods,   :   .   :   .

Thus

13. Marshy ground, .   •

or Thus

14. Brushwood,   ⸰   ⸰   ⸰

15. Railways, • ⸰   ⸰   ⸰

16. Railway on an embankment,

17. Railway in a cutting, .   •

18. Quarries or broken ground, . .

19. Gravel pits,   ⸰   ⸰

20. Parish boundaries   ⸰   ⸰

4 Ft R.H.

i.e., 4 feet from the root of the hedge on the side shown.

21. County boundaries,   .   .
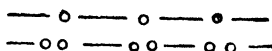
22. Number of a line and direction in which it is chained, .   .   .

23. Survey station,   .   .   .
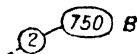
24. Plots of land which are included in the same area are joined by a brace, thus :—

Clearing the Ground.—On completing the field work, insist on the chainmen clearing the ground of all pegs, laths, pieces of paper, etc., so that the ground may be left as nearly as possible in its original condition.

If, however, the survey has been made for engineering or building purposes, the pegs should be left in the ground, as they will be of considerable assistance in setting-out the proposed works at a later period.

Plotting to keep Pace with the Survey.—In order to keep a close check on the accuracy of the field work, the survey lines and offsets measured on a particular day should be plotted at the close of that day's work. In this way any errors made in the measurements or details omitted will be detected before the field work has got very far. Such errors and omissions must be attended to before the survey party proceeds to new ground.

The plotted plan need only be finished in pencil, as it is almost certain to get soiled and stained by the constant reference to it as the work proceeds, but this plan should show all the features of the ground surveyed. Later on, it will be found useful for reference in preparing the final plan in the office.

Correcting the Final Plan.—Although every care and precaution may have been taken against errors and omissions in the field work, if the ground is of an extensive or complicated character errors and omissions are sure to be made. Hence, before publication of the final plan, either the plan, or a tracing of it, should be taken on the ground surveyed for the purpose of checking and correction.

The surveyor engaged in checking should be provided with a scale, tape, optical square, and compass. It is his duty to check

and correct the plan line by line, observing, for example, if any crooked fences have been plotted as straight; if the positions of the extremities of straight fences as given by the plan agree with the corresponding measurements on the ground; if the faces of two or more blocks of buildings, which converge at a particular point on the plan, do the same on the ground, and so on. Footpaths—often ill defined on the grass and easily overlooked—may be put in by a combination of traversing and sketching, since the correct position of such footpaths in country districts is not generally of much importance. Similarly, the positions of isolated trees may be fixed by taking their bear-
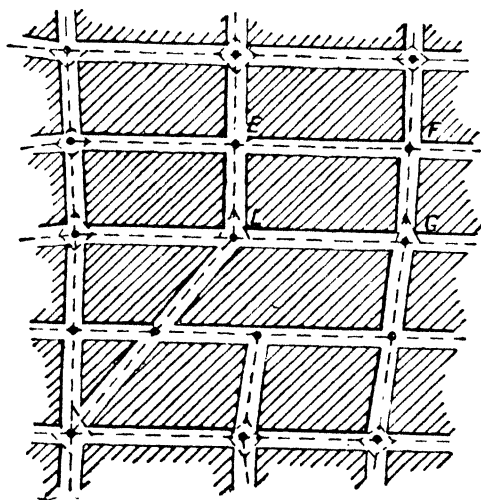
Fig. 41.

ings from some well defined point and taping or pacing their distances from it.

In every way the plan must be subjected to rigorous tests, so that the checker—at the completion of his work—may be able to guarantee its accuracy.

Town Work.—Town surveying with the chain follows the same general method as that in the country, with the exception that long tie lines cannot be used.

As, in the main, the blocks of property bounded by streets are quadrilateral in form, the tie lines are restricted to short

diagonal lines at the corners of the streets, and are too short in comparison with the length of the main lines.

This is well shown in Fig. 41, where the necessary tie and check lines are shown by the short diagonal lines at the corners of the different streets. Certain of the long lines, however, act as proof lines; for example, in the quadrilateral L E F G (Fig. 41), the lines L E and G F in plotting are drawn and checked in position by means of the triangles at stations L and G. On scaling off the lengths of the lines L E and G F, the measured length E F should join their extremities. Thus, E F acts as a proof line of the figure. It is obvious from Fig. 41 that any slight error in the measurement of the tie lines will cause a much greater error at the extremities of the corresponding main lines. Hence, the greatest care must be taken in obtaining these measurements, and if great accuracy is required, the chain should be discarded and all measurements made with a steel tape.
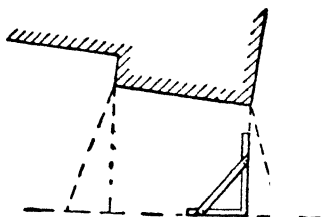
Fig. 42.

To get the best results in town surveys, all the figures should be tied-in by measured angles at their corners, the angles being measured with some accurate angle measuring instrument, such as a theodolite or box sextant.

Offsets.—If the object of the survey is to fill in the detail of a topographical map drawn to a small scale, the accurate determination of the relative positions of the streets is of greater importance than the dimensions of the separate buildings abutting on them, and the offsets may be taken in the usual way with the linen tape. But, when the land is very valuable—as it is in the centres of all large towns—and the object of the survey is to determine the boundaries of the different plots of land and their contents, the offsets should be set out by means of a large wooden set square (about 6 feet by 4 feet, see Fig. 42), and measured with a steel tape to the nearest $\frac{1}{4}''$ or $\frac{1}{8}''$, an oblique offset being put in as a check. While these dimensions may be finer than can be shown on the plan, this degree of accuracy is necessary in order to calculate the exact contents of each plot of land.

Amount of Detail necessary.—If the scale of representation is large (say $\frac{1}{120}$), two lines, from which offsets are taken, should be used in each street, in order to keep the offsets short. With

such a scale $(\frac{1}{120})$ the frontage plan of each building on a street, together with all outbuildings, areas, gullies, manholes, coal shoots, lamp-posts, etc., can easily be shown, and their positions and dimensions must be determined.

In small scale work this amount of detail is not required, and one line down each street is sufficient.

Marking Stations.—Stations are marked either by cutting a cross in the pavement or by driving an iron spike (about 4 inches long and $\frac{1}{2}$ inch round or square) between the sets. The positions of the stations are fixed by measurements to the corners of the nearest buildings, a careful note accompanied by a dimensioned sketch of each case is made in the field book.

The rods are held in position over the stations by clamping them in a wooden or metal tripod.

Booking the Measurements.—The record of the measurements is entered up in the same way as in country work. As a great amount of detail, involving sketches and written dimensions, has to be dealt with, care must be taken to leave a fair amount of space between the entries of chain measurements, crowded bookings only lead to slovenly and inaccurate work.

In measuring buildings consisting of a series of frontages, care must be taken to see that the sum of the separate frontages is equal to the total length of the series.

The names of all streets, churches, chapels, schools, and other public buildings must be carefully noted.

A note should be made of the nature of the road coverings, whether wood, granite, macadam, asphalte, etc., and also the nature of the pavement.

Should a street cross a river or stream, the name of the river or stream, its direction of flow, and any other useful particulars must be noted.

The positions of boundaries (if any occur within the limits of the survey), whether parliamentary, parish, municipal, or ward must be ascertained and shown on the plan.

Best Time to Work.—It is quite obvious that survey work—involving, as it does, the use of rods, chains, and tapes—cannot be carried out in the crowded streets at the centre of a large town during the busy part of the day. The best time for such work is in the early morning before the streets become crowded, the remainder of the working day being utilised in the measurement of the quieter side streets.

Checking Town Plans.—When the outlines of the streets have been plotted, tracings of sections of the plan should be made

and compared with the streets they represent. Each detail is examined separately, so that nothing may be omitted, and a few measurements taken here and there will enable the surveyor to satisfy himself that the plan is accurate.

For the better filling-in of the details of houses and outbuildings, small sections of the plan, about 18 inches square, are drawn to a large scale and mounted on a board, so that the plans of the houses, etc., may be drawn to scale as the measurements proceed. These sectional plans are afterwards reduced, and the detail fitted in on the general plan.

EXAMPLES

1. State what apparatus you would require in order to make a survey of a farm of about 20 acres in area. Assuming that the ground is fairly flat and presents no special difficulties, describe in sequence your programme of operations.

2. Draw a sketch plan of an imaginary farm, consisting of five enclosures (one of which must be a plantation), with farm buildings near the centre. Make your sketch cover half a sheet of foolscap paper, and suppose the scale to be 3 chains to 1 inch. Assuming the ground represented to be fairly level, and you are about to make a survey of the farm, draw the survey lines you would use. The stations must be lettered, lines numbered, and chaining direction indicated in the usual way.

3. State the principles which should guide you in determining the best positions for the main and filling-in lines of a chain survey. Illustrate your answer by such sketches as you consider necessary.

4. What do you understand by the term "representative fraction of a scale"? If you were asked to survey a plot of land, what is the nearest fraction of a foot you would measure to, if you are later to plot the plan of the survey to a scale of (a) $\frac{1}{96}$ and (b) 1 inch to 100 feet? $\qquad$ $(Ans. \ \frac{1}{24}'; \ \frac{1}{2}'.)$

5. Illustrate by sketches the chief conventional signs which have been adopted by surveyors to indicate special features on a plan.

6. Suppose that you are requested to make a survey of about 5 acres of town land on which streets run in roughly rectangular directions. State what instruments you would require, and what precautions you would take in order to ensure an accurate survey. If the object of the survey is mainly to show the relative positions

of the streets and the blocks of property on them, to what fraction of a foot would you measure the offsets if the plan is to be plotted to a scale of 100 feet to 1 inch ? (*Ans.* $\frac{1}{2}'$.)

7. In question (2), write up one line of the field book, assuming distances and offsets.

8. Suppose you were making a chain survey of an estate, which is bounded on one side by a winding river (say) 200 feet wide. The river may be passed by a convenient ford ; but for the rest of its course it is too deep for chaining across. Explain by aid of a sketch how you would make a survey of the far bank of the river, and how you would tie-in the survey lines to those in your general scheme.

9. Let the probable error in chaining a line be *e* per chain, then it may be shown that for errors which tend to compensate, the total probable error in chaining N chains is equal to $e \sqrt{N}$. If the probable error in placing an arrow in position at the end of the chain is $\frac{1}{4}$ inch, find the probable error caused thereby in measuring a line 1 mile long with (1) a gunter chain, (2) a 100-feet chain, and (3) a 500-feet steel band.

(*Ans.* $2.23''$; $1.83''$; $0.81''$.)

10. A surveyor finds he can lay out the direction of an offset by the eye in the field with a maximum error of 5° from the true perpendicular. With what degree of accuracy must he measure the length of the offset in order that the error on the paper from this source may not exceed that from angular error ? Under these conditions, if the plan is to be plotted to a scale of 200 feet to 1 inch, what must be the maximum length of offset so that the displacement of a point on the paper from the two sources of error combined may not exceed $0.01''$? (B.Sc., London, 1907.) (*Ans.* 0.38 per cent. ; 263 feet.)

CHAPTER IV.

TRAVERSING.

Preliminary Remarks.—Suppose we wish to find the distance between the two points A and B (Fig. 43). Naturally, if the line joining the two points is unobstructed, we should measure the required distance direct; but if the line is obstructed, we take the shortest open course and connect the two points by a chain of lines, such as A C, C D, D E, and E B.
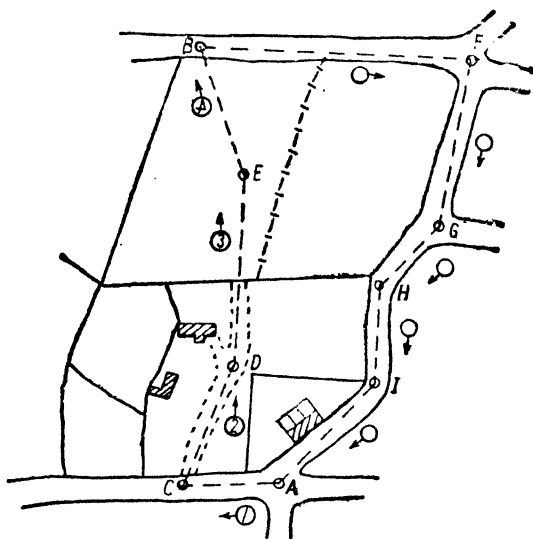
Fig. 43.

The connecting lines A C, C D, etc., are chained in the usual way, and if the angles at C, D, and E are determined by some means, we should be able to plot the chain of lines and thus determine the required distance, either by measurement from the plan or by calculation.

Closed and Unclosed Traverses.—An operation of this kind is called a traverse. If the operations stopped at B, the figure formed by the chain of lines would be an unclosed polygon, and the traverse would be spoken of as an *unclosed traverse*, but if we worked back again to A by some other route, as B F, F G, G H, etc. (Fig. 43), the traverse would be a *closed traverse.*

Obviously, as the chaining of the lines proceeds, offsets may be taken from them to neighbouring objects, and a plan of the route showing the topographical features in its immediate neighbourhood could be prepared.

Definition.—Hence, the term traversing may be defined as the art of surveying a long, narrow, winding strip of country.

This method of surveying is very largely used for filling-in the detail work of a survey, the traverse closing on points which are fixed in position by the larger operations. It is also used for surveying the ground required for a proposed road, canal, or railway, and it forms the basis of most underground surveys.

Necessity for Closing a Traverse.—Wherever possible all traverses should be closed, as without this there can be no rigid check on the accuracy of the work.

The check on the accuracy of a closed traverse lies in the fact that on plotting the survey the chain of lines must close accurately on the starting point, and since the lines form a closed polygon, we can check the sum of the interior angles by the well-known rule, *the sum of the interior angles of any polygon equals twice as many right angles as the figure has sides, less four right angles.* Or, if $n =$ the number of sides in the figure, then the sum of interior angles $= 2n \times 90 - 4 \times 90 = 180(n - 2)$.

Different Methods of Traversing.—There are several methods of traversing, depending on the instruments employed in determining the relative directions of the main lines. Three methods only will be dealt with at present—viz., chain traversing; traversing with the chain and cross staff; and traversing with the chain and compass.

Chain Traversing.

In this method the whole of the work is done with the chain and tape. The angle at the intersection of any two lines is fixed by measuring the three sides of a triangle, as shown at B, Fig. 44. The point c is taken in A B, 1, 2, or 3 chains from B, the line C B is produced backwards to d, B d being made equal to B c, and the chord $c\,d$ is measured. As a check on this, the sides of the triangle B ef are determined.

The same mode of procedure is employed at each station, one triangle tying in the two lines and the second triangle serving as a check on the accuracy of the first.

It should be noticed that if the tie and check triangles are made isosceles, there will be two similar triangles at each station, and consequently the chords *d c* and *e f* will be directly proportional to the sides B *c* and B *e*. Hence, if we make B *c* equal to B *e*, the chord *d c* will be equal to the chord *e f*. Thus, by comparing the lengths of the measured chords at a station, any inaccuracy can at once be detected and the error corrected before leaving the station.

Calculating Interior Angles.—If required, the interior angle at any station may be calculated as follows :—
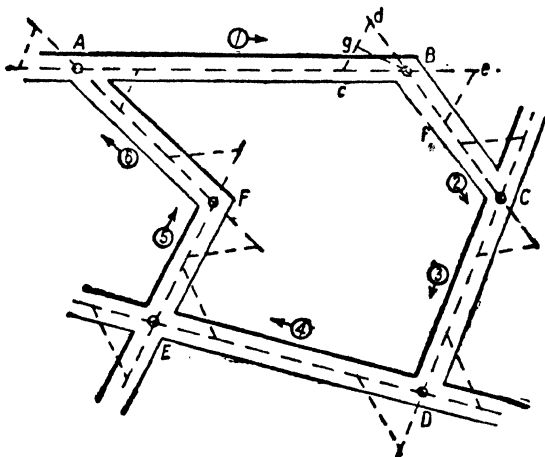
Fig. 44.

Let A B and B C (Fig. 44) represent the main lines meeting at B, and *c d* B the triangle tying-in the angle A B C. Bisect *c d* at *g* and join *g* B.

Then,
$$\sin \frac{c\,\mathrm{B}\,d}{2} = \frac{c\,g}{c\,\mathrm{B}},$$

or
$$\angle\, c\,\mathrm{B}\,d = 2\sin^{-1}\frac{c\,g}{c\,\mathrm{B}},$$

and
$$\angle\, \mathrm{A\,B\,C} = 180° - \angle\, c\,\mathrm{B}\,d$$
$$= 180° - 2\sin^{-1}\frac{c\,g}{c\,\mathrm{B}}.$$

This method of traversing should not be used unless the

ground is fairly level and there is plenty of ground, free from obstacles, in the neighbourhood of the stations.

Plotting.—The traverse is plotted by building up the tie and check triangles at each corner of the figure. The former fixes the direction of any line relative to the preceding one, and the latter checks the accuracy of the plotting and field measurements.

Traversing with the Chain and Cross Staff.—In this method the main lines are set out either at right angles or at angles of 45° with each other, a check line being thrown across the angle at each change of direction. Thus, in Fig. 45 the line B C is tied to the line A B by setting out (with the cross staff) B C at an angle of 45° with A B, the accuracy of the work being tested by the measurement of the line a b. The third and succeeding lines shown in Fig. 45 are tied on to the lines immediately preceding them by right angles set out with the cross staff, the accuracy of the setting out being tested by the lines c d, d e, etc.

Offsets are taken from the check lines when they are suitably placed for this purpose.

The accuracy of the setting out may be tested by calculation as the work proceeds. Thus, when the intersection angle is 45°, as at station B (Fig. 45), we have :—

Fig. 45.

$$a\,b^2 = a\,\mathrm{B}^2 + \mathrm{B}\,b^2 - 2\,a\,\mathrm{B}\,.\,\mathrm{B}\,b\cos 45°.$$

If we make $a\,\mathrm{B} = \mathrm{B}\,b = 1, 2, 3$, etc., chains,

then
$$a\,b^2 = 2(a\,\mathrm{B})^2 - 2(a\,\mathrm{B})^2 \cos 45°$$
$$= 2\,a\,\mathrm{B}^2\,(1 - \cos 45°)$$
$$= 2\,a\,\mathrm{B}^2\left(1 - \frac{1}{\sqrt{2}}\right)$$
$$= \sqrt{2}\,a\,\mathrm{B}^2\,(\sqrt{2} - 1)$$

or
$$a\,b = a\,\mathrm{B}\,\sqrt{2 - \sqrt{2}}$$
$$= \cdot7655\,a\,\mathrm{B}.$$

If $a\,\mathrm{B} = 2$ chains,

then $a\,b = 1$ chain 53 links.
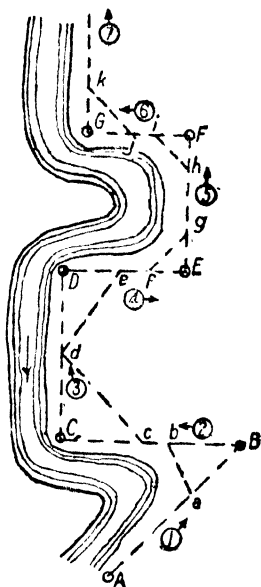
When the included angle is a right angle, as at C (Fig. 45),

then	$c\,d^2 = C\,c^2 + C\,d^2,$
if we make	$C\,c = C\,d,$
we get	$c\,d^2 = 2\,C\,d^2,$
or	$c\,d = C\,d\,\sqrt{2}$
	$= 1\cdot414\ C\,d.$
If	$C\,d = 3$ chains,
then	$c\,d = 4\cdot242$ chains.

When work of this kind is in contemplation, a short table should be prepared, giving the length of the proof line for distances of 1 to 5 chains measured on the main lines for intersection angles of 45° and 90°. By referring the length of a proof line as measured to the corresponding tabular length, the accuracy of the setting out can at once be tested.

Plotting.—This method of traversing lends itself to easy and accurate plotting, since the angle between any two consecutive lines is either a right angle or an angle of 45°. The check lines should be drawn on the plan, even though no offsets have been taken from them, as by their use slight inaccuracies in plotting the main lines can be detected.

Use Short Lines.—In making use of this system, it must be remembered that the sighting base is very short (not, as a rule, more than 3 or 4 inches), and that the sights are of a crude description. Hence it is necessary to keep the main lines fairly short, in order that the work may be accurate.

Traversing with the Chain and Compass.—In this system the lines A B, B C, etc., are set out to follow the boundary or the centre of the strip of ground to be surveyed. The fore-bearing R A Q of the line A B (Fig. 46) having been obtained with the compass, the line is chained and offsets taken as usual. On arriving at station B, the back-bearing S B T of A B and the fore-bearing S B V of the line B C are obtained. The line B C is next chained and the offsets taken as required. At C the back-bearing U C W of C B and the fore-bearing U C X of C D are observed. These operations are repeated until the end of the traverse is reached.

Error in Fore- and Back-Bearings.—An error of more than one-quarter to one-half a degree between fore- and back-bearings should not be permitted. If a greater error is obtained on any

ine, the fore- and back-bearings of the line should be redetermined, and if the error persists magnetic disturbance should be suspected and dealt with as explained on p. 48.

Traversing with the compass is rapid and simple in its operation. It is largely used for filling in the detail of surface surveys, and forms the basis of most underground surveys.

In using this system, it must be remembered that the accuracy of the work depends chiefly on the accuracy with which the bearings of the lines are obtained, and as the work proceeds the
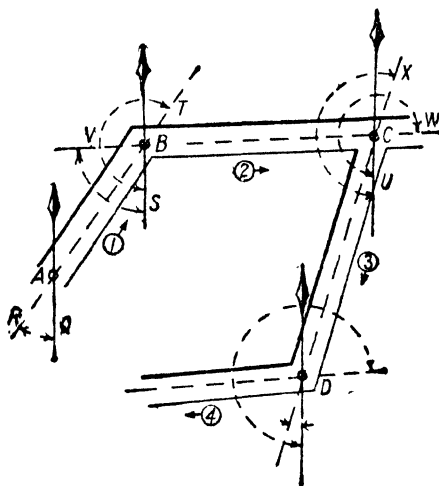
Fig. 46.

only check on this is the agreement of the fore- and back-bearings in each particular case. It follows that as few bearings as possible should be put in. Hence the lines should be as long as possible, and traverses, consisting of short lines varying rapidly in direction, should—so far as possible—be avoided.

For convenience in plotting, in addition to the ordinary bookings of distances and offsets, the fore- and back-bearings should be recorded in tabular form as they are determined. A convenient form of table, exhibiting the method of entering the notes, is given below.

Line.	Fore-bearing.	Back-bearing.	Distance
A B	42° 30′	222° 30′	840
B C	94° 15′	274° 15′	1,230
C D	210° 45′	30° 30′	654
Etc.	. .	. .	. .

Plotting a Compass Traverse.—There are two methods of plotting a compass traverse in general use—viz., the method of plotting by (1) " *latitudes* " and " *departures*," and (2) by the use of the protractor.

The former method, although more troublesome to carry out, is more accurate in its results than the latter, and is the method most generally adopted.

" **Latitudes** " **and** " **Departures.**"—The meaning of the terms " latitude " and " departure," as used in plotting a compass traverse, may be best understood by an example. Let A B and N S (Fig. 47) represent the directions of a traverse line and the magnetic meridian respectively. The fore-bearing of the line is equal to the angle S A Q or a. An observer in walking from A to B would go a distance A C north of his starting point, and a distance A D east of the same point. The distance A C is the observer's *change of latitude* (or, more shortly, the " *latitude* "), and A D is his *departure from the meridian* through A, or is his " *departure*." Thus, the terms " latitude " and " departure " are applied to the rectangular components of a traverse line. It is quite immaterial in what direction a line proceeds, hence some latitudes will be measured towards the North, others towards the South ; some departures are to the East, and others to the West of the first station. Consequently the total " latitude " or " departure " on any traverse is the algebraic sum of the " latitudes " or " departures " of all the lines. When the traverse closes on its starting point, both these sums should be zero ; if not, the quantity left gives the closing error in " latitude " or " departure," or both.

Northings and Southings, Eastings and Westings.—Many writers on this subject apply the terms " *Northing* " and " *Southing* " to the north or south projections of a line on the magnetic meridian, through one extremity, and the terms " *Easting* " and " *Westing* " to its east or west projections on a line at right angles to the meridian, through the same extremity. Thus, in

Fig. 47, A C is the Northing and A D the Easting of the line
A B ; also A C′ is the Southing and A D′ the Westing of the
line A B′.

Plotting the Traverse.—In order to plot a traverse by this
method we must know the total " latitude " and " departure "—
i.e., the *independent co-ordinates*—of every station of the traverse.
Thus, in Fig. 48, if we know the distances C E and C F, the
point C may be plotted by laying off these distances at right
angles to the axes of reference. Similarly, if D I and D J be
known, the point D may be plotted, and hence the position
of the line C D is fixed. In the same way the positions of all
the other stations may be plotted. The check on the plotting
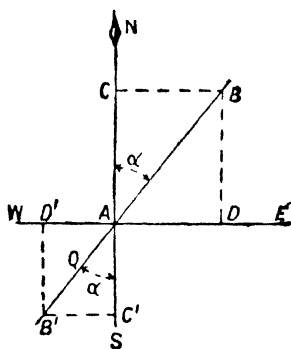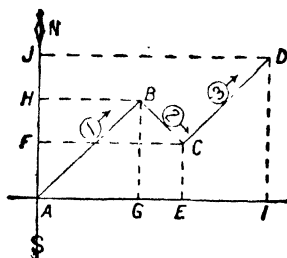
Fig. 47.   Fig. 48.

is obtained by scaling off the length of any line which ought to
agree with its chained length.

It will be noticed, on referring to Fig. 48, that the total de-
parture A I is equal to A G + G E + E I, and A J is equal to
A H − H F + F J. The distances A G, G E, E I, and A H,
H F, F J are spoken of as the *consecutive co-ordinates* of the
stations B, C, and D.

Referring to Fig. 47, let a be the reducing bearing (see p. 46)
of any traverse line such as A B, then the

<div align="center">

" Latitude " of A B = A B cos a,

</div>

and the " departure " of A B = A B sin a.

Thus the " latitude " and " departure " of any traverse line

I. Line	II. Fore-bearing	III. Back-bearing	IV. Reduced Bearing	V. Distance Links	VI. Included Angles	VII. Consecutive Co-ordinates Lat. N	VII. S	VIII. Dep. E	VIII. W	IX. Independent Co-ordinates Lat. N	IX. S	X. Dep. E	X. W
A B	30° 30'	213° 30'	N. 30° 30' E.	1542	EAB, 12° 10'	1328·0	...	782·7	...	1328·0	...	782·7	...
B C	123° 15'	303° 10'	S. 56° 47·5' E.	978	ABC, 87° 17·5'	...	535·6	818·1	...	792·4	...	1600·8	...
C D	225° 40'	45° 30'	S. 45° 35' W.	1230	BCD, 77° 37·5'	...	861·0	...	878·4	...	68·6	722·4	...
D E	286° 20'	106° 10'	N. 73° 45' W.	645	CDE, 119° 20'	180·5	...	...	619·3	111·9	...	103·1	...
E A	222° 40'	42° 40'	S. 42° 40' W.	152·3	DEA, 243° 35'	...	111·9	...	103·1	0	0	0	0
Sums,					540°	1508·5	1508·5	1600·8	1600·8				
						1508·5	...	1600·8	...				
Diffs.,					·	0·0	...	0·0	...				

is obtained by multiplying the length of the line by the cosine
and sine respectively of its reduced bearing.*

The method of preparing a compass traverse for plotting is
shown in the table opposite.

Explanation of the Table.—Columns I. to V. have already
been explained. The included angles given in Column VI. are
best obtained from a diagram. By inspection of Fig. 49 it will
at once be seen that the included angle A B C is equal to the
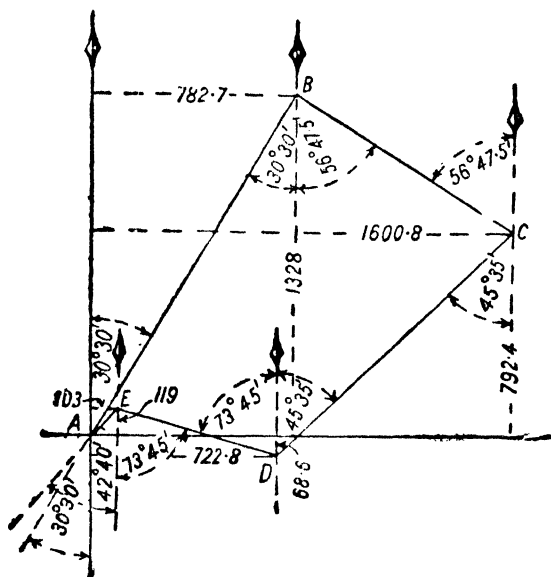
Fig. 49.

sum of the reduced bearings of the lines A B and B C. Also,
the included angle

$$B \, C \, D = 180° - \text{sum of reduced bearings of B C and C D}$$
$$= 180° - (56° \, 47·5' + 45° \, 35')$$
$$= 77° \, 37·5'.$$

* The labour of computing "latitudes" and "departures" is much
reduced by the use of traverse tables, the most accurate of which are
those computed by Mr. R. L. Gurden, and published by Charles Griffin &
Co., Ltd. These tables are calculated up to 100 of distance and for every
minute of angle.

The included angle

$$\text{C D E} = \text{sum of reduced bearings of C D and D E}$$
$$= 45^\circ\ 35' + 73^\circ\ 45'$$
$$= 119^\circ\ 20'.$$

The interior angle

$$\text{D E A} = 360^\circ - (73^\circ\ 45' + 42^\circ\ 40') = 243^\circ\ 35'.$$

Lastly, the interior angle

$$\text{E A B} = \text{the difference of the reduced bearings of E A}$$
$$\text{and A B}$$
$$= 42^\circ\ 40' - 30^\circ\ 30'$$
$$= 12^\circ\ 10'.$$

The sum of the interior angles $= 540^\circ$.

This satisfies the rule, sum of interior angles $= 180^\circ\ (n - 2)$, since in this case $n = 5$. ∴ sum of interior angles $= 180^\circ\ (5 - 2) = 540^\circ$.

The " latitude " and " departures " in Columns VII. and VIII. have been calculated by the rules already given. Thus,

$$\text{Lat. A B} = 1,542 \cos 30^\circ\ 30'\ \text{N.}$$
$$= 1,328\ \text{links N.}$$

$$\text{Dep. A B} = 1,542 \sin 30^\circ\ 30'\ \text{E.}$$
$$= 782\!\cdot\!7\ \text{links E.}$$

The " latitudes " and " departures " of the other lines are obtained in the same manner, and are entered in their proper columns.

The independent co-ordinates of the stations of the traverse are obtained by taking the algebraic sum of the " latitudes " and " departures " of the several lines. As the traverse is reduced to a meridian and latitude through station A, the " latitude " and " departure " of this station are zero. To obtain the co-ordinates of station B, transfer the latitude and departure of B to the corresponding columns of the independent co-ordinates. As the line B C goes S.E. and its latitude is 535·6, subtract this amount from 1,328 (the Northing of B), and the remainder 792·4 gives the distance that C is north of A. The departure of B C (818·1) being of the same sign (E.) as the departure of A B, we add this number to the departure of B, and thus the number 1,600·8—giving the distance that C is E. of A—is obtained. Proceeding in this way, we find that the last entries in all the independent co-ordinates' columns are zeros. This

must be so if both the survey and arithmetical work have been accurately carried out, since the traverse closes on its starting point. As a check on the arithmetical work, the latitudes and departures in Columns VII. and VIII. should be totalled and their differences taken. These differences should be respec- tively equal to the last entries in the corresponding columns of the independent co-ordinates.

The plotting is done from the numbers in the columns of the independent co-ordinates, and should present no difficulty.

Methods of Dealing with Closing Errors.—It is seldom that a compass traverse will close accurately on its starting point. This may be due either to faulty chain work or faulty bearings, or both. If the scale of representation is small, and the closing error so small that it cannot be easily discerned on the plan, it may be disregarded. For example, if the scale is 3 chains to 1 inch, and the accuracy of the plotting ·01 inch, a closing error of 3 or 4 links in the " latitudes " and " departures " may be ignored.

Small inaccuracies may be allowed for by a judicious altering of the " latitudes " and " departures," the governing idea being to close the traverse with the least possible alteration of figure, as given by the field bookings. In other cases some assumption must be made as to the relative accuracy of the compass and chain work.

When the bearings and chain work are considered to be equally in error, we use the method due to Bowditch (given in a treatise on the " Adjustment of Observations," by T. W. Wright, B.A.), and correct each " latitude " and " departure," as follows :—

Correction in " latitude " or " departure "

$$= \frac{\text{length of line}}{\text{sum of lines}} \times \text{whole error in " latitude " or " departure."} \quad \textbf{(1)}$$

This method of correcting the positions of the plotted points alters both the lengths and bearings of the lines. If it is neces- sary to make the bookings agree with the plan, the new lengths and bearings may either be measured on the plan or calculated by the following rules :—

New length of line

$$= \sqrt{(\text{corrected " dep."})^2 + (\text{corrected " lat."})^2}. \quad \textbf{(2)}$$

The corrected reduced bearing (θ) is given by

$$\text{Tan } \theta = \frac{\text{corrected " departure "}}{\text{corrected " latitude "}} \quad \textbf{(3)}$$

$$= \frac{\text{corrected " Easting " or " Westing "}}{\text{corrected " Northing " or " Southing "}}.$$

I.	II.	III	IV	V.	VI.
Line.	Fore-bearing.	Back-bearing.	Reduced bearing.	Distance (Links).	Included Angles.
A p	86° 50'	267° 0'	N. 86° 55' E.	87	X A p, 100° 35'
p q	126° 20'	306° 15'	S. 53° 42·5' E.	208	A p q, 140° 37·5'
q r	136° 10'	316° 5'	S. 43° 52·5' E.	127	p q r, 170° 10'
r s	164° 30'	344° 30'	S. 15° 30' E.	194	q r s, 151° 37·5'
s t	198° 30'	18° 30'	S. 18° 30' W.	351	r s t, 146° 00'
t u	173° 30'	353° 0'	S. 6° 45' E.	383	s t u, 205° 15'
u v	140° 20'	320° 0'	S. 39° 50' E.	237	t u v, 213° 5'
v w	179° 50'	359° 40'	S. 0° 15' E.	227	u v w, 139° 55'
w x'	233° 20'	53° 20'	S. 53° 20' W.	226	v w x', 126° 25'
x' y	270° 30'	90° 30'	N. 89° 30' W.	402	w x' y, 143° 50'
y X	255° 20'	75° 30'	S. 75° 25' W.	135	x' y X, 194° 5'
X A	7° 30'	187° 30'	N. 7° 30' E.	1,707	y X A, 67° 55'
			Sums, . .	4,284	1,799° 30'
					Error in lat., . . .
A p	86° 55'	266° 55'	N. 86° 55' E.	87	X A p, 100° 35'
p q	126° 18'	306° 18'	S. 53° 42' E.	210	A p q, 140° 37'
q r	135° 57'	315° 57'	S. 44° 3' E	127	p q r, 170° 21'
r s	164° 15'	344° 15'	S. 15° 45' E.	195	q r s, 151° 42'
s t	198° 14'	18° 14'	S. 18° 14' W.	352	r s t, 146° 1'
t u	173° 5'	353° 5'	S. 6° 55' E.	384	s t u, 205° 9'
u v	140° 6'	320° 6'	S. 39° 54' E.	238	t u v, 212° 59'
v w	179° 30'	359° 30'	S. 0° 30' E.	228	u v w, 140° 36'
w x'	232° 55'	52° 55'	S. 52° 55' W.	226	v w x', 126° 35'
x' y	270° 18'	90° 18'	N. 89° 42' W.	401	w x' y, 143° 13'
y X	255° 24'	75° 24'	S. 75° 24' W.	135	x' y X, 194° 16'
X A	7° 30'	187° 30'	N. 7° 30' E.	1,707	y X A, 67° 55'
			Sums, . .	4,290	1,779° 59'
					Diffs., ...
					Error in lat., ...

A.

VII		VIII.		IX.		X.	
Consecutive Co-ordinates.				Independent Co-ordinates.			
Lat.		Dep		Lat.		Dep.	
N.	S.	E.	W.	N.	S.	E.	W.
5	..	87	..	5	..	87	..
..	123	168	..	..	118	255	..
..	91	88	..	..	209	343	..
..	187	52	..	..	396	395	..
..	333	..	111	..	729	284	..
..	380	45	..	..	1,109	329	..
..	182	152	..	..	1,291	481	..
..	227	1	..	..	1,518	482	..
..	135	..	181	..	1,653	301	..
3	..	..	402	..	1,650	..	101
..	34	..	131	..	1,684	..	232
1,692	..	223	..	8	..	..	9
1,700	1,692	816	825				
1,692			816				
8		Error in dep.,	9				
5	..	87	..	5	..	87	..
..	124	169	..	..	119	256	..
..	91	88	..	..	210	344	..
..	188	53	..	..	398	397	..
..	334	..	110	..	732	287	..
..	381	46	..	..	1,113	333	..
..	183	153	..	..	1,296	485	..
..	228	2	..	..	1,524	488	..
..	136	..	180	..	1,660	308	..
2	..	..	401	..	1,658	..	93
..	34	..	131	..	1,692	..	224
1,692	..	223	..	0	0	0	1
7	1,699	598	822				
	7		598				
	1,692		224				
	1,692		223				
	0	Error in dep.,	1				

As an example of this method, we will take the compass traverse to fix the position of the centre line of the river embankment in the Wigglesworth Hall Survey, the record of the traverse is given in the field notes.

The method of correcting the traverse is shown in the foregoing table.

On examining the table, it will be observed that the closing error in latitude is 8 links North, and in departure 9 links West. The positions of the points A and X are fixed by the general chain work of the survey, and as the bearing of the line A X was obtained with a 5-inch theodolite, the remaining bearings being taken with a prismatic compass, it is assumed that both the length and bearing of the line A X are correct. Hence the traverse is considered to close on the point X, and not on the point A.

The latitudes and departures shown in the lower part of the table are obtained by applying rule (1). The sum of the lines (omitting the line X A) is 2,577 links, hence the correction to be applied to the latitude of A $p = \dfrac{87 \times 8}{2,577} = \cdot 27$ link, and the correction in departure $= \dfrac{87 \times 9}{2,577} = \cdot 306$ link.

As both these corrections are less than half a link, they have been ignored.

The correction in latitude of $p\,q = \dfrac{208 \times 8}{2,577}$

$$= \cdot 646 \text{ link,}$$

and the correction in departure $\quad = \dfrac{208 \times 9}{2,577}$

$$= \cdot 725 \text{ link.}$$

Both these corrections, being greater than half a link, 1 link has been *added* to both the latitude and departure of $p\,q$.

In a similar way, the latitudes and departures of the remaining lines have been corrected, care being taken to *add* the corrections to the Southings and Eastings, and to *subtract* them from the " Northings " and " Westings," since the latter are in excess.

The final closing error obtained in this way is zero in " latitude " and 1 link in " departure." As 1 link cannot be shown in plotting this error in " departure " is ignored.

In the lower part of the table the bearings, distances, and included angles are also shown corrected.

The corrected distances and bearings have been obtained by applying rules (2) and (3). Thus, in the case of the line $p\,q$, the corrected distance $= \sqrt{124^2 + 169^2} = 210$ links.

The corrected reduced bearing is given by

$$\text{Tan } \theta = \frac{169}{124}$$
$$= \tan 53° 42',$$

∴ the corrected reduced bearing of $p\,q = $ S. 53° 42′ E.

The corrected fore-bearing $= 180° - 53° 42'$
$= 126° 18',$

and the corrected back-bearing $= 180° + 126° 18'$
$= 306° 18'.$

It rarely happens that it is necessary to make the field bookings
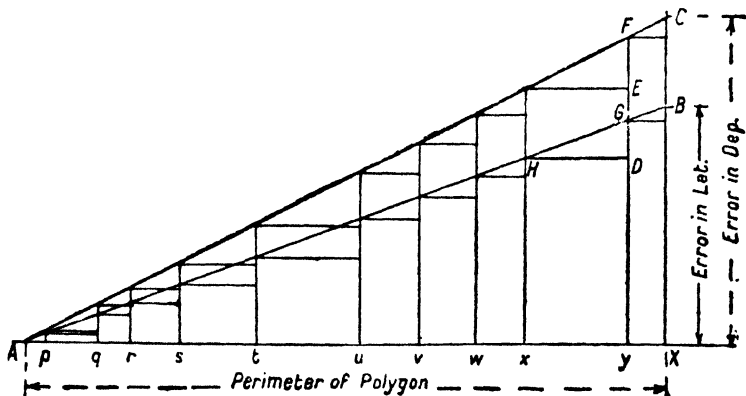
Fig. 50.

agree with the corrected " latitudes " and " departures," hence the labour involved in correcting the figures in Columns II. to VI. inclusive is obviated.

The calculations necessary in correcting the " latitudes " and " departures " may be much simplified by using a slide rule, as a 10-inch slide rule will give results well within the limits of permissible error.

The corrections of " latitude " and " departure," according to Bowditch's method, may also be obtained from a diagram constructed as follows :—Lay off along a base line the sides of the figure to any convenient scale (see Fig. 50), at one extremity

of the base line erect a perpendicular and mark on this perpendicular (to a large scale) the errors in " latitude " and " departure." Join the points thus obtained to the other extremity of the base. At each point of division of the base line erect a perpendicular to cut the hypothenuses of the triangles, and through the points of intersection draw horizontal lines to cut the next perpendicular in order. The required correction in " latitude " or " departure " at any station is given by the intercept on the perpendicular drawn through the point corresponding to that station. Thus the correction in " latitude " at station y is given by the intercept D G, and equals 1·3 links, similarly the correction in departure equals E F or 1·45 links.

The construction is based on the fact that all the triangles under each hypothenuse are similar, hence, if we consider any small triangle, such as D G H (Fig. 50), we have :—

$$\frac{D\,G}{D\,H} = \frac{X\,B}{X\,A},$$

$$\therefore D\,G = \frac{D\,H}{X\,A} \cdot X\,B$$

$$= \frac{x\,y}{X\,A} \cdot X\,B.$$

Correction in " latitude " $= \dfrac{\text{length of line}}{\text{sum of lines}} \times \begin{array}{c}\text{whole error in}\\ \text{" latitude."}\end{array}$

The same reasoning applies to the correction in " departure."

Correcting the Traverse Polygon when the Errors are assumed to be due to Chaining only.—As we have already seen, Bowditch's method of correcting a traverse alters both the bearings and lengths of the sides. In some cases (where the bearings have been obtained by a reliable observer using a good theodolite, or where the ground is very irregular in contour) the closing error may be considered as due to chaining alone.

Axis Method of Adjustment.—In a case of this kind the axis method of adjustment, in which the lengths of the sides only are altered, is the best for the purpose.

Suppose the diagram in Fig. 51 to represent a traverse, the closing error of which is shown by A A'. Join A' A, and produce the line to cut the polygon in some point H. Bisect A A' at the point G. Consider the polygon to be divided into two parts at the point H, and commencing at H, replot the figure, increasing the lengths of the sides H C, C B, B A (without altering the

angles) in the ratio of H G to H A, the point A will coincide
with G. Similarly, if we replot the sides H D, D E, E F, F A',
diminishing each in the ratio of H G to H A'. the point A' will
coincide with G, and the polygon will close.

The effect on the polygon in Fig. 51 is shown by the dotted lines.
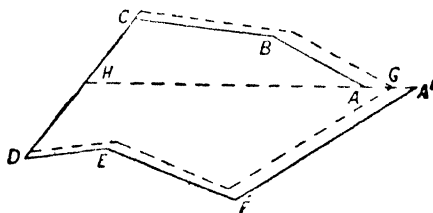
Fig. 51.

Obviously, it may happen that the line A A' when produced
will not cut the polygon at all, or it may cut it so that the axis
of correction H A is very short, necessitating a large percentage
of correction. The axis of correction need not, however, be in
the line A A' produced, but it must be parallel to it.

When applying this principle to a traverse plotted with the
scale and protractor, measure from the plan the error in " lati-
tude " and " departure." In a convenient position on the paper,
draw two lines meeting at B (Fig. 52) parallel and at right angles
to the meridian On these lines lay off—to a larger scale—the
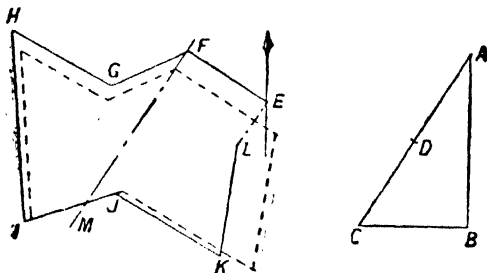
Fig. 52.

error in latitude B A and departure B C. Join A C and bisect
the line at D. The line A C is parallel to the axes of closure and
correction, and A D or D C is the closing correction.

With the parallel ruler or set squares, draw the axis of correc
tion in position, parallel to A C, and cutting the traverse polygon

in the points F and M, so placed that F M is the longest line which can be drawn parallel to A C across the polygon. To close the polygon measure F M and A D, shorten the sides M I, I H, H G, and G F, in the ratio of F M/(F M + A D), and lengthen the sides M J, J K, K L, and F E in the ratio of (F M + 2 A D)/(F M + A D).

In Fig. 52 (the polygon has been drawn intentionally with a large percentage of correction), A D equals ·27, and F M 1·92 units. The sides M I, I H, H G, and G F have been shortened in the ratio of 1·92/2·19, and the sides M J, J K, K L, F E lengthened in the ratio 2·46/2·19. The effect of this is shown in the figure by the dotted lines.

Application of Axis Method to " Latitudes " and " Departures." —As this method of correction alters the lengths of the lines only, the " latitudes " and " departures " will be proportionate to the corresponding distances. Hence, in the case of a traverse to be plotted by co-ordinates, the lengths of the axis of correction and half the axis of closure are obtained as already described, and the " latitudes " and " departures " are corrected in the same way that we should treat the sides.

Line.	Consecutive Co-ordinates.				Independent Co-ordinates.			
	Lat.		Dep.		Lat.		Dep.	
	N.	S.	E.	W.	N.	S.	E.	W.
A p	5·04	..	87·6	..	5·04	..	87·6	..
p q	..	123·8	169·0	..	..	118·76	256·6	..
q r	..	91·6	88·6	..	..	210·36	345·2	..
r s	..	188·2	52·3	..	..	398·56	397·5	..
s t	..	335·0	..	111·8	..	733·56	285·7	..
t u	..	382·5	45·3	..	..	1116·06	331·0	..
u v	..	183·2	153·0	..	..	1219·26	484·0	..
v w	..	228·5	1·0	..	..	1527·76	485·0	..
w x'	..	134·0	..	179·5	..	1661·76	305·5	..
x'y	2·98	..	..	399·0	..	1658·78	..	93·5
y X	..	33·7	..	130·0	..	1692·48	..	223·5
X A	1692·0	..	223·0	..	..	·48	..	·5
Sums,	1700·02	1700·5	819·8	820·3				
		1700·02		819·8				
Diffs.,		·48		·5				

The length of the axis of correction of the traverse given in Table A, as obtained from an uncorrected plan, is 814 links, and half the axis of closure is 6·02. The co-ordinates of the points p to u are shown in the foregoing table increased in the ratio of 826/820, and the co-ordinates of the points u to X decreased in the ratio of 814/820. The effect on the traverse lines will be seen by comparing the corrected co-ordinates in Table A with those given in the table opposite, in which the essential parts only are repeated.

Adjustment by Weighting.—It is improbable that all the traverse measurements will have been made under conditions equally conducive to accuracy. The conditions under which the measurements were made having been noted, the lines which are likely to be most in error will be known, and an estimate formed of the probable contribution of each line to the total error. This estimate is expressed by attaching a number (called its " weight ") to each line, the numbers indicating in the surveyor's opinion the relative trustworthiness of the several lines. This operation is known as " weighting " the lines. It is usual to give unit weight to the line or lines considered to be least in error, the weights of the others being assigned by comparison. The numbers 1, 2 and 3 only are employed, it being understood that a line with a weight of 3 is considered to have contributed 3 times as much error to the total as a line whose weight is unity.

In compass traverses the estimation of the " weight " must include both bearing and length of each line.

In applying this system to the method of Bowditch, the closing errors are obtained as described in the preceding pages, after which the corrections to be applied to the latitude and departure of each line are obtained from the rule :—

$$\frac{\text{Weighted length of line}}{\text{Sum of weighted lines}} \times \text{Whole error in lat. (or dep.)}.$$

The weighted length of each line is obtained by multiplying the length of the line by its " weight."

Let the weights of the several lines in the traverse given in Table A be 3, 3, 2, 3, 2, 1, 2, 2, 3, 3, 3, 1. Multiplying the length of each line by its " weight " and summing the products we find the sum of the weighted lines to be 7,730, and applying the above rule, the correction to (say) pq is

$$\frac{3 \times 208}{7,730} \times 8 = 0{\cdot}65 \text{ link in lat.}$$

and in departure is

$$\frac{3 \times 208}{7,730} \times 9 = 0 \cdot 73 \text{ link.}$$

In the case of theodolite traverses, each consecutive latitude, or departure, is multiplied by the weight of the corresponding line and the correction in latitude or departure of any line is then given by :—

$$\frac{\text{Weighted lat. (or dep.) of the line}}{\text{Arithmetical sum of the weighted lats. (or deps.)}} \times \frac{\text{Whole error in}}{\text{lat. (or dep.).}}$$

The sum of the weighted latitudes and departures is obtained without reference to sign.

If we assume the traverse in Table A to be a theodolite traverse, and weight the lines as in the preceding section, the sum of the weighted latitudes is 5,199 and of the weighted departures 4,035, and the correction to (say) pq will be

$$\frac{3 \times 123}{5,199} \times 8 = 0 \cdot 57 \text{ link in lat.,}$$

and

$$\frac{3 \times 168}{4,035} \times 9 = 1 \cdot 1 \text{ link in dep.}$$

The co-ordinates of the other lines are treated in the same way. In applying the corrections we add or subtract each correction according as the co-ordinate dealt with is in defect or excess.

Missing Quantities.—As we have seen, when there is no error of closure, the algebraic sum of the latitudes and of the departures is zero, *i.e.*,

$$l \cos \alpha + l_1 \cos \alpha_1 + l_2 \cos \alpha_2 + \text{etc.} = 0 \quad . \quad . \quad (\beta)$$
$$l \sin \alpha + l_1 \sin \alpha_1 + l_2 \sin \alpha_2 + \text{etc.} = 0 \quad . \quad . \quad (\varphi)$$

where l, l_1, etc., are the lengths of the traverse lines and $\alpha, \alpha_1, \alpha_2$, etc., their respective bearings. Hence if it has not been possible to obtain all the data necessary to close the traverse, the missing quantities may be found, provided they are not more than two in number. The missing quantities may comprise (1) the length of one side ; (2) the bearing of one side ; (3) the lengths of two sides ; (4) two bearings ; (5) the length and bearing of the same side ; (6) the length of one side and the bearing of another.

Let L and D be equal in magnitude but opposite in sign to the

respective algebraical sums of the latitudes and departures of the known quantities. In case (1) the required length is

$$\sqrt{L^2 + D^2}$$

and in case (2) the unknown bearing (θ) will be given by the equation

$$\tan \theta = \frac{D}{L}$$

In the other cases the required quantities will be obtained by the simultaneous solution of the equations (β) and (φ). In case (4) it will be necessary to know the approximate bearings of the lines in order to decide in which quadrants they lie, and in case (6) the simplest solution is as follows :—

Let l^1 and δ be the unknown quantities ; the general equations (β) and (φ) will finally reduce to

$$l \cos \delta + l^1 \cos \alpha = L \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$\text{and} \quad l \sin \delta + l^1 \sin \alpha = D . \quad . \quad . \quad . \quad . \quad (2)$$

Multiplying equation (1) by $\sin \alpha$ and equation (2) by $\cos \alpha$ we get :—

$$l \cos \delta \sin \alpha + l^1 \cos \alpha \sin \alpha = L \sin \alpha$$
$$\text{and} \quad l \sin \delta \cos \alpha + l^1 \sin \alpha \cos \alpha = D \cos \alpha$$

Hence $l (\sin \alpha \cos \delta - \cos \alpha \sin \delta) = L \sin \alpha - D \cos \alpha$

That is, $l \sin (\alpha - \delta) = L \sin \alpha - D \cos \alpha$

or $$\alpha - \delta = \sin^{-1} \left(\frac{L \sin \alpha - D \cos \alpha}{l} \right)$$

Hence $$\delta = \alpha - \sin^{-1} \left(\frac{L \sin \alpha - D \cos \alpha}{l} \right)$$

δ is now known, and on substituting its value in either equation (1) or (2) the required length may be computed.

In solving problems dealing with missing quantities the student is recommended to lay down the traverse from the known data approximately to scale ; the unknown quantities will thus appear as gaps in the traverse polygon, from which the approximate lengths and bearings may be inferred, thus assisting in the solution of the problem.

In cases (1) and (2) the solution supplies an approximate check on the accuracy of the traverse ; in the other cases, since the solution is dependent on the whole of the measured quantities, and as these contain the errors propagated throughout the

traverse, these errors are brought into the solution of the missing data. The Surveyor is, therefore, not justified in omitting measurements to save field work. When it is known that certain quantities cannot be obtained in the field, the greatest care should be taken to ensure that the remaining data are accurate ; wherever possible cross bearings (Fig. 172) and the bearings of referring objects outside the course of the traverse (Fig. 173) should be obtained so that the accuracy of the measured quantities is fully assured.

Plotting with the Protractor.—The best form of protractor for this purpose consists of a ring of cardboard or horn-paper, the inner or outer edge of which is accurately divided into degrees and half-degrees, the divisions being numbered clockwise from zero. The diameter of the divided circular edge should be 10 or 12 inches.

In plotting a compass traverse with this instrument, the direction of the magnetic meridian is first laid down by a fine line drawn on the plan, near that part of the paper on which the traverse is to be plotted. A fine pencil line is then drawn at right angles to the meridian line, and the protractor adjusted in position so that the zero and 180° divisions coincide with the south and north ends of the meridian line, and the 90° and 270° divisions coincide with the line drawn at right angles thereto. The centre of the protractor will now coincide with the point of intersection of the two lines, as shown in Fig. 53.
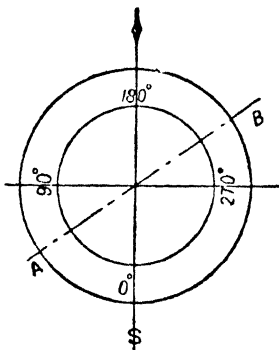
Fig. 53.

Having adjusted the protractor in position, load it with paper weights, and proceed to prick off the fore- and back-bearings of the traverse lines, taking care to mark the points with their appropriate letters. Thus, if the fore-bearing of a line A B is 45° and its back-bearing 225°, the mark at 45° is denoted by "A," and that at 225° by "B" (see Fig. 53).

Remove the protractor when all the bearings have been laid down and check the work by joining each pair of points with a straight edge. If the fore- and back-bearings have been accurately marked off, the straight edge will—in every case—pass through the point which marked the position of the centre of the protractor.

To lay down the traverse, through the point on the plan marking the position of the first station, draw a line parallel to the direction of the first line (A B, Fig. 53), as marked out with the protractor, and scale off A B (in the direction given by the fore-bearing) equal to the length of the line as given in the field book. Through B draw a line parallel to the direction of the second line (B C) as laid down with the protractor, and from it cut off B C equal to its measured length. This process is repeated until all the lines of the traverse have been plotted in position.

It should be noted that the greatest care is necessary in drawing the lines parallel to their directions as given by the protractor, and in scaling off their lengths, since if any line be inaccurately plotted, all the lines which follow it will be thrown out of their true positions.

If the scale of the plan is small, by using the internal protractor and parallel ruler the traverse may be laid down at once from the field records. To do this, the protractor is adjusted, and weighted, in position with the line joining the zero and 180° divisions accurately in the meridian line, the latter being placed so that the vacant circle will include the first station and a considerable part of the traverse.

The parallel ruler is placed on the protractor with its edge cutting the divisions marking the fore- and back-bearings of the first line (Fig. 54). The ruler is then moved until its edge passes through the first station, the line is drawn in position and its length laid off. Through its forward extremity a parallel is drawn to the bearing of the second line, and is cut off to the requisite length. This process is repeated without moving the protractor, so long as the lines fall within it, after which the protractor is set in a new position and the plotting continued.

If in plotting a traverse by " latitudes " and " departures " the position of a station is inaccurately plotted, the error will disturb the positions of two lines only. The error may be detected on the plan by scaling the lengths of the lines meeting at the point, when both lines would show an error in length; but, in plotting with the protractor, an error can only be detected by going over the work a second time. The closing error of a traverse plotted by this method will, in consequence, be made up of errors of draughtsmanship and errors in the field. To allow for these errors, and to cause the traverse to close, proceed as follows :—On any convenient part of the paper draw an indefinite line *a* X (Fig. 55), and mark off *a b, b c, c d*, etc. (to any scale),

equal to A B, B C, C D, etc., the sides of the traverse. At *a'*
draw *a' f equal* to the closing error A A', join *a f*, and erect the
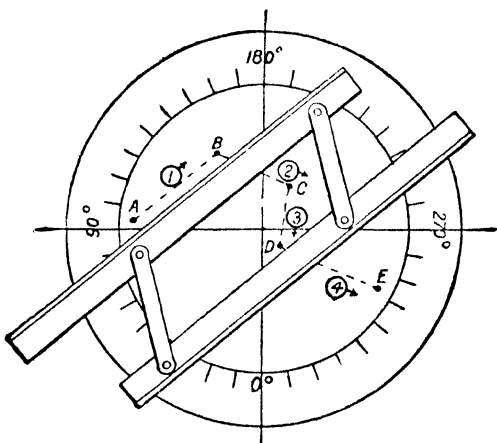
Fig. 54.

perpendiculars *b g*, *c h*, *d i*, etc., meeting *a f* in the points *g*, *h*, *i*,
etc.

Through the points B, C, D, and E of the traverse polygon
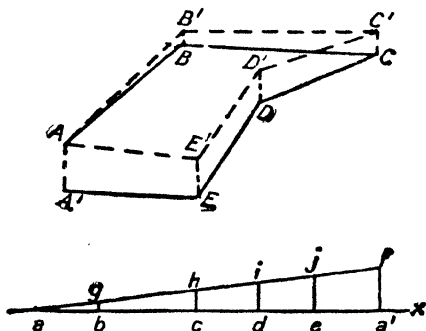
Fig. 55.

draw lines parallel to the axis of closure A A'. From the line
drawn through B, cut off a length B B' equal to *b g*. Similarly,

make C C′, D D′, and E E′ equal in length to *c h*, *d i*, *e j* respectively. The lines joining the points A, B′, C′, D′, and E′ represent the positions of the corrected traverse lines.

This method of correcting the traverse alters both the lengths and bearings of the lines forming it. If it is desired to make the field record agree with the plotting, the new lengths and bearings may be determined by measurement on the plan.

It should be noted that, if the bearings have been obtained with a theodolite, the protractor should be set on the paper with its zero at the north and not at the south end of the meridian line. With this exception, the plotting is carried out precisely as in the case of an ordinary compass traverse.

EXAMPLES.

1. The main lines of a closed traverse form a rectangle A B C D. If the bearing of A B is 30°, find the whole circle and reduced bearings of the remaining sides. Also, if A B is 1,000 feet and B C 500 feet long, determine the latitude and departure of the station D relative to A. C is north of B.

	(*Ans.* Whole circle Bearings.	Red. Bearings.
	B C, 300°	N. 60° W.
	C D, 210°	S. 30° W.
	D A, 120°	S. 60° E.
	Lat. of D = 250′ N.	
	Dep. of D = 433′ W.)	

2. Surface and underground traverses have been run between two mine shafts A and B. The co-ordinates of A and B as given by the underground traverse are 8,560 N., 24,860 W., and 10,451 N., 30,624 W., respectively. The surface traverse gave the co-ordinates of B as 10,320 N., and 30,415 W., those of A being as before. Assuming the surface traverse to be correct, find the error in both bearing and distance of the line A B, as given by the underground traverse.

(*Victoria University, B.Sc.Tech.*, 1927.)

(*Ans.* 0° 35·5′ ; 238 feet.)

3. Find the co-ordinates of the point at which a line run from

A on a bearing of N. 10° E. will cut the given traverse and the length of this line :—

TOTAL CO-ORDINATES (IN FEET).

Line.	Lat.		Dep.	
	N.	S.	E.	W.
A B	1650	...	440	...
B C	2875	...	120	...
C D	3643	...	...	220
D E	...	1450	...	376
E A	0	0	0	0

(*Victoria University, B.Sc.Tech.*, 1932.)

(*Ans.* 1,991 N., 351·1 E. ; 2,021 feet.)

4. The bookings of part of a traverse are as follows :—

Line.	Reduced Bearing.	Distance (feet).
A B	N. 10ᶜ E.	4000
B C	N. 7° W.	3000
C D	N. 4ᐤ E.	1000
D E	N. 12ᐤ W.	700

The bearings of an inaccessible point F were taken at A and E and found to be N. 40° W. and S. 30° W. respectively. Compute the distance from F to C.

(*Victoria University, B.Sc.Tech.*, 1930.)   (*Ans.* 4,756 feet.)

5. It is proposed to connect a point A in an underground gallery with the surface by means of a vertical shaft. The traverse from A to the shaft (C) of the mine, and from the shaft to a point D on the surface is given below. You are required to find the length and bearing of the line from D to A[1], the point A[1] being the point on the surface where sinking must commence.

Line.	Bearing.	Distance (feet).
A B	360°	500
B C	45°	1,500
C D	210°	900

(*Ans.* Bearing of D A[1] = S. 38° W. ; length, 991·2′.)

6. From the following bearings and distances find the length of D E so that A E and F may be in the same straight line :—

Ine.	Bearing.	Distance.
B	95° 30′	1.000 ft.
C	75° 18′	500 ft.
D	18° 45′	400 ft.
E	330° 15′	...
F	64° 10′	750 ft.

(*Victoria University, B.Sc.Tech.*, 1929.)   (*Ans.* 332·4 feet.)

7. The position of a station C in a line A B cannot be fixed from either A or B owing to the nature of the ground ; but there is no difficulty in chaining its distance from an off station D. To fix the point C the traverse A B D was worked out on the ground and the data given in the following table obtained. From these data find the lengths of A B, A C, and D C.

Line.	Bearing.	Distance (feet).	Lat. N.	Lat. S.	Dep. E.	Dep. W.
A B	53° 31′	...	...	...	...	...
B D	159° 40′	450	...	422	156·3	...
D A	279° 40′	600	100·8	...	...	591·4
D C	300° 20′	...	...	...	...	...

Consecutive Co-ordinates.

(*Ans.* 540·8 feet ; 230·4 feet ; 470·6 feet.)

CHAPTER V.

LEVELLING.

Levelling Instruments.

Preliminary Remarks.—As defined on page 1, the term levelling is applied to "the art of ascertaining and representing the relative elevations of different points on the earth's surface, and of the objects upon it."

The relative heights of points on the earth's surface are determined by ascertaining their vertical distances from a surface, all points on which are at the same altitude. Thus, in Fig. 56, if A B represents the surface of the water in a pond, the heights
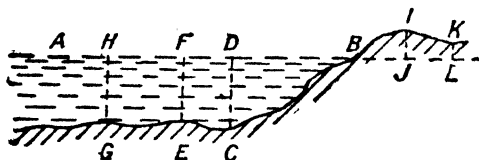
Fig. 56.

of the points G, E, C, B, I, and K relative to each other are known when their vertical distances G H, E F, C D, etc., from A B have been determined.

Datum Surface and Datum Point.—A surface of reference such as A B is termed a *datum surface*, and any point in it, such as the point B, used for reference, is called a *datum point*.

The surface of reference is sometimes called a level surface, and it may be defined as "a surface such that a plane tangential to it at any point is parallel to an accurately set up spirit level at the point, or is at right angles to a plumb line suspended at the point." Hence, a level surface is not a plane, but a spheroidal surface. The surface of a still lake is a level surface, so also is the surface passing through the mean level of the sea at various places on the earth.

As a plumb line always comes to rest in the direction of the resultant force acting on the plumb-bob, in approaching, or receding from, a large mountain mass, the plumb line would be slightly deflected from its normal position towards the mountain. Hence, under such conditions the level surface at right angles to the plumb line would rise on approaching and fall when receding from the mountain mass.

If a water pipe were laid through the mountain, perfectly level, its centre line would appear quite straight on the section, but in reality it would be a curved line, the highest point on the curve being vertically under (or over) the centre of mass of the mountain. The flow of water through the pipe would not, however, be interfered with in the slightest degree, since in a hydrostatic sense the pipe would be everywhere level.

In actual levelling work the line of sight of the instrument used is—by suitable means—placed horizontal, and is consequently a tangent to the level surface passing through it.

Line of Collimation.—This line of sight is commonly called the line of collimation of the instrument.

Simple and Compound Levelling.—If the object of the levelling operations is to determine the difference of altitude (commonly called the "difference of level") of two points A and B (Fig. 57), and the points are so conveniently placed that this difference may be

Fig. 57.

obtained by *one* setting of the instrument (*i.e.*, by using one line of collimation only), the operation is spoken of as "*simple levelling.*"
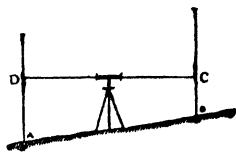
If A D and B C are the readings on graduated staves held at A and B, obviously the difference of level of A and B is equal to A D — B C.

It is seldom, however, that the operations can be completed without moving the instrument into one or more new positions, and consequently several lines of collimation must be used. Levelling work of this kind is termed "*compound levelling.*"

In the older forms of levelling instruments the line of collimation was placed horizontal by taking advantage of the fact that the free surface of water at rest is horizontal.

Water Level.—This principle is still made use of in the case of the instrument known as the water level, one form of which is shown in Fig. 58.

This instrument consists essentially of two open glass vessels

7

A and B, whose lower extremities are joined by a metal tube C. The tube is connected to the head of a supporting tripod, or staff, by a ball and socket joint, so that it may be set roughly horizontal. On pouring water into one of the vessels, it will flow along the connecting tube and rise in the other. When the fluid is at rest the free surface of the water will stand at the same height in both vessels, and consequently the line (of collimation) joining these surfaces will be horizontal. The water used in the instrument is generally coloured in order that the free surface of the fluid may be more easily seen.

In using the instrument, the eye is placed in the plane containing the surface of the liquid in the vessels and the edge of a sliding vane—carried on a graduated staff held by an assistant —is sighted. The observer by signals causes his assistant to elevate or depress the vane until its edge is cut by the line of
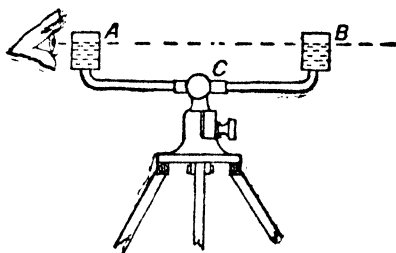
Fig. 58.

sight. The reading on the staff is then observed by the assistant and recorded in a suitable manner.

It is obvious that the water level cannot give very accurate readings, since the sighting base is short compared with the distance of the staff. Further, the observer is at the mercy of his assistant in taking the staff readings, as in most cases he is not able to read the staff himself. This instrument is now rarely used in ordinary levelling work ; its use is chiefly restricted to rough determination of contours, for which it is well adapted, but the principle underlying the instrument is often made use of in mining work.

Spirit Level.—All the chief forms of modern levelling instruments are provided with a telescope, to which is attached a spirit level.

The axis of the telescope forms the line of collimation, and is placed in a horizontal position by reference to the spirit level.

The spirit level consists of a sealed cylindrical glass tube nearly filled with some mobile liquid, such as ether or alcohol. The axis of the tube is bent to an arc of a circle, in order to reduce the sensitiveness of the level. The space in the tube not filled with liquid is commonly called the " bubble," which always occupies the highest part of the tube. The greater the radius to which the level tube is bent, the greater will be the displacement of the bubble for a given change of inclination of the level tube.

To protect the glass tube from injury it is mounted in a brass tube, the upper portion of which is cut away near the centre, so that the bubble may be seen. In all cases the spirit level tube is mounted so that the plane containing its axis is vertical, and the plane tangent to the vertex of the curve to which the tube is bent is parallel to some standard line or plane of the instrument. In instruments provided with a telescope the tangent plane is parallel to the axis of the telescope; in a builder's level it is parallel to the base of the wooden block in which the
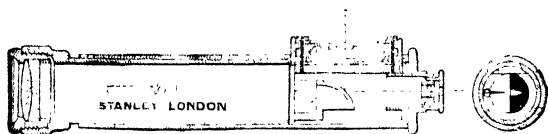
Fig. 59.

tube is mounted. Since the tangent plane is tangent to the tube at its highest point, it follows that, when the centre of the bubble coincides with this point, the tangent plane will be horizontal, and the standard line or plane on the instrument will also be horizontal.

Hand Level.—The simplest form of instrument provided with a telescope used in levelling is the hand level.

The instrument consists of a small telescope (Fig. 59), to which is attached a small and not very sensitive spirit level. The brass tube in which the spirit level is mounted is cut away at the centre on both its upper and lower surfaces. Immediately below the centre of the spirit level a slot is cut in the telescope tube, vertically over a small curved mirror. This mirror occupies one-half the width of the tube. A horizontal sight wire is placed in the axis of the telescope at the common focus of the eye lens and the object glass.

The instrument when in use is held in the hand and sighted

towards the staff held by an assistant, and the forward end of the telescope is elevated or depressed until the image of the bubble as seen in the mirror is bisected by the cross wire. The required reading is then obtained by noting where the cross wire appears to cut the staff.

Though not very accurate, this instrument is useful in obtaining short cross-sections, and sections on steep ground.

Dumpy and " Y " Levels.—The instruments used for accurate spirit levelling fall into two main types—(*a*) " Dumpy " levels, and (*b*) " Y " levels—the former receives its name from its compact shape, and the latter from the shape of the bearings of the telescope tube. In the Dumpy level type the telescope is fixed in position, and in the " Y " level it is loose in its bearings. The Dumpy level is the more popular form of the two, as it contains fewer loose parts, and is less liable to get out of adjustment.

Tripods.—All instruments for accurate levelling work are supported on tripods when in use. These tripods are generally constructed with solid legs, made of mahogany, or in some cases where lightness is desired, they are made of yellow pine. The legs when closed form a frustrum of a cone, tapering from the foot towards the top (or " head ") of the tripod, the cross-section of each leg being a sector of a circle. When not in use the tripod is kept closed by a brass ring, which is slipped over its foot, and is pushed upwards until it binds the legs firmly together. To prevent the tripod slipping, the foot of each leg is turned to a point, and is strengthened by a flat, pointed steel plate, fixed by screws to its inner surface.

The " head " of the tripod consists of a brass casting circular in plan. The upper portion of the head is provided with an external screw to fit the corresponding internal screw or the lower part of the instrument. The external screw is protected from injury by a brass cap, which is also provided with a corresponding internal screw, the cap being screwed into place when the tripod is not in use. Below the external screw is a flange about 3 inches in diameter, which carries on its under surface three pairs of projecting lugs. Firmly fixed to the top of each tripod leg is a brass fitting, which ends in a projecting tongue. This tongue accurately fits between a pair of the lugs on the tripod head, and is united with them by a screw bolt, thus forming a hinge joint.

It is absolutely necessary that there should be no slackness of fitting in the joint between the head of the tripod and the leg, since any looseness in the joints will disturb the position of the level and lead to inaccurate readings.

Many tripods are now built with framed legs, which have the advantage of being lighter in weight than those with solid legs, possessing an equal degree of rigidity. They do not, however, close into such a compact form, and hence are not so convenient to carry about. The solid leg tripod is the form which is most generally used by British surveyors.

Four-Screw and Three-Screw Levels.—The internal screw already referred to—by means of which the level is fixed to the tripod—is cut either in a circular plate attached to the base of the instrument or in a separate plate having three projecting arms. In the former type, the circular plate is the lower of a pair of plates (commonly called the parallel plates, see Fig. 60), which are united by a ball and socket joint, the distance between the plates being regulated by four screws—the levelling screws
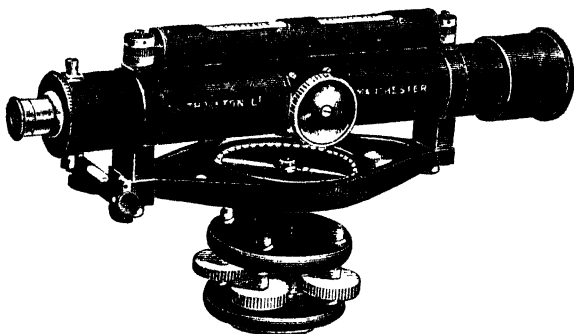
Fig. 60.

of the instrument. The ball of the ball and socket joint is attached to the lower end of the vertical axis.

In the second type, the hollow boss through which the axis of the instrument passes is provided with three arms, each carrying a levelling screw, in line with the centre of the (tribrach) plate carrying the internal thread. The lower ends of the levelling screws are mushroom-shaped, and pass through the enlarged ends of key-hole-shaped slots cut in a swivelling plate fitted to the top of the tribrach plate. When the instrument has been placed in position the swivelling plate is pushed back so that the narrow part of the slots engage with the shoulders on the ends of the levelling screws, thus firmly uniting the level to the tripod. The swivelling plate is prevented from working back by tightening a locking screw. In the instrument shown in

Fig. 61 the locking plate is dispensed with, the levelling screws being inserted from the ends of the arms of the tribrach plate.

The three-screw type possesses the following advantages :—

(a) In levelling the instrument the screws are not antagonistic, hence they are always easy to turn, which is a great convenience, especially when using the instrument in cold weather.

(b) For the same reason, the central axis cannot be strained by screwing up too tightly.

(c) The instrument can be levelled with one hand.
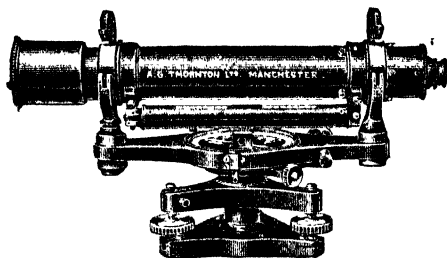
Fig. 61.

The Telescope.—Fixed to the upper extremity of the vertical axis of the instrument is a horizontal bar, to which the telescope is attached. In the dumpy level the outer telescope tube is soldered to two blocks of brass, which are fixed to the bar by screws. The telescope tube of the Y level is encircled by two shouldered brass collars (see Fig. 61), which are accurately ground to the same diameter. These collars rest in Y-shaped bearings, one of which is fitted at each extremity of the horizontal bar, the Y bearings being made adjustable for height by screws and lock-nuts. The telescope is kept in its bearings by means of stirrups hinged on one of the branches of each Y support, and fixed to the other by a pin joint, the pin being easily removable. A flat spring fixed to the concave side of the stirrup presses on the upper part of the bearing collar, and ensures an equal bearing pressure at the two bearings. The telescope may be rotated about its own axis in the Y bearings, but longitudinal movement is prevented by the shoulders on the bearing collars.

At the forward end of the telescope is fitted an object glass about 1¼ to 1½ inches in diameter, consisting of a convexo-concave and a bi-convex lens combination. The object glass is protected from clouding by rain or shielded from the direct rays of the sun

by a short sliding tube (the dew cap), which may be drawn out as required.

At the opposite end of the telescope a magnifying eyepiece is placed. The eyepiece is a short tube fitted with two small plano-convex lenses at its ends, the convex surfaces of the lenses being towards each other. The eyepiece may be adjusted in position—for focussing—by sliding in an outer tube in which it is fitted.

In some telescopes the object glass is fitted to a tube, which slides within the outer telescope tube; in others it is the eyepiece which is fitted in this way. The motion of the inner tube is governed by a pinion and rack, the former being turned as required for focussing the object glass by a milled thumb grip.

The Diaphragm.—When the telescope is accurately focussed on any object, the foci of the object glass and eyepiece coincide in a certain plane, at right angles to the axis of the telescope, and in this plane the sighting webs of the instrument are placed. These webs may be either spider threads, fine lines engraved on a thin glass plate, or very fine iridium points, but in all cases they are carried on the flat end of a cylindrical ring (the diaphragm ring), which is suspended within the telescope tube by two vertical capstan-headed screws in the dumpy level, or by four equidistant screws in the Y level. These screws are seen projecting from the telescope tube near the eyepiece in Figs. 60 and 61.

In the dumpy level the sighting webs consist of a central horizontal wire crossed by two vertical wires placed symmetric-ally; the arrangement in the case of the Y level is a simple cross formed by central horizontal and vertical wires.

Two additional horizontal (stadia) webs are sometimes fitted, in order to read the distance of the levelling staff from the instru-ment.

When properly fitted and the instrument is in careful hands, spider webs are very satisfactory; so also are glass diaphragms, the only objection to their use is that dust or moisture some-times works its way into the tube, and dims the glass. Iridium points are difficult to replace and somewhat troublesome in use, as the staff must be held so that its image appears in contact with the points, before a reading can be obtained.

The Spirit Level.—The main spirit level with its protecting case is fixed to the telescope by adjusting screws (see Figs. 60 and 61) with its axis parallel to the axis of the telescope. A second and much shorter spirit level (known as the cross level)

is fitted at right angles to the main level, either on the telescope
(Fig. 60) or on the horizontal bar.   The cross level is useful
when levelling the instrument.

If required, the instrument may be obtained fitted with a
compass, as shown in Fig. 61, which also shows a clamp and
tangent screw fitted to the vertical axis, by means of which
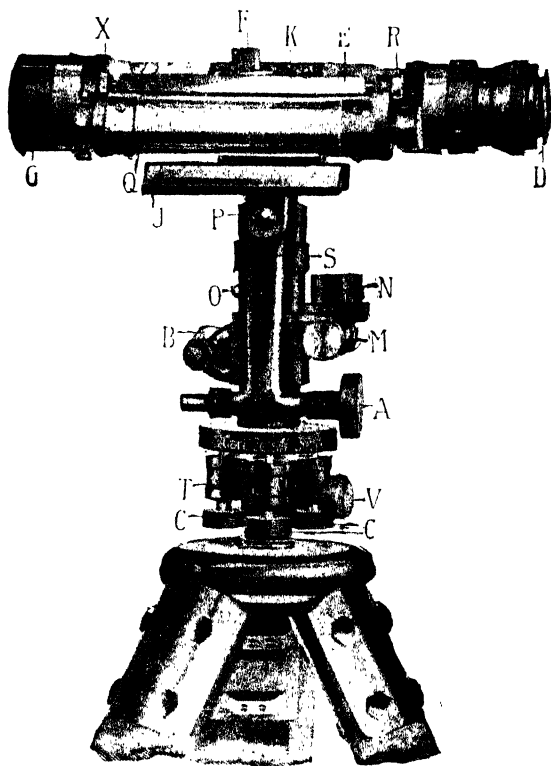
Fig. 62.

the telescope may be accurately set on a distant point.   These
additions are, however, seldom required in practice.

The Zeiss Level.—An instrument possessing several novel
features has lately been introduced by Carl Zeiss, of Jena.

In this instrument the tripod head is provided with a pro-

jecting pin—the stock-head pin—which fits in a cylindrical socket T (Figs. 62 and 63), the level being fixed in position by a clamping screw V. The instrument is levelled primarily by bringing the bubble of the circular level N to the centre, by means of the three levelling screws C. This places the vertical axis of the instrument approximately vertical. The whole of
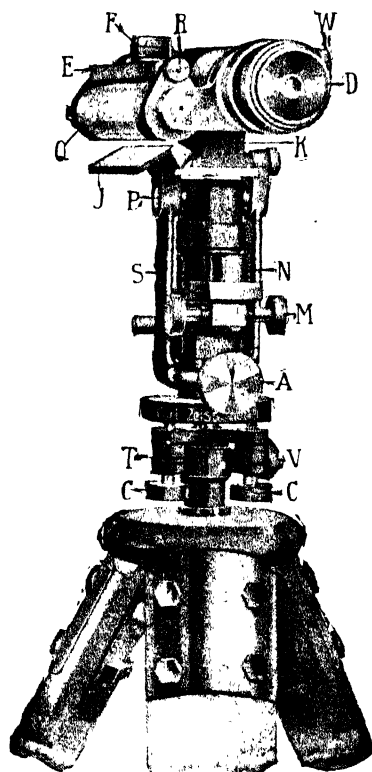
Fig. 63.

the upper part of the instrument may be fixed to the vertical axis by the clamp M and its tangent screw B.

The telescope is attached to a horizontal axis Γ, about which it may be rotated through a limited angle by a forked lever S, whose motion is controlled by an elevation and depression screw

A.   The telescope is connected to the upper extremity of the forke 1 lever either by a short dovetailed bar, or by a screw and lock-nut, and is capable of turning about its own axis (in bearings) within t ie limits fixed by two stops.   A reversible bubble tube * Q is attached to the telescope by adjusting screws.

The telescope tube is of invariable length, the focussing being provided for by the addition of a sliding lens inside the telescope. The motion of this lens is governed by a rack and pinion in the usual way.

The final focussing of the eyepiece on the cross webs is effected with the aid of a screw, the collar of which is graduated in terms of diopters.

The reversible bubble is encased in a glass cylinder, to protect it somewhat from changes of temperature, and its movements are seen through a combination of prisms contained within the casing E.   This combination of prisms forms two tangent images of the bubble at F, which may be viewed from the eyepiece or the objective end of the telescope, illumination of the bubble is provided for by a reflecting mirror J.   The final levelling of the instrument is effected by turning the screw A until the two images of the ends of the bubble are coincident.

When the telescope is turned on its own axis, so that the bubble tube is on the right, the images of the ends of the bubble are still seen in the prism F.

The prism casing E may be moved parallel to the bubble tube by means of the screw R, this movement is provided for the purpose of adjusting the instrument.

The webs (lines on glass) are in the form of a cross, two stadia webs being also provided for the reading of distances.

The instrument is very light, the total weight including tripod and case is only 8½ lbs.

The Levelling Staff.—This is a graduated wooden rod, and is constructed in many forms ; those in most common use are made either on the telescopic principle or, of solid lengths of wood, united by screws.   For general work the telescopic form is the most suitable.

This form of staff consists of a hollow box 5 feet long (forming the bottom length), within which slides another hollow box (the middle length) 4′ 6″ long ; within this a third and solid

* In this kind of bubble tube the axis is straight, and the interior of the tube is accurately ground spindle or barrel shaped ; hence, so long as the axis of the tube is horizontal, the tube may be rotated on its own axis without disturbing the position of the bubble.

length slides, so that when the middle and top lengths are pulled out, the staff is 14 feet long.

When extended, the middle and top lengths are held in position by spring catches.

The staff is made of mahogany, the foot of the bottom length is shod with brass, and the separate lengths are prevented from warping by suitably placed brass clamps.

For underground work, the length of the staff is about 6 feet, closing to 2 feet or 2 feet 6 inches. For engineering purposes the length varies from 10 to 20 feet, the 14-feet staff being the best length for general purposes. The appearance of the staff when closed is shown in Fig. 64.

On the face of the staff is a scale, either printed on paper, glued in position and varnished, or painted in position on the separate lengths and reading continuously from the foot to the top of the extended staff. The divisions of the scale depend on the unit adopted and the purpose for which the levelling operations are intended. In this country the unit is the foot, and this is divided on the staff in inches and eighths, or into tenths and hundredths, the latter being the most common. A few of the more common scales are shown in Fig. 65.

The divisions of the scale are represented on a white ground by black lines, and the spaces between them. On the staff shown in Fig. 64 the feet lengths are marked with red, and the tenths of a foot by black figures. The *top* of a red figure coincides with the number of feet denoted by the figure as measured from the foot of the staff. To avoid overcrowding the black figures, alternate tenths of a foot only are figured, and *the length of each black figure is made one-tenth of a foot*. It is a good plan to have small red figures denoting feet placed at the left-hand side of the scale, and repeated every two or three-tenths of a foot, as these figures facilitate the reading of the staff when it is near the telescope.
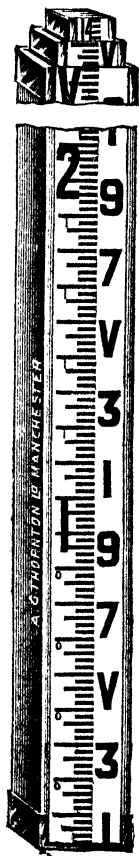
Fig. 64

Reading the Staff.—In reading the staff the beginner must remember (1) the black divisions on the scale mark the even and the white divisions the odd hundredths; (2) the length of each black figure is one-tenth of a foot, the top of the figure coincides in position with the tenth denoted by the figure, and
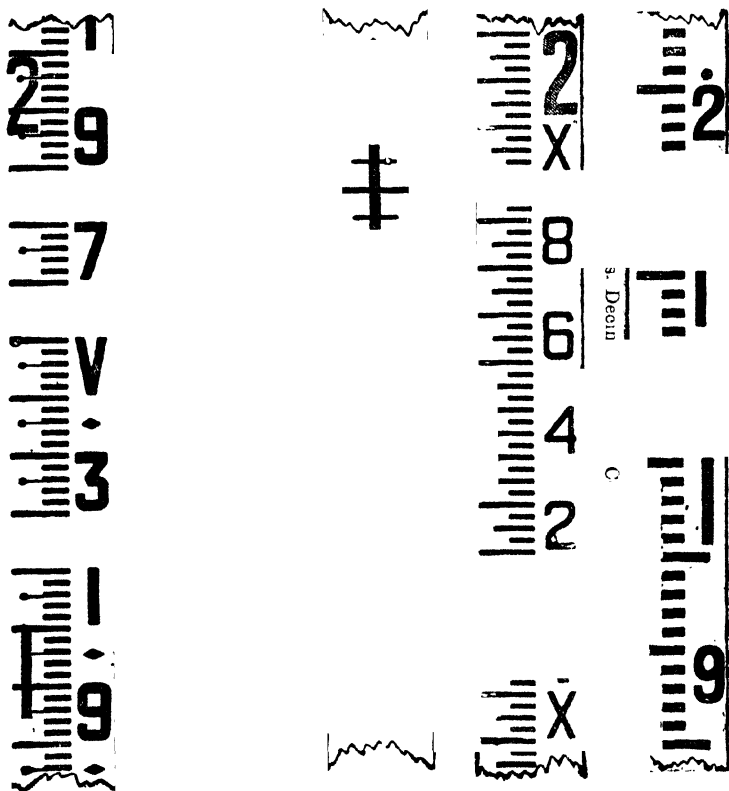
Fig. 65.

the bottom of the figure with the tenth next lower. Thus, the bottom of the black figure 3 marks ·2 foot, and the bottom of the V (5) is ·4, etc. When the cross web cuts a black figure, record the tenths as one lower than that denoted by the figure intersected; thus, if the latter is 7 enter ·6, if 9 enter ·8 plus

the hundredths counted from the bottom of the figure. If the cross web cuts between two figures, the first decimal place in the reading is given by the lower of the two; thus, if the scale is intersected between 7 and 9, the reading is ·7, plus the hundredths counted from the top of the figure.

Some difficulty will at first be experienced in reading the staff, partly due to the fact that its image is seen inverted, and partly due to the scale being unfamiliar. A little experience in the field will, however, soon remove these difficulties.

The beginner should cultivate the habit of taking level readings in the following order:— (1) See that the bubble is in the centre of its run, if not bring it central by a slight turn on one of the levelling screws; (2) read and record the feet and tenths; (3) read and record the hundredths; (4) observe if the bubble is still central; (5) again read the staff and check the figures already recorded.

Staff to be held Vertical.—It is essential that the staffman should be trained to hold the staff quite vertical when a reading is being observed. To assist him in doing this, the staff may be fitted with a plumb-bob, or with a small circular level, as shown in Fig. 66. When not in use the level folds flush with the
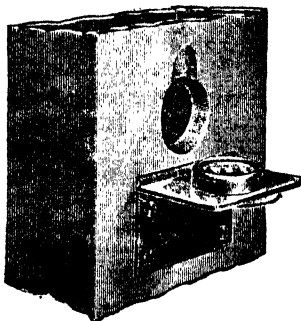
Fig. 66.
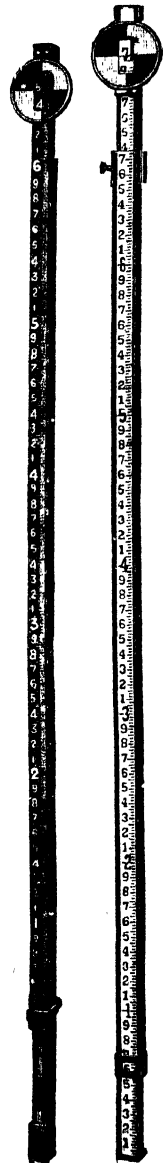
Fig. 67.

back of the staff. If the staff is not fitted with either of these aids to accuracy, and the leveller is in doubt about the staff being held vertical, he should signal the staffman to slowly oscillate the staff about its foot, in the vertical plane containing the line of collimation. This will cause the reading at the instrument to decrease and increase as the staff approaches and recedes from the vertical. The minimum reading is the one observed and recorded.

This method of swinging the staff should always be resorted to when a reading is being taken at an important staff station.

In America and in the Colonies many engineers still use the vane staff, two types of which are shown in Fig. 67.

In America the levelling instrument is left in charge of a subordinate, who directs the staffman to raise or lower the vane as required. When the vane is set in its proper position the chief surveyor—standing near the staff and chainmen—reads the staff and records both the staff reading and the distance on the chain. In this way the chief surveyor is able to supervise the work of the chainmen and staffman more closely. The assistant at the instrument records his level readings, which are afterwards checked by reference to the record of his chief.

Other Levelling Instruments.

The Aneroid Barometer.—This instrument consists of a thin cylindrical metal box hermetically sealed, and from which the air has been exhausted. The ends of the box are corrugated in circular corrugations, and as the pressure of the atmosphere increases or decreases, they slightly approach or recede from each other. This slight movement is magnified by a train of levers, the last lever in the train being an index arm, the end of which moves over an arc graduated in feet or inches of mercury, or both. The working parts of the instrument are enclosed in a brass box having a glass face, through which the movements of the index can be observed. The external appearance of the instrument is shown in Fig. 68.

The atmospheric pressure at any place depends on the weight of the column of air above it. Hence, it follows that the higher the place of observation the less will be the atmospheric pressure. If we neglect the variations in the size of the sealed box due to changes of temperature, the movement of the index will be directly proportional to the height or depth to which the instrument is carried, provided the atmospheric pressure at the place

of departure remains constant. In the more delicate instruments, readings may be obtained showing a difference of level of 1 foot, the smaller instruments give readings from 20 to 50 feet, smaller differences of level being obtained by estimation.

Levelling with the Aneroid Barometer.—In using this instrument to determine the difference of level of two stations, the scale on the instrument is either set to zero at the station of departure or the readings of the altitude and barometric scales are recorded. On arriving at the second station, the altitude
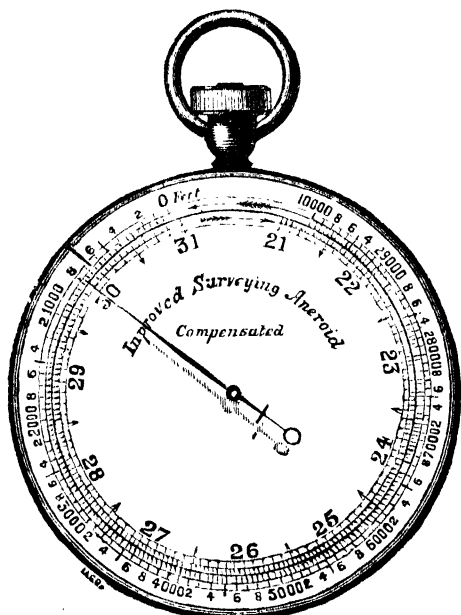
Fig. 68.

scale is again read, and the difference of level of the two stations is then obtained by taking the difference of the readings of the altitude scale.

As the barometric pressure may not have remained constant in the interval between the two observations, the instrument should be returned to the departure station, in order that the barometric pressure may be again recorded and a correction introduced (if required) in the observed readings.

A better and much more accurate method of using the baro-
meter is to make use of two instruments, one of which—after
comparing the two—is kept at the station of departure and is
read every quarter of an hour, the temperature being also re-
corded. The second instrument is carried to the stations, the
heights of which are required, and its readings for height and
pressure, and the temperature at the time are recorded at these
stations.

From the data thus obtained the altitude may be corrected,
and a more exact result obtained.

The difference of level between two stations, allowing for
change in barometric pressure and temperature, may be obtained
with sufficient accuracy for all practical purposes in the case
of aneroid readings from the formula :—

$$H = 60345 \cdot 5 \log \frac{B}{b} \{1 + \cdot 00111 (T + T_1 - 64°)\},$$

where H = difference of level in feet,

　　　　B = barometrical reading at the lower station,

　　　　b = barometrical reading at the upper station,

　　　　T = the temperature (Fah.) at the lower, and

　　　　T_1 = the temperature (Fah.) at the upper station.

In addition to the corrections for temperature and pressure,
the altitude will generally require correction for the instrument.
The amount of this correction can only be determined by experi-
ment and comparison with a good standard instrument.

By using two instruments differences of level under 5,000 feet
may be determined within 5 or 10 feet, but the errors may be
as large as 30 feet if one instrument only is used.

The Mercurial Mountain Barometer.—This portable barometer
(Fig. 69) is generally constructed on Fontin's principle, in which
the level of the mercury in the reservoir is adjustable by means
of a thumb screw at its base. The mercury is completely en-
closed, and by turning the thumb screw the volume of the
reservoir may be reduced until the mercury completely fills it,
and the barometer tube. By this means the instrument is
rendered extremely portable.

In carrying the instrument between stations it should be
carried inverted.

When in use, the barometer is suspended from gimbals on a
tripod, and the level of the mercury is raised or lowered by the
thumb screw until its surface is exactly level with a fixed point

in the reservoir. This point is made the zero of the scale on which the height of the mercury is determined.
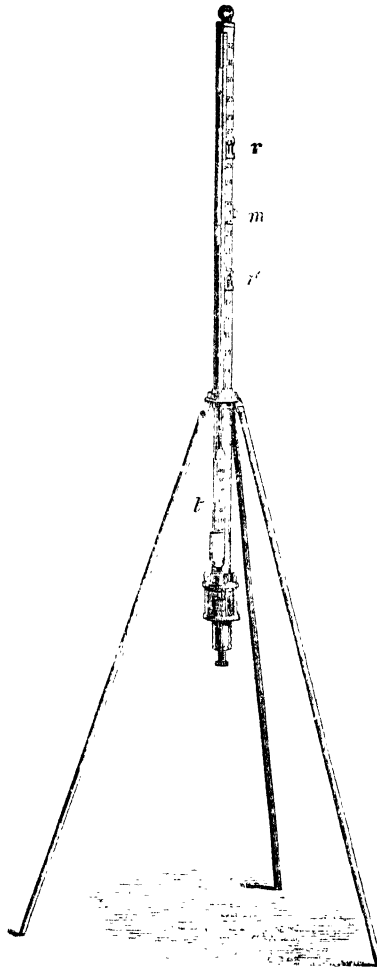
Fig. 69.

As in the case of the aneroid, two instruments are necessary, one being kept at the lower station and read, so far as possible,

synchronously with the other, which is transported to the stations whose heights are required.

This type of instrument gives the most accurate means of determining heights by barometric observations. The results, however, require correction for alteration in the length of the mercury column, due to changes in temperature, and to the variation in pressure due to the same cause. Hence, in addition to reading the height of the mercury column, it is necessary to read both the temperature of the air and of the mercury at the time of observation. The latter is obtained by reading the thermometer attached to the barometer tube case. An allowance must also be made for the variation in the force of gravity due to change in latitude.

Various formulæ have been devised for calculating the difference in height between two stations from barometric observations.

The following is given by Laplace:—

The difference of height in feet

$$= 60{,}345{\cdot}5\{1 + \cdot 00111111\,(T + T_1 - 64°)\}$$
$$\times \log \left\{ \frac{B}{b} \times \frac{1}{1 + \cdot 0001\,(t - t_1)} \right\}$$
$$\times \{1 + \cdot 002695 \cos 2\,\theta\},$$

where T and T_1 are the readings of the air thermometers in Fahrenheit degrees at the lower and upper stations,

B and b the barometric heights at the lower and upper stations,

t and t_1 the readings of the " attached " thermometers in Fahrenheit degrees at the lower and upper stations, and

θ the mean latitude of the stations.

In a formula of this description, in correcting for the mean density of the air, the mean of the readings on the air thermometers at the two stations is assumed to be the mean temperature of the intervening column of air. This assumption is not always correct, hence levels obtained from barometric observations are more or less uncertain.

The Hypsometer.—Since the temperature at which water boils varies with the pressure to which it is subjected, a method is thereby suggested of determining relative heights by means of a delicate thermometer. The instrument for this purpose is known as the hypsometer, and consists of a cylindrical metal

vessel, the upper portion of which is telescopic, and is drawn out when the instrument is in use. The lower part of the vessel contains water in the vapour from which the bulb of the thermometer is immersed. The stem of the thermometer passes through a hole in the top of the vessel, and through a second hole the steam escapes.

The water is heated by means of a spirit lamp.

The temperature of ebullition and of the air at the two stations being determined as nearly as possible at the same time; by reference to tables, the corresponding atmospheric pressures are obtained, and the difference of height of the stations may be calculated by the preceding formula.

In applying the formula it must be noted that the factor containing t and t_1 corrects for the effects of temperature on the mercury and scale. This correction is not required in the case of the aneroid or the hypsometer, and the formula thus becomes :—

Difference of level in feet

$$= 60,345 \cdot 5 \left\{ 1 + \cdot 00111111 \, (T + T_1 - 64^\circ) \right\}$$
$$\times \log \frac{B}{b} \, (1 + \cdot 002695 \cos 2 \, \theta).$$

Fig. 70.

Reflecting Level.—In levelling on very steep ground the ordinary level is at a great disadvantage, as it would be necessary to move the instrument into a new position every few feet. If the object of the operations is to determine an approximate section only, the ordinary level may be replaced by the hand level (Fig. 59) or a simple reflecting level (Fig. 70) may be used.

The reflecting level consists of a piece of silvered plate glass about $1\frac{1}{2}$ inches diameter. This glass is fixed in a heavy square metal frame, so that a diagonal of the square is horizontal. The silvering on the glass above this line is removed, so that objects may be seen through the clear glass. The glass is movable in its frame, and, if preferred, it may be set with the edge of the silvering vertical. The frame is connected to a ring by a pin and swivel joint, so arranged that when the instrument is suspended from the ring the plane of the half-mirror is vertical.

When in use, the instrument is suspended from the thumb or from a projection on a short staff, which keeps it at a constant height above the ground. On looking at the mirror the reflected image of the eye is seen, and the observer moves his head until
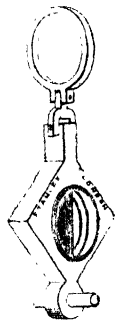
the image of the pupil is bisected by the horizontal edge of the
silvering, when any distant object seen through the clear glass
and apparently cut by the same edge is at the same level as the
eye.

The method of using the reflecting level to determine the differ-
ence of level between the two points A and P is shown in Fig. 71.

The levelling staff is first held at
A, and the reflecting level is set
up at some convenient point C
in the line A P, and the reading
(A B) on the staff is obtained.
The staff is then moved to C,
the level set up at some point
H, and the staff reading C G is
observed. The staff is now
moved on to H and the level
to the point L, the operations
being repeated until the point P
is reached. If the section on the
line A P is required, the hori-
zontal distances of the points occupied by the staff must be
measured from A.

Fig. 71.

The difference of level of

A and P = A D + C F + H J + L N
    = A B − B D + C G − F G + H K − J K + L O − N O
    = A B + C G + H K + L O − (C E + H I + L M + P Q)
    = sum of staff readings − four times the height of the
      instrument.

Or, generally, the difference of level between two points
    = sum of staff readings − N times the height of instru-
      ment,

where N = number of staff readings.

If in working downhill the instrument precedes the staff,
the difference of level between any two points = N times height
of instrument − sum of staff readings.

It is better that the staff should precede the instrument in
working downhill, so that longer sights may be used.

Boning Rods.—These are used by excavators for continuing
a line of levels previously fixed on by the engineer. They con-
sist of a vertical piece of deal about 3″ × 1″, to the top of which
a cross batten, about a foot long and of the same cross-section,
is fixed at right angles. Three rods for a set; the height of the

rod is immaterial, but all the rods of the same set must be of the same length. When in use, two of the rods are held upright on points whose levels have been determined, and the third rod is held upright at some other point. When the tops of the three rods are in the same plane, as determined by sighting over their cross battens, the level of the third point is correct.

Sounding Cords or Lead Lines.—These are used for finding the depth of water over known points, and consist of hemp cords for moderate depths, steel or brass wires, or thin steel or brass wire ropes for greater depths. They are loaded with weights at their lower ends and divided by knots or attached tallies into fathoms, and read to quarter-fathoms for nautical purposes, or divided in 5-feet lengths, and read to feet and tenths for engineering purposes.

The Adjustments of the Level.

These may be divided into two classes—(*a*) temporary and (*b*) permanent.

The temporary adjustments are (1) to make the vertical axis of the instrument truly vertical, and (2) to make the foci of the eyepiece and object glass coincide at the cross wires.

The first adjustment—technically known as " setting up the instrument "—is effected by the levelling screws, and is best learned by imitation.

The mode of procedure is as follows :—Firmly plant the tripod with the axis of the instrument approximately vertical. In the four-screw instrument turn the telescope until its axis is in the vertical plane containing a diagonal pair of levelling screws and turn these screws as required until the bubble is central. The telescope is then turned through a right angle, and the second diagonal pair of screws are manipulated until the bubble is again central. If the instrument is in good condition the bubble should now remain central for all positions of the telescope, otherwise the final adjustment of the position of the bubble must be made by a judicious tightening up of the levelling screws.

In the three-screw type, the telescope should be set parallel to the line joining a pair of the levelling screws, and one or both of the screws turned until the bubble is central. The telescope is then placed over the third screw, which is turned as required until the bubble is again in the middle of its run, the final setting being accomplished by turning one or other of the levelling screws.

This adjustment has to be repeated each time the instrument

is moved into a new position, and is one of the chief sources of delay in levelling.

Adjustment for Parallax.—The second temporary adjustment is commonly called the adjustment for parallax, and is carried out as follows :—Direct the telescope towards some distant object, light in colour, such as a white cloud, and draw out the eyepiece until the cross wires are in perfect focus. If the adjustment is correct, on sighting the staff the image of the cross wires will not appear to move relative to the image of the staff when the eye is moved from side to side. If relative motion of the images occurs when tested in this way, the position of the eyepiece must be further adjusted until the relative motion disappears.

Permanent Adjustments.

(1) *To place the cross wires in the axis of the telescope.*

In the Y-level, after carefully setting up the instrument, direct the telescope to some well-defined distant point, and bring its image into exact coincidence with the point of intersection of the cross wires. Without turning the instrument on its vertical axis, rotate the telescope in its bearings through 180°. The image of the distant point will remain in coincidence with the point of intersection of the cross wires during the rotation if the instrument is in correct adjustment. If the image moves sideways relative to the cross wires, take up half the displacement by turning the telescope on its vertical axis, and the remaining half by means of the horizontal diaphragm screws. If the image moves vertically, take up half the displacement by the vertical diaphragm screws. Repeat until perfect.

In the dumpy level the cross wire is placed in the axis of the telescope by the maker, and the diaphragm screws should not afterwards be touched.

It may be shown that if the line of collimation is set parallel to the spirit level, no error will be caused in the staff readings by the cross wire being slightly out of the axis of the telescope.

(2) *To set the line of collimation parallel to the spirit level.*

This is commonly known as " the adjustment for collimation error."

This is the most important adjustment, a level which is out of collimation is absolutely unreliable.

In instruments of the Y-level type, the test is made as follows :—Set up the instrument on a firm foundation and bring the bubble

exactly to middle of its run. Now take the telescope out of its bearings, and replace it thereon with its ends reversed, replacing the clips after reversal. If the bubble is still in the centre of its run, the instrument is in correct adjustment. If the bubble is out of the centre, take up half the deviation with the levelling screws, and the other half by the screws fixing the level to the telescope.

This adjustment places the spirit level parallel to the axis of the bearing collars. It is the duty of the maker of the instrument to make this axis coincident with the axis of the telsecope. Repeat until the instrument satisfies the test.

In the case of the dumpy level, the instrument must be taken into the field and set up (at C, Fig. 72) *exactly midway* between two firm pegs A and B. The distance apart of the pegs will depend on the power of the instrument. For an ordinary 12″ level this distance would be from 200 to 300 feet. The instrument having been set up and carefully levelled, a reading is
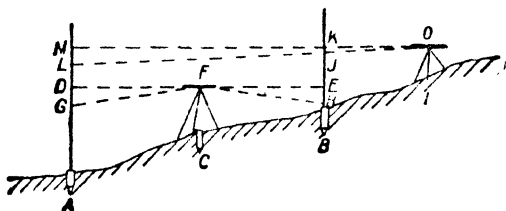
Fig. 72.

taken on a levelling staff held at A. The telescope is then directed to a second staff at B and the reading noted. Provided the bubble is exactly central when reading on the staves, the true difference of level of A and B is given by the difference of the staff readings. This is true whether the instrument is in adjustment or not, for, let D E be a truly horizontal line passing through the instrument and cutting the staves at D and E ; then the true difference of level of A and $B = A D - B E$. If the line of collimation is not parallel to the spirit level, it will make a constant angle with the level, and will appear to cut the staves in the points G and H (whether above or below D E is immaterial), and from symmetry, the triangles F D G and F E H are equal in all respects.

∴ D G = E H, consequently the difference of level of A and B
$$= A D - D G - (B E - E H)$$
$$= A G - B H.$$

Having obtained the difference of level of **A** and **B**, remove
the instrument to some point I beyond one of the staves **and**
nearly in line with A and B. Carefully level the instrument **and**
again take readings on the staves ; if the difference of these
readings equals the difference of level of the two points, as already
determined, the instrument is in correct adjustment ; if not,
elevate or depress the telescope by the levelling screws until the
difference of the staff readings is equal to the difference of level
of the points. The line of collimation will now be horizontal,
and the bubble must be brought to the centre of its run by the
screws fixing it to the telescope. The operations should be
repeated until the instrument satisfies the test.

An instrument adjusted in this way will be corrected for the
effects of " curvature " on the length of sight taken, and for the
effects of " refraction " under the atmospheric conditions pre-
vailing at the time.

Instead of altering the direction of the telescope by the levelling
screws, the line of collimation may be equally well adjusted by
raising or lowering the diaphragm by its screws, until the differ-
ence of the staff readings is equal to the difference of level of
the points A and B. This moves the line of collimation into
parallelism with the spirit level, without disturbing the position
of the bubble.

The amount (J K, Fig. 72) by which the reading on the nearer
staff must be altered, either by the levelling screws, or by moving
the diaphragm is easily found if we know the distances A B
and B I (it is convenient to make B I some fraction of A B) ;
for, let O K M be a truly horizontal line passing through the
instrument, A L and B J the staff readings at A and B, then it
is clear that the collimation error is equal to L M — J K, and by
similar triangles we have

$$\frac{L M - J K}{J K} = \frac{O M - O K}{O K}.$$

From this it follows that,

$$J K = \frac{B I}{A B} (L M - J K),$$

and the required staff reading at

$$B = B J \pm \frac{B I}{A B} \times \text{collimation error.}$$

Suppose the observed error of the instrument is $+ 0.04$,

A B = 3 chains, B I = $\frac{1}{2}$ chain, and the nearer staff reading is
C 25 ; then the level reading on the nearer staff must be made
equal of $6 \cdot 25 - \dfrac{0 \cdot 5 \times 0 \cdot 04}{3}$, or $6 \cdot 243$, in order to place the line
of collimation horizontal.

Zeiss Level.—This instrument is tested and adjusted in the
following manner :—The level is carefully set up with the revers-
ible bubble on the left and a reading is taken on a staff, held
at a distance of about 300 feet. The telescope is then rotated
in its bearings until the reversible bubble is on the right and a
second reading on the staff is obtained. The eyepiece is now
withdrawn and placed in the sleeve on the objective cap, which
is then placed in position over the object glass, and the cross
wires are again focussed. The telescope is turned round so that
the (ordinary) eyepiece end is towards the staff and two more
readings are obtained with the bubble on the left and right
respectively. If the four staff readings are not identical the
telescope (with the eyepiece in its proper position and bubble
on the left) is set to their mean reading, and the prism casing
E (Fig. 62) is displaced by the screw R, so as to bring
the images of the two ends of the bubble into coincidence.
When this is the case the instrument should be in perfect
adjustment.

(3) *To place the telescope at right angles to the vertical axis, or to
make the instrument traverse,*

Set up the instrument in the usual way with the telescope
over a pair of levelling screws, or over one screw in the tribrach
type. Reverse the direction of the telescope by turning it about
its vertical axis, through an angle of 180°. If the bubble after
rotation is still central, the adjustment is correct. If the bubble
deviates from the centre, take up half the deviation with the
levelling screws and the remaining half by the screws fixing the
telescope to the horizontal bar.

This adjustment is relatively unimportant, and very few levels
will traverse exactly. The leveller can easily guard against
errors of dislevelment by observing that the bubble is central
before and after each reading, any slight movement of the bubble
being corrected before reading the staff, by slightly turning one
of the levelling screws.

The Sensitiveness of the Spirit Level.—This is estimated by
the distance the bubble moves for a given change of inclination.
The more sensitive the level, the greater is the displacement
of the bubble for a given change of inclination.

The sensitiveness of a spirit level is determined as follows :—

Let R = radius to which the bubble tube is bent (or internally ground).

L = length of arc through which the bubble moves for a change of inclination equal to one second.

Then

$$\frac{L}{2\pi R} = \frac{1}{360 \times 60 \times 60},$$

or

$$L = \frac{2\pi R}{360 \times 60 \times 60}$$

$$= \frac{R}{206,265}.$$

The sensitiveness is directly proportional to the radius, and is equal to that of a plumb line of equal radius.
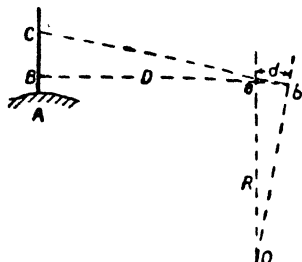
Fig. 73.

To determine R the instrument is taken into the field and a reading A B (Fig. 73) is obtained on a staff at a known distance D from the level. The bubble is then displaced through d divisions of the bubble scale, and the staff reading A C obtained.

By reference to Fig. 73 it is seen that the triangles $o\,a\,b$ (o being the centre of curvature of the bubble tube) and b C B are approximately similar.

∴ $\dfrac{o\,a}{a\,b} = \dfrac{b\,B}{B\,C}$ (very nearly),

or $o\,a = \dfrac{b\,B\,.\,a\,b}{B\,C}$,

∴ $R = \dfrac{d\,.\,D}{A\,C - A\,B}$,

i.e., $R = \dfrac{\text{displacement of bubble in feet} \times \text{distance of staff}}{\text{difference of staff readings}}$

EXAMPLES.

1. Sketch and describe the dumpy level.

2. Describe the chief tests and adjustments of the dumpy level.

3. How would you test and (if necessary) adjust a dumpy level for collimation error ? At what distance does the effect of curvature become appreciable when using an ordinary levelling staff reading to ·01 foot ?

4. What do you understand by the term " collimation error " as applied to a level ? How would you test and adjust a Y-level for collimation error ?

5. Sketch and describe the construction of the ordinary levelling staff. Give a detail sketch of about 6 inches of the scale, and indicate where the line of collimation would cut it when the reading is 4·28.

6. Explain what you understand by the term " level surface." Find the angle between the lines of collimation of two accurately set up spirit levels 40 miles apart. You may assume the radius of the earth to be 4,000 miles.   (*Ans.* 179° 25′ 37·4″.)

7. In determining the difference of level of two points with the reflecting level the following readings were obtained :— 12·8, 10·3, 11·4, 9·3, 8·7, and 12·1. The height of the instrument was 5 feet, find the difference of level of the two points.

(*Ans.* 34·6 feet.)

8. On a certain level the bubble moves 2 millimetres for a change of inclination of 20 seconds. Find the sensitiveness and radius of the spirit level.   (*Ans.* ·1 mm. ; 20·63 metres.)

9. Describe with sketches the differences in manipulation when levelling a dumpy level fitted with (*a*) four-foot screws, and (*b*) three-foot screws. What advantages has the latter over the former type of instrument ?

10. Explain why the sighting webs are arranged differently in the " dumpy " and " Y " types of instrument.

11. Sketch and describe the water level. To what kind of levelling work is the use of this instrument chiefly restricted ?

12. In testing a dumpy level for collimation error, the staff readings on two pegs, when the instrument was midway between them, were 8·94 and 8·65, the distance between the pegs 200 feet. The staff readings given by the instrument, when moved to a position 40 feet beyond the higher peg, were 7·92 and 7·58. What was the collimation error of the instrument ? To what reading must the instrument be set on the nearer staff, in order to place the line of collimation truly horizontal ?

(*Ans.* − 0·05 ; 7·57.)

CHAPTER VI.

LEVELLING OPERATIONS.

THE operations of levelling are determined by the object for which the work is intended, and may be roughly classified as follows :—

1. To determine the relative height of two points.
2. To obtain the levels of a series of distant points (bench marks) in predetermined positions, to be used subsequently for reference.
3. To determine the outline of the ground along a predetermined line, with or without reference to lateral outline, as in the case where the levelling work is required for a line of communication, or for drainage work.
4. To obtain sufficient information over a portion of country for producing either a contoured plan, or a spot level plan, from which the level of any point may be approximately determined.

Class 1.

The relative height of two given points may be determined by (*a*) a single line of sight when the distance between the points
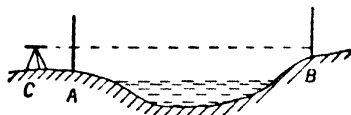
Fig. 74.

and their difference of level is not too great ; or (*b*) by flying levels when the distance or the difference of level between them is considerable.

If A and B (Fig. 74) are the points whose difference of level is required, and the distance from A to B is not more than 700 feet, the difference of level is obtained by setting up the level at some

convenient point C and reading on staves held at A and B, when the required difference of level will be given by the difference of the staff readings.

Curvature and Refraction.—If the distance between A and B is greater than 700 feet, an error will be introduced which will be perceptible on an ordinary levelling staff. This error is partly due to the curvature of the earth, and partly to the refraction of the ray of light coming from the more distant staff. To deter·mine the correction for curvature we proceed as follows :—

Let D = distance of the staff from the instrument in miles,
R = mean radius of the earth,
C = required correction.

Referring to Fig. 75, let the instrument and the staff be set up at points on the same level surface, and A C a level surface passing through the telescope and cutting the staff at C, E A
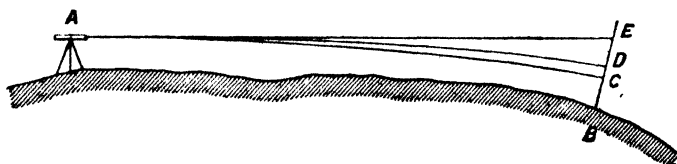
Fig. 75.

the ray of light passing from the staff to the instrument, and O (not shown) the centre of the earth.

Since instrument and staff are over points in the same level surface, the reading obtained on the staff would equal B C if the earth's surface were plane, but owing to the curvature of the earth's surface, the actual reading is B E, consequently the error due to curvature = B E − B C, or C = E C,

but
$$A E^2 = E C (2 . O C + E C)$$
$$= 2 . O C . E C + E C^2.$$

Since E C is a very small quantity compared with A E or O C, its square may be neglected,

∴
$$A E^2 = 2 . O C \times E C$$

or
$$C = \frac{A E^2}{2 . R}.$$

The mean equatorial radius of the earth $= 20,926,202$ fee^t, $\left.\right\}$ *
and the polar radius $\qquad\qquad\qquad\qquad = 20,854,995$ feet. $\left.\right\}$

∴   Mean radius $= 20,890,548$ feet,

consequently $C = \dfrac{D^2 \times 5,280^2}{2 \times 20,890,548}$ feet,

$$= 0\cdot6672 \; D^2,$$

where D is in statute miles.

For most cases occurring in practice, this may be taken as $\frac{2}{3} D^2$ feet.

The ray of light from the staff to the instrument is not straight, as we have supposed in the previous investigation, but follows a curved path which is convex downwards. Thus in Fig. 75 the line of collimation of the instrument will appear to cut the staff at D and not at E. Hence, the correction for refraction is opposite in sign to that for curvature, and the combined correction C D (Fig. 75) $= C - R'$, where R' is the error due to refraction.

The error due to refraction cannot, however, be exactly estimated, as it varies very much with the state of the atmosphere. Usually it is taken at one-seventh of that due to curvature, hence the combined correction—

$$C - R' = \tfrac{2}{3} D^2 - \tfrac{1}{7} \tfrac{2}{3} D^2$$
$$= \tfrac{4}{7} D^2.$$

On a sight of 700 feet the combined correction

$$= \tfrac{4}{7} \left(\tfrac{700}{5280} \right)^2 = \cdot0105 \text{ foot,}$$

an amount perceptible on an ordinary levelling staff.

Reciprocal Levelling.—If it is necessary that the difference of level of the two points be determined with great precision, the method known as reciprocal levelling should be adopted. In this method, the level is set up over one of the points (A, Fig. 76) and the staff at the other (B), and the staff reading is obtained. The level and staff are then interchanged, care being taken to set up the level so that the line of collimation is the same height above the ground in its two positions. The reading on the staff at A having been determined, the difference of level of the points, freed from the effects of curvature and refraction, is given by the rule—

$$\text{True difference of level} = \frac{\text{difference of staff readings}}{2}.$$

This rule may be determined as follows :—Referring to Fig. 76, let A and B represent the two stations ; O the earth's centre ; B D and A F the staff readings, G and E the respective positions of the telescope. Through G and A strike arcs with O as centre, cutting the staff at B in the points K and C ; also through E and B strike arcs cutting F A produced in the points H and I.

Since A G = B E, E K = B C = difference of level of A and B ; also, B E + E C + C K + K D − (H F − H A) = D B − A F, but K D = H F = correction for curvature and refraction ; also E C = H A, ∴ (B E + E C) + (C K + C E) = B D − A F, or B C + E K = B D − A F.

But, B C = E K = difference of level of A and B, ∴ twice difference of level of A and B = B D − A F, or true difference of level

$$= \frac{\text{difference of staff readings}}{2}.$$

The accuracy of the results in the above method depends on the accuracy with which the level is set at the same height above the two stations, and as it is difficult to set up a level in two different positions with its line of collimation exactly at the same height above the ground, it is better to use the following method, which obviates this difficulty :—
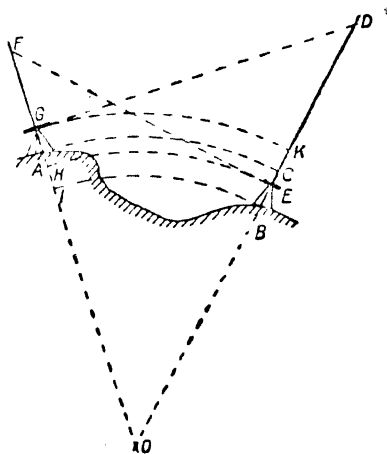
Fig. 76.

The level is set up at a point M (Fig. 77) nearly in line with levelling staves held at A and B, and the readings A E and B D are obtained.

The instrument is then removed to a point M′ (Fig. 78), so placed that B M′ = A M, and the staff readings B I and A H are determined.

Suppose level surfaces are drawn through the instrument when in its two positions, cutting the levelling staves in the points F and C, J and G.

In Fig. 77, the difference of level of
A and B = B C − A F,
and   B C − A F = B D − C D − $\overline{\text{A E} - \text{E F}}$.

In Fig. 78, the difference of level of

A and B = $BJ - AG$,

and $BJ - AG = BI - IJ - \overline{AH} - \overline{GH}$.

∴ true difference of level of

$$A \text{ and } B = \frac{BD - CD - \overline{AE} - \overline{EF} + BI - IJ - \overline{AH} - \overline{GH}}{2}.$$

Since $BM' = AM$,

$GH = CD$ = correction for curvature and refraction on the longer sight, and

$EF = IJ$ = correction for curvature and refraction on nearer sight,

.. difference of level of

$$A \text{ and } B = \frac{BD - AE + BI - AH}{2}$$

$$= \frac{\text{sum of differences of staff readings}}{2}.$$
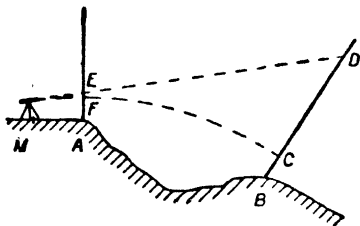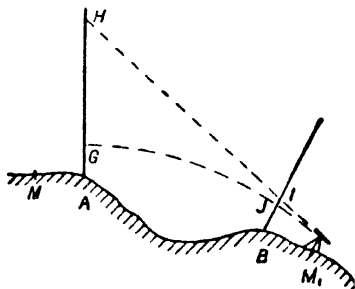
Fig. 77.   Fig. 78.

In using this method it is necessary that the second pair of staff readings be obtained immediately after the first, otherwise the refraction may change in the interval.

When the distance between the two points or their difference of level is considerable, the operations must be carried out by compound levelling.

Back-sights, Fore-sights, and Intermediates.—In levelling work generally certain terms are applied to the staff readings, in order to distinguish between them. The first reading after setting up the level is termed a *back-sight*, the last reading obtained immediately before removing the level into a new position is called a *fore-sight*. As all other readings on the same line of collimation are intermediate between these, they are termed

intermediates. To prevent confusion, it is necessary to enter the back-sight, intermediate, and fore-sight readings in their respective columns of the level book.

In the case that we are considering, since the object of the operations is simply to determine the difference of level of two points, back-sight and fore-sight readings only are necessary, and the shortest convenient course between the points is selected. The level is set up as far forward as possible consistent with the necessity of obtaining a clear sight of the staff at the first point A, and the back-sight reading is obtained. The staffman then moves forward and erects the staff on a firm point as many paces forward of the level as A is behind it, and the fore-sight reading is obtained. The instrument is then carried forward and set up as far beyond the staff as the power of the telescope and local obstacles will permit. Meanwhile the staffman—keeping the foot of the staff in contact with the ground—turns the staff round to face the instrument. The second back-sight having been obtained, the staffman again moves forward as many paces beyond the level as he was previously behind it, and the second fore-sight reading is determined. These proceedings are repeated until the point B is reached.

The required difference of level of the two points is given by the difference of the sums of the back-sights and fore-sights if B is higher than A, or by the difference of the sums of the fore-sights and back-sights if B is the lower point.

Flying Levels.—Work carried out as described above is frequently spoken of as " taking a flying level " between the points.

Checking Result.—The only satisfactory way to check the results obtained by the above process is by taking a flying level (not necessarily by the same course) back again from B to A, when the two differences of level should agree—*i.e.*, the algebraical sum of the back-sights and fore-sights for the whole round should either be zero, or fall within the limits of permissible error.

Class 2.

A chain of bench marks is generally placed along the site of a proposed work, for future reference. The bench marks are placed from a quarter to half a mile apart, and near the site of all intended structures, such as bridges, culverts, etc., but in positions where they will remain undisturbed during the progress of the work. Their heights are all determined either relative

9

to some fixed point arbitrarily chosen or to the ordnance datum.

Ordnance Datum.—The datum fixed point to which the levels marked on all ordnance plans are referred is that of the mean tide level at Liverpool, or the height of a point 4·67 feet above the Old Dock sill at Liverpool.

Ordnance Bench Marks.—In referring the levels of the bench marks to ordnance datum, it is usual to commence the levelling from an ordnance bench mark. These marks are in the form of a broad arrow, and are cut on the plinths of public buildings, walls of bridges, gate posts, etc. Their positions and heights (in feet) are indicated on the 6-inch ordnance plans, thus— ⤊ B.M. 450·7.

The shape of the mark cut in a wall or other object is shown in Fig. 79, the height referred to is the height of the centre of the horizontal V-shaped incision. Hence, in taking a reading, at an ordnance bench mark, it is necessary to hold the foot of the staff level with the centre of the horizontal mark.

In selecting points suitable for bench marks, due regard must be paid, not only to their permanent stability, but also to their ease of location. The doorsteps of churches or other public buildings, the tops of mile or gate posts, marks cut on the roots of trees, are all suitable positions for bench marks; but in all cases great care must be taken to exactly define their position, or great waste of time may ensue in afterwards locating them.

Failing suitable points as described above, the bench marks may be set on the tops of stout stakes firmly driven into the ground.

Having decided on the positions of the bench marks, their heights relative to the datum point may be obtained by flying levels, or determined while the preliminary levelling operations are in progress.

Class 3.

Section on a Given Line. Line of Section. Cross-sections.— In this class, the object of the operations is to determine an accurate outline of the surface of the ground along a predetermined line. The outline of the ground thus obtained is spoken of as " the section on the given line," and the latter is termed the " line of section."

When information regarding the lateral outline of the ground is required, cross-sections, at right angles to the line of section, are put in at known distances from its commencement. If the section is required for the laying down of a main drain, or a water

main, cross-sections are not required, but in the case of a line of communication, cross-sections are very necessary.

The levelling operations commence at a bench mark which may or may not be near the line of section. In the latter case a flying level is run from the bench mark to the first point on the section, and from this point distances along the line of section must be chained. The level is set up so as to command as many points on the section as possible, and about midway between the back- and fore-sights on the line of collimation, staff readings are taken at each point where the outline of the ground on the line of section changes, the horizontal distance at each point from the commencement of the section is noted from the chain measurements. When the leveller considers that it is desirable to change the position of his instrument, he warns the staffman by a pre-arranged signal, and the staffman selects a point suitable
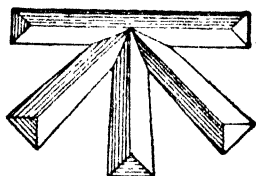
Fig. 79.
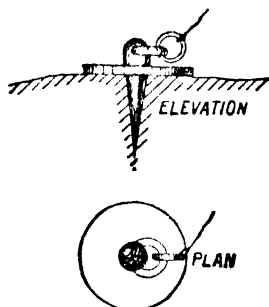
Fig. 80.

for a fore-sight reading. The point selected may be in any convenient position, not necessarily on the line of section, but must be of a firm, unyielding character. Failing any local object firm enough for the purpose, a brick or stone should be driven into the ground, and on this the staff must remain while the fore- and back-sight readings are obtained.

Too much care cannot be exercised at this point, as it should be remembered that any error in a fore- or back-sight reading, caused either by the settlement of the staff into the ground, or by the lack of perpendicularity, will cause an equal error in every reading afterwards obtained, and as such errors are all of the same sign, the resulting effect may be very serious.

Foot Plates.—If much soft ground has to be worked over, a metal foot plate (Fig. 80) should be used at change points. The

foot plate is stamped firmly into the ground, the solid hemisphere uppermost, and on this the staff is held while the change of level is made.

The fore-sight reading having been obtained, the level is moved forward and set up in a commanding position; meanwhile the staffman turns the staff to face the instrument. By a signal the leveller informs the staffman that he has obtained the back-sight reading, and the latter (if necessary) moves back into the line of section. The chaining and determination of intermediate readings are resumed, the work proceeding in this way until the reading (a fore-sight) is obtained at the last point on the section.

Checking the Levels.—If the last reading on the section is taken on a bench mark, the levels can at once be checked by taking the difference of the sums of the back- and fore-sight readings. This difference must agree (within predetermined limits) with the difference of level of the bench mark and the datum point. If the level readings do not end on a bench mark, the only way to check them is to run a flying level back to the starting point; when the algebraic sum of the back- and fore-sights for the whole round should be zero.

Before commencing a long section, a bench mark should be established at the end of every ten chains or thereabouts. If this be done, then each day's work may end on a bench mark and be fully checked, thus localising any error which may arise.

Ending a Day's Work.—At the end of a day's work the point at which the chaining stopped must be marked with a peg. It is convenient to have this point at the end of a chain, a careful note being made of the exact distance along the section, so that the leader may have his proper complement of arrows on recommencing the work. The last level reading for the day may be taken on a temporary bench mark, from which the following day's operations must commence. The bench mark selected should be easy of access and location, and a careful note of its position made in the field book.

Equality of Sights on Long Gradients.—In levelling on a long upward gradient, the tendency to place the instrument further from the staff for a back-sight than for a fore-sight reading must be guarded against. Neglect of the precaution to place the instrument midway between the back- and fore-sight positions of the staff will cause the ground to appear steeper than it really is. On an upward gradient the back-sight readings will be more

affected by curvature and refraction than the fore-sight readings, and on a downward gradient the fore-sights will be similarly affected. In the former case the rise, and in the latter the fall, will be too great, hence the errors due to inequality of sights may not be discovered from the check levels.

Where to take Readings.—In taking staff readings the beginner should bear in mind that it is better to take too many than too few, and he should take a reading wherever it is likely to be of service. When crossing a road with footpaths, a reading should be obtained at the boundaries, on the curbs, in the gutters and at the centre of the road; in passing a ditch or stream, take readings at the tops of the banks and the centre; in the case of a wall or fence, its height must be noted, and full notes must be made in the remarks column of the level book, of the nature and position of all objects encountered on the line of section.

Obstacles in Levelling.—When the line of section passes through a building, the gap in the chainage may be filled in by one of the methods already described, and the levels continued by a flying level round the building to the line of section on the further side.

If a high wall is encountered, an elevated position for the level may often be found from which the leveller may obtain a fore-sight reading with the staff on the further side of the wall; failing this, a flying level may be run from one side of the wall to the other through the nearest gate. If neither of these methods can be used, a fore-sight reading should be obtained with the staff inverted, the foot of the staff being held flush with the underside of the coping. The level and staff are then carried to the other side of the wall, and the back-sight reading obtained with the staff again inverted, the foot of the staff being held flush with the underside of the same coping stone. Back-sight and fore-sight readings obtained with the staff inverted must be considered negative, and are entered in their respective columns prefixed with a negative sign.

In crossing a shallow pond or stream, the readings are obtained in the usual way, the staff- and chain-men wading through the water.

River Soundings.—In the case of a broad, deep river, the levels may be set on the far bank, either by a flying level across the nearest bridge, or by reciprocal levelling as described on p. 122, the gap in the chaining being filled in by one of the methods described in Chapter II. When it is necessary to obtain the section of the river bed, the level of the water surface is carefully

obtained. The levels of other points in the bed of the river relative to the surface of the water are obtained by soundings taken from a boat.

If the river is sluggish the boat may be held in line with two rods on the far bank by the oars, while each sounding is obtained ; if the stream is rapid the boat must be anchored up-stream for each sounding and the anchor line paid out until the sounding cord is in line with the sighting rods.

The distance of each sounding from a standard point on the near bank is obtained either by paying-out a wire from the near bank, the wire being hauled tight at each sounding, and suitably
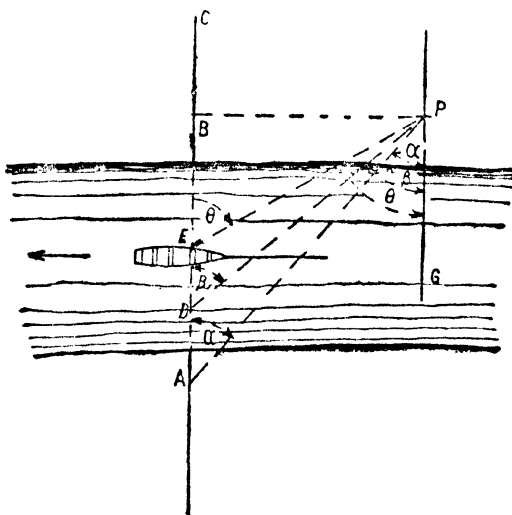
Fig. 81.

marked with a file or paint for distance, which is afterwards measured by the chain or tape, or a line B P (Fig. 81) is set out and measured at right angles to the line of section A B and the angle subtended by B P at each sounding is obtained by means of a box-sextant, or by taking the difference of the bearings of line joining the point of sounding to P and of the line of section A B. The positions of points of sounding are readily found by plotting. It is only necessary to draw through P a line P G parallel to A B, and to set out lines from P making angles with P G equal to the observed angles. The points of intersection

of these lines and the line of section give the positions of points of sounding, their distances from the standard point A are then obtained by measurement from the plan. These distances may be readily calculated, for let

B P subtend the angles α, β, θ, etc., at the points A, D, E, etc., then

$$A B = B P \cot \alpha,$$
$$D B = B P \cot \beta,$$
$$E B = B P \cot \theta, \text{ etc.,}$$

$\therefore$

$$A D = A B - D B$$
$$= B P (\cot \alpha - \cot \beta),$$

similarly

$$A E = B P (\cot \alpha - \cot \theta), \text{ etc.}$$

Forms of Field Book.—The calculations required in ordinary levelling are of a simple character, and consist chiefly of the addi‑ tion of long columns of figures.

Two systems of completing the field notes, preliminary to plotting the section, are in common use. The oldest and the one most used by British surveyors is known as the " Rise and Fall " system, and the other as the " Collimation " system. The latter is simpler in use, and has one column of figures less than the former, its chief disadvantage is that in computing the reduced levels, those of the intermediate readings are not checked, whereas, in the rise and fall system the reduced level of every point is fully checked in the computation.

The form of field book on the rise and fall system is shown on the next page, together with the method of casting.

Meaning of Terms Rise and Fall.—It should be noted that the terms " Rise " and " Fall " mean the rise or fall of any point on the section relative to the point immediately preceding it. A rise being denoted if the staff reading is less than that of the preceding point, and *vice versâ;* thus, in the following field book, the first back-sight is 13·25, and the next reading, a fore-sight, is 1·32 ; as the second reading is less than the first, the second point is 13·25 — 1·32, or 11·93 feet higher than the first. At 25·00 feet on the section, the staff reading is 7·84 the preceding reading 5·16 ; this indicates a fall of 7·84 – 5·16, or 2·68 feet, and so on. At 340·0 feet on the section, the fore-sight reading is 6·54 feet *above* the line of collimation, and the preceding reading is 7·63 feet *below* the same line, hence the rise is 7·63 — (— 6·54), or 14·17 feet. Similarly, the back-sight reading at 342·0 feet is 5·32 feet *above* the collimation line, and the next reading 8·21 feet *below* that line, hence the fall between the two points is — 5·32 — 8·21,

Back sight.	Intermediat	Fore sight.	Rise.	Fall.	Reduced Levels.	Distance.	Remarks.
13·25					20·00		B.M. cut thus ⋏ on right-hand curb, at corner of Smith St. and Martin Lane.
		1·32	11·93		31·93		
12·84							
		2·65	10·19		42·12		
8·43							
	5·16		3·27		45·39	00·0	At peg, beginning o section.
	7·84			2·68	42·71	25·0	
	10·32			2·48	40·23	75·0	
		11·45		1·13	39·10	120·0	
1·24							
	5·83			4·59	34·51	205·5	Centre of footpath, 4' wide.
	8·94			3·11	31·40	250·0	
	12·32			3·38	28·02	310·0	
	7·63		4·69		32·71	340·0	Foot of wall.
		−6·54	14·17		46·88	340·0	Staff inverted, foot flush with coping. Wall 2 feet thick.
−5·32						342·0	
	8·21			13·53	33·35	342·0	Foot of wall and edge of road.
	7·70		0·51		33·86	352·0	Centre of road.
	8·10			0·40	33·46	360·0	In gutter.
	7·60		0·50		33·96	360·0	Edge of curb.
	7·30		0·30		34·26	369·0	Bottom of fence. Fence 4' high.
	12·82			5·52	28·74	420·5	
		13·61		0·79	27·95		
2·14							
	4·21			2·07	25·88	500·0	Top of bank.
	8·42			4·21	21·67	503·5	Centre of stream.
	3·92		4·50		26·17	506·0	Top of bank.
	1·23		2·69		28·86	550·0	
		0·95	0·28		29·14	584·0	At peg. End of section.
37·90		29·98	53·03	43·89			
−5·32		−6·54	43·89				
32·58		23·44	9·14				
23·44							
9·14							

or— 13·53 feet. The computation of the remaining rises and falls should present no difficulty.

Reduced Levels.—The numbers in the reduced levels column show the heights of all the points on the section relative to some fixed datum. The latter may be arbitrarily chosen so that the whole of the section lies above the datum line in plotting, or it may be fixed relative to the ordnance datum. The reduced levels are obtained by adding algebraically the numbers in the rise and fall columns to the reduced level of the datum point.

Distances.—The entries in the distance column give the distance —usually in feet—of each point from the commencement of the section.

Checking the Level Book.—In this system, all the arithmetical work involved in casting-up the levels is completely checked, since, if the work is correctly done, the difference of the sums of the back-sights and fore-sights equals the difference of the sums of the rises and falls, equals the difference of the first and last reduced levels.

Before proceeding to fill in the reduced levels, the rises and falls should be checked by the above rule, otherwise any error which may have arisen in computing them will be carried forward to the reduced levels column.

If the booking fills several pages of the level book, enter the last staff reading booked on the page as a fore-sight and the same reading as a back-sight at the head of the next page. By doing this each page may be worked out and checked independently, thus localising arithmetical errors. The sums of the back-and fore-sights and the sums of the rises and falls on any page must be carried forward and entered at the heads of their proper columns on the next page.

Collimation System.—In this system the reduced level of each line of collimation is first computed by adding the back-sight to the reduced level of the corresponding fore-sight, and the reduced level of each point on the section is then obtained by subtracting the staff reading at the point from the reduced level of the corresponding line of collimation.

.The method of casting up the level book is illustrated in the following table :—

Explanation of Table.—The figures in the first three columns have been already explained. In column four, the reduced level of the first line of collimation is clearly 2·34 feet higher than the bench mark, and its value is 2·34 + 54·50, or 56·84. The point on the ground at which the first fore-sight was obtained is 10·32

Back-sight.	Inter-mediate.	Fore-sight.	Collimation Levels.	Reduced Levels.	Distances (Feet).	Remarks.
2·34			56·84	54·50		On Ordnance B.M. 54·5, at P on plan.
		10·32		46·52		
1·54			48·06			
	2·32			45·74	00 {	On spike, centre of Middleton Road.
	2·83			45·23	15	In gutter.
	2·21			45·85	15	On curb.
	1·90			46·16	21	At fence, 3' high.
	3·45			44·61	50	
	6·71			41·35	110	Brow of ditch.
	9·23			38·83	114	Centre of ditch.
	6·21			41·85	118	Brow of ditch.
	5·82			42·24	150	
	3·21			44·85	175	
	2·89			45·17	200	
		1·25		46·81		
13·42			60·23			
	10·54			49·69	250	
	5·32			54·91	300	
	2·15			58·08	350	At fence. Height, 4'.
		1·41		58·82	410	
1·32			60·14			
	3·41			56·73	425	Edge of occupation road.
	3·01			57·13	430	Centre of occupation road.
	3·52			56·62	435	Edge of occupation road.
	6·85			53·29	500	
	10·23			49·91	580	
		13·56		46·58	594	Edge of cutting.
1·45			48·03			
	8·92			39·11	612	
		12·84		35·19	625	Top of retaining wall.
0·93			36·12			
	4·14			31·98	625	Foot of retaining wall.
		3·64		32·48	636	Spike centre of Cross Street. End of section.
21·00		43·02		54·50		
		21·00		32·48		
	Fall,	22·02		22·02		

below the line of collimation, hence the reduced level of this point is 56·84 — 10·32, or 46·52. On changing level the back-sight reading shows that the line of collimation is 1·54 above the foot

of the staff ; hence the reduced level of the line of collimation
is 46·52 + 1·54, or 48·06. This number may also be obtained
from the difference of the back- and fore-sight readings, since
the change in level of the line of collimation is 10·32 − 1·54, or
8·78. The foresight being greater than the back-sight shows
that the line of collimation has been moved down by this amount,
therefore the new collimation level is 56·84 − 8·78, or 48·06.
The computation of the reduced levels of the remaining readings
should present no difficulty.

Cross-sections.—When the object of levelling operations is
to provide sufficient information for the construction of a line
of communication, in addition to the section along the proposed
centre line, it is necessary to obtain information regarding the
lateral outline of the ground. For this purpose cross-sections
at right angles to the centre line are put in at each chain, or at
each 100 feet peg, on the centre line. The length of the cross-
sections vary with the purpose for which the work is intended.
In the case of an ordinary road, the length may be from 50 to
100 feet on each side of the centre line ; for railroads the
length varies from 200 to 300 yards on each side of the centre
line.

The cross-sections are numbered consecutively from the com-
mencement of the centre line, and are set out at right angles
to the main line of section with the chain and tape, the cross-
staff or the optical square, and the distances are measured left
and right from the centre peg.

The longitudinal and cross-sections may be worked together
or separately. In the former case two additional columns are
required in the field book, giving distances left and right
of the centre line. The staff readings on the cross-section
are entered with the readings obtained on the longitudinal
section.

A convenient form of record illustrating this method of entering
the field notes is given on the next page.

Usually, however, the longitudinal section is first determined,
the cross-sections being obtained at a later period. In this case,
several cross-sections are marked out, and the points at which
the surface outline changes on each cross-section are marked
with cleft sticks, or pegs (Fig. 82). The distances of these pegs
are then measured right and left of the centre peg on each cross-
section, and this information, together with the number of the
cross-section, is written on slips of paper, which are inserted in
the clefts of the pegs.

Back-sight	Inter-mediate.	Fore-sight.	Rise	Fall.	Reduced Level.	Distance.			Remarks.
						Left	Centre.	Right.	
13·50					64·83		4,000		At peg No. 40.
	6·34		7·16		71·99		4,035		
	3·12		3·22		75·21		4,060		{ At fence, Height, 3'.
	6·71			3·59	71·62		4,080		
	7·32			0·61	71·01		4,100		At peg No. 41.
	8·45			1·13	69·88	20	4,100		
	10·32			1·87	68·01	50	4,100		
	13·64			3·32	64·69	100	4,100		
	9·76		3·88		68·57	140	4,100		
	6·32		3·44		72·01	170	4,100		
	7·15			0·83	71·18	200	4,100		{ At Wall, Height, 4'.
	9·65			2·50	68·68		4,100	30	
	11·72			2·07	66·61		4,100	48	
		13·92		2·20	64·41		4,100	69	
1·64									
	4·32			2·68	61·73		4,100	100	
	8·95			4·63	57·10		4,100	150	
	6·73		2·22		59·32		4,100	200	
	4·23		2·50		61·82		4,100	220	
		12·65		8·42	53·40				
10·23									
	8·42		1·81		55·21		4,150		
	9·17			0·75	54·46		4,175		
	10·13			0·96	53·50		4,200		At peg No. 42.
	12·64			2·51	50·99	75	4,200		
	8·43		4·21		55·20	120	4,200		
	9·51			1·08	54·12	200	4,200		
	11·15			1·64	52·48		4,200	60	
	12·32			1·17	51·31		4,200	110	
	13·12			0·80	50·51		4,200	150	
	8·64		4·48		54·99		4,200	200	By oak tree.
	10·85			2·21	52·78		4,250		
		13·21		2·36	50·42		4,300		At peg No. 43.
25·37		39·78	32·92	47·33					
		25·37	..	32·92					
	Fall,	14·41		14·41					

This is shown at A and B (Fig. 82). In the former the information given by the slip is, Section No. 40, point marked is 24′ 6″ to the left (L) of the centre line; the latter is that of a point 45′ 0″ to the right (R) of the centre line on Section No. 96.

When sufficient sections have been marked out the levelling operations commence either from the nearest bench mark, or the previously determined level at each centre peg is accepted as correct, and the first back-sight on each cross-section is taken at the centre peg on the section. In either case, the leveller sets up his instrument to command as many cross-sections as possible, the staffman places his staff at the centre peg on the first cross-section to be worked, and the leveller obtains the staff reading. The staffman next places his staff at the first marked point on the section, picks up the slip of paper, and calls out the information written thereon. The leveller records this in his level book, takes the staff reading, and the staffman moves out to the consecutive points on the same cross-section, at each point calling out his distance, number of cross-section, and right or left as the case may be. When as many points on the first
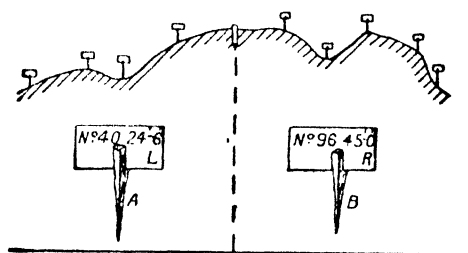
Fig. 82.

cross-section have been obtained, as is possible without moving the instrument, the other cross-sections commanded by the instrument are obtained in a similar manner working from the centre peg outwards. A fore-sight is then taken to a favourable point, the leveller moves his instrument to a new position, takes the corresponding back-sight reading, and the levelling of the cross-sections is continued as before. The fore-sight and back-sight readings must be entered in the bookings of *all* the cross-sections dealt with from the same position of the instrument.

When the sections on one side of the centre line have been obtained the instrument is moved to the other side of the centre line, and the levels of the marked points on the cross-sections are obtained as before, the last fore-sight reading should be

taken either on a bench mark, or on the point at which the first back-sight reading was obtained.

To avoid confusion, the bookings of each cross-section should be entered on a separate page of the level book, and full information as to the number of the cross-section, whether on the left or right of the centre line, with any other matter which may be useful, should be recorded.

Use of Stadia Wires.—As already pointed out on p. 99, distances along the line of section may be obtained without using a chain by the addition of two extra horizontal (stadia) webs to the diaphragm. These webs are symmetrically placed about the centre web. On sighting the staff three readings are noted, the reading at the centre web gives the staff reading for level, and the difference of the readings at the stadia wires gives the data from which the horizontal distance from the staff to the instrument can be obtained.

In all instruments fitted with stadia wires it may be shown (see p. 387, Part II.) that when the line of collimation is horizontal, the horizontal distance (D) from the instrument to the staff is given by the formula,

$$D = \frac{f}{i} S + (f + d),$$

where f = the focal length of the object glass for parallel rays,

    S = the difference of staff readings given by the lower and upper stadia wires,

    i = distance between the stadia wires,

and d = distance from the object glass to the vertical axis of the instrument.

Many makers now fit instruments with stadia wires, so that the ratio of f to i = 100 to 1. The term $(f + d)$ varies from about 1 foot to 1·5 feet, and depends on the size of the telescope.

In the Zeiss level described on p. 100, the term $(f + d)$ is got rid of, and the distance of the staff is obtained by multiplying the staff reading by 100.

Determining the Constants of the Instrument.—The constant terms in the distance equation for any instrument may be obtained in the following way :—Select a piece of flat ground and

drive, in a straight line, a series of pegs at distances of 0, 25, 50, 75, etc., to about 400 feet, the total distance depending on the power of the telescope. Set up the level over the zero peg, and obtain the stadia reading at each peg in the series. The results are then plotted on squared paper to a large scale, the distances (D) being plotted vertically and stadia readings (S) horizontally. The plotted points—allowing for personal errors—will lie nearly on a straight line. The average straight line is drawn through the points and its equation is determined. This equation is the distance equation for the instrument.

If the coefficient of S is 100, the distance of the staff is readily determined ; it is only necessary to move the decimal point two places to the right, and add the value of the constant $(f + d)$; but if the coefficient of S is not 100 the computation is more troublesome. In this case the product of f/i and S may be quickly determined by aid of a slide rule, a graph constructed as described above, or by means of a table.

The Distance Table.—To construct a distance table, prepare three columns of distances without the addition of the constant $(f + d)$, and a fourth column in which the constant is added. The distances in the second, fourth, and sixth columns are computed for stadia readings of ·01 to ·09, ·1 to ·9, 1 to 9 feet respectively, and the figures in the eighth column are obtained by adding the constant $(f + d)$ to each of the numbers in the third distance column.

The table given below is constructed for an instrument whose distance equation is $D = 97·88 S + 1·25$ feet.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.
Stadia Reading S.	Distance. D feet.	Stadia Reading S.	Distance. D feet.	Stadia Reading S.	Distance. D feet.	Stadia Reading S.	Distance. D +1·25 feet.
·01	·979	·1	9·79	1	97·9	1	99·15
·02	1·958	·2	19·58	2	195·8	2	197·05
·03	2·936	·3	29·36	3	293·6	3	294·89
·04	3·915	·4	39·15	4	391·5	4	392·75
·05	4·894	·5	48·94	5	489·4	5	490·65
·06	5·873	·6	58·73	6	587·3	6	588·55
·07	6·852	·7	68·52	7	685·2	7	686·45
·08	7·830	·8	78·30	8	783·0	8	784·25
·09	8·810	·9	88·10	9	881·0	9	882·25

We will illustrate the use of the table by two examples.   Let the stadia reading be 2·84, required the distance of the staff.

From column VIII., D for 2 feet = 197·05
     ,,   IV., D for ·8 ,, =   78·30
     ,,   II., D for ·04 ,, =   3·91

Required distance = 279·26 feet.

If the stadia reading is less than 1 foot, care must be taken to add the constant when using the table; for example, let the stadia reading be 0·63 foot.

From column IV., D for ·6 foot = 58·73
     ,,   II., D for ·03 ,, =   2·94
         Add 1·25 ,,   1·25

Required distance = 62·92 feet.

Since the distances obtained from stadia readings are in all cases the distances from the instrument to the staff in its successive positions, when determining the section along a given line, *the instrument* (when not fitted with a compass) *must in all cases be set up in the line* of *section*, and when a cross-section is to be determined, *the instrument must be set up at the point of intersection of the cross and longitudinal sections*, if the distances on the cross-section are determined with the instrument.   The staff must either be held in the line of section when changing level, or the first intermediate after changing level must be read at a point whose distance on the line of section has already been determined.   Further, *it is essential that the staff be held quite vertical at each staff station*, hence the staff must be fitted either with a spirit level or a plumb-bob.   With ordinary care the distances obtained on the section are quite as accurate as those obtained by chaining, and the work can be more quickly done.

It is obvious that the stadia readings will diminish as the staff is moved from a back-sight station towards the instrument, and increase as it is moved from the instrument to the fore-sight station.   Readings obtained with the staff held between the instrument and the back-sight station should be recorded with a negative sign, as these readings must be subtracted from the distance of the instrument from the corresponding back-sight

station, in order to obtain the distance of the staff position from the same point. After the staff has passed the instrument these distances are to be added to the distance of the instrument from the preceding back-sight station.

Care at Change Points.—The greatest care is necessary when taking a fore- or back-sight reading, not only to obtain an accurate reading for level, but also an accurate reading for distance. An error in collimation distance at either of these points will disturb the position (in distance) of all the following points on the section.

A convenient form of record for this system of levelling is given on pp. 142 and 143.

In calculating the distances given in the table, the equation $D = 97 \cdot 88 \, S + 1 \cdot 25$ feet has been used.

Plotting the Sections.—In plotting the longitudinal section a horizontal line is first drawn with an accurate straight edge across the paper as a datum line, and along this line the distances recorded in the field book are carefully laid down and figured. At each plotted distance a line is drawn at right angles to the datum line, and on each line the appropriate height—taken from the reduced levels column of the field book—is laid down and figured along the line of section. The outline of the surface of the ground is then obtained by joining the plotted points, either by straight lines or by a fair curve.

Horizontal and Vertical Scales.—The horizontal scale used in laying down the distances along the datum line should be the same as the scale of the plan. In order that both heights and distances may be equally apparent on the section, it is necessary that the vertical should be greater than the horizontal scale, the relation between the two scales and the degree of exaggeration shown depends very largely on the length of the section. For comparatively short sections, the vertical varies from 2 to 5 times the horizontal scale, but in long sections the amount of exaggeration is much greater.

The datum level for the section and the position of the datum point should be specified in writing below the datum line, and the information given in the remarks column of the level book should be written, in its appropriate position, on the section.

The cross-sections are plotted precisely as the longitudinal sections, except that they are generally plotted to a larger scale.

The longitudinal and cross-sections given in the field book on p. 136 are shown in Fig. 83.

10

Back-sight.	Inter-mediate.	Fore-sight.	Rise.	Fall.	Reduced Level.	Reading on Bottom Wire.	Reading on Top Wire.
2·45				..	30·00	3·46	1·43
	3·69			1·24	28·76	4·49	2·89
	6·82			3·13	25·63	7·42	6·22
	8·54			1·72	23·91	8·94	8·14
	5·60		2·94		26·85	5·85	5·35
	2·87		2·73		29·58	3·02	2·72
	4·62			1·75	27·83	4·92	4·32
	8·54			3·92	23·91	9·04	8·04
	10·12			1·58	22·33	10·97	9·27
	7·63		2·49		24·82	8·79	6·47
	5·34		2·29		27·11	5·47	5·21
	6·21			0·87	26·24	6·58	5·84
	8·42			2·21	24·03	9·18	7·66
	11·51			3·09	20·94	12·61	10·41
	5·62		5·89		26·83	5·83	5·41
	7·34			1·72	25·11	8·05	6·63
		10·45		3·11	22·00	11·57	9·33
3·63						4·65	2·61
	5·63			2·00	20·00	5·95	5·32
	7·34			1·71	18·29	7·47	7·22
	9·62			2·28	16·01	9·85	9·39
	10·89			1·27	14·74	11·39	10·40
	12·36			1·47	13·27	13·31	11·41
		11·20	1·16		14·43	12·46	9·95
6·08		21·65	17·50	33 07			
		6·08		17·50			
	Fall,	15·57		15·57			

Class 4.

Ridge and Valley Lines.—On examining the outline of the surface of an extended tract of ground, two important systems of lines may be traced. One system joins all the points of highest elevation, and the other all the points of lowest elevation in their immediate neighbourhood. The lines forming the first system are spoken of as ridge lines and as all points on a ridge line are higher than points on either side of the line,

Stadia Reading.	Distance.				Remarks.
	Instrument.	Staff.			
		Left.	Centre.	Right.	
2·03	200·0		00·0		B.M. cut thus + on plinth
—1·60	—157·9		42·1		of wall near station P.
—1·20	—118·7		81·3		
—0·80	— 79·5		120·5		
—0·50	— 50·2		149·8		Cross-section No. 1.
0·30	30·6	30·6	200·0		Height of instrument,
0·60	60·0	60·0	200·0		4·7′. Reduced level of
1·00	99·1	99·1	200·0		ground at 200′ = 30·00
1·70	167·7	167·7	200·0		+ 2·45 — 4·7 = 27·75.
2·32	228·4	228·4	200·0		At fence, 4′ high.
0·26	26·7		200·0	26·7	
0·74	73·7		200·0	73·7	◂
1·52	150·0		200·0	150·0	
2·20	216·6		200·0	216·6	At foot of wall, height 3·5′.
0·42	42·3		242·3		
1·42	140·3		340·3		
2·24	220·6		420·6		
2·04	201·0		621·6		Height of instrument 5·12′
—0·63	— 62·9		558·7		Reduced level of ground
—0·25	— 25·7		595·9		at 621·6 = 25·63—5·12
0·46	46·3		667·9		= 20·51′.
0·99	98·2		719·8		
1·90	187·3		808·9		
2·51	247·0		868·6		At peg. End of section.

surface water flows away from it in both directions. Hence ridge lines form the boundaries of watersheds and gathering grounds. Ridge lines are seldom straight for any considerable distance in either plan or elevation. Generally one or more principal lines may be traced from which secondary ridge lines diverge; these again give rise to a third set, and so on, the final set ending in the sea. In some rare instances, as in the case of the crater of a volcano, an inland sea or lake, from which water disappears by evaporation, or a coral atoll, ridge lines become closed curves.

A Valley Line is such that all points on it are lower than points immediately on the right and left of the line. Hence surface water flows towards a valley line, and the positions of valley lines are marked out by rivers, streams, and lakes. A valley line commences at the point of intersection of two ridge lines, at a bend in a ridge line, or at a *pass—i.e.*, the lowest point on a ridge line. Like the ridge lines, on which its shape depends, a valley line is seldom straight for any considerable distance in either plan or elevation.

The positions of valley lines are indicated on a plan by the positions of rivers, streams, lakes, etc., but the positions of ridge lines and elevations generally can only be roughly inferred by the information given on an ordinary plan.

To aid an engineer in forming an opinion with regard to the elevations and depressions on the surface of the ground, various methods have been adopted, depending on the purpose for which the plan is required.

A method used largely for topographical purposes consists of short lines (hachure lines) drawn in the direction of greatest slope, the thickness of the lines and their closeness are made to depend on the steepness of the ground, ridge lines being thus shown in relief. A good general idea of relative elevations and depressions may be obtained from a plan shaded in this way, but accurate estimations of elevations and depressions cannot be made.

Method of Figured Heights.—A simple method is that adopted on the 6″ and 25″ ordnance plans of this country, on which points are marked on the plans of the more important roads, and the heights of these points are given by the figures adjoining them, thus giving an approximate idea of the rise and fall of the ground. The same purpose is also served by the ordnance bench marks. Obviously the difference of level of two figured points may be accurately determined from their figured heights, but the relative heights of other points cannot be obtained by this method.

Method of Contour Lines.—The method universally adopted by engineers for accurately exhibiting elevations and depressions of the surface of the ground on a plan is the method of contour lines.

A contour line is the line of intersection of a level surface and the surface of the ground. Hence all points on the same contour line are at the same height above the datum surface.

Vertical Interval and Horizontal Equivalent.—The vertical

distance between any two consecutive contour lines is called the *vertical interval*, and the horizontal distance between any two consecutive contour lines is termed the *horizontal equivalent.*

For a given vertical interval the horizontal equivalent will depend on the steepness of the ground ; on a vertical cliff it is zero—*i.e.*, the contour lines are coincident—and on a level surface the horizontal equivalent is infinite. In some few instances—as in the case of an overhanging cliff—contour lines overlap, but generally they are (in plan) non-intersecting closed curves.

Contour lines cut ridge and valley lines at right angles. The
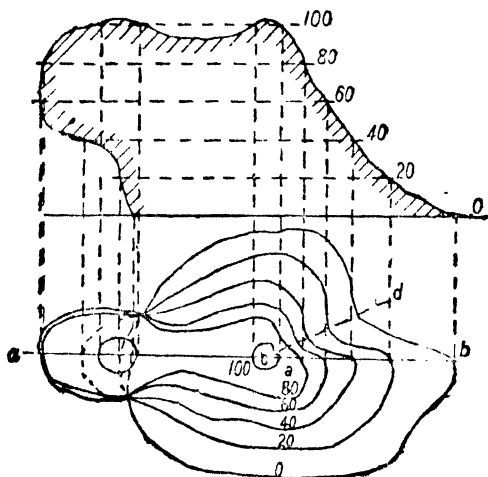
Fig. 84.

position of a ridge line is shown on the plan when contour lines are convex, and a valley line when the contours are concave as viewed from a lower level. Thus, in Fig. 84, *a b* indicates the position of a ridge line and *c d* that of a valley line.

The great value of an accurately contoured plan, to an engineer, lies in (1) the information it gives him of the elevations and depressions of the surface of the ground in relation to the more important surface features ; (2) the facility with which sections may be drawn in any direction from the information given by the plan only ; (3) the data it gives for the calculation of quantities either of earthwork or impounded water in reservoir

construction; (4) the extent of watersheds and gathering grounds.

The vertical interval between two contour lines depends on the nature of the work for which the ground plan has been prepared, the scale of the plan, and the figure of the surface of the ground. In drainage work, for example, the vertical interval will be smaller on flat than on hilly ground.

On the 6-inch ordnance plans of Britain, contour lines are drawn at a vertical interval of 25 feet, the principal contours are determined with a greater degree of precision than the others, and occur at every 50 feet of elevation in the flatter parts of the country, and at every 100 feet in the more hilly districts. The closest contours occur on the plans of some towns which have been prepared with a vertical interval varying from 2 to 8 feet

Methods of Determining Contours.—There are several methods of determining contour lines, depending on the degree of accuracy required and the instruments available. The methods may be divided into two classes; in one class points are found and marked on fixed straight lines (preferably near ridge and valley lines), at the given vertical interval apart; in the other points are found (and marked) on each contour line by following its outline on the surface of the ground. The former is the quicker of the two methods, as the leveller has the advantage of working along definite lines, and the contour points are more easily surveyed and plotted. The second method is, however, the most accurate, as it more easily lends itself to the determination of local surface irregularities.

System of Radiating Lines.—If the ground to be contoured is not too extensive, a convenient method is obtained by arranging all the lines—**on** which it is proposed to carry out the levelling work—radiating from a common centre, the lines being marked out, in directions of greatest utility, and their relative positions fixed either by measuring the angles between them or by chained triangles. To save time in checking the levels, bench marks are arranged at the centre and near the outer extremities of the radiating lines, the heights of these bench marks being determined relative to a common datum.

In finding the positions of the contour points, it is immaterial whether we work outward or inward along the radiating lines, but the work on each line should begin and end on a bench mark, so that the levels on each line may be checked independently. The contour points are marked with numbered pegs, the numbers agreeing with those in the level book. The pegs

are afterwards chained and their positions plotted on the plan, fair curves drawn through the points of equal altitude then give the required contour lines (see Fig. 85).

It will be observed that the levelling operations are the inverse of those which usually obtain, since the object of the operations is to find the positions of points whose heights are known, whereas in ordinary levelling work the heights are required of points
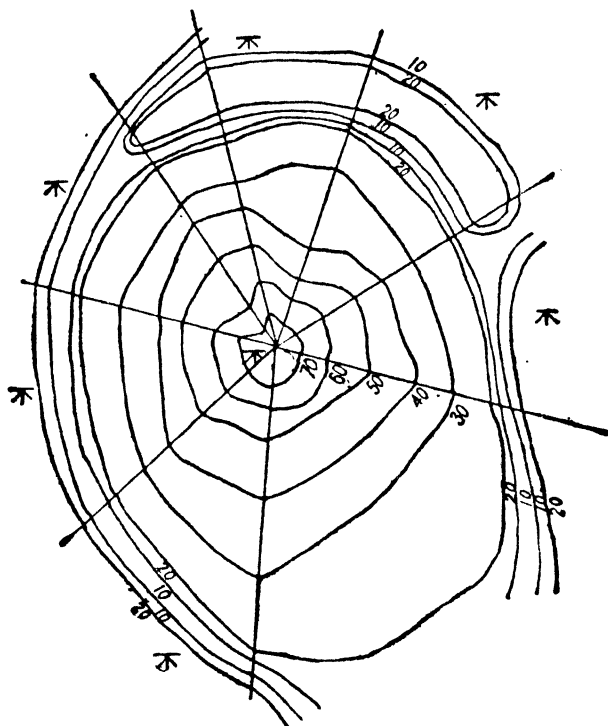
Fig. 85.

whose positions are fixed (by the irregularities of the ground and the position of the line of section).

The method to be followed is illustrated in the following level book, in which it is assumed that the vertical interval is 10 feet, the contour lines lie between the limits of 0 and 100 feet, and the levels commence on a secondary bench mark at 105·34 feet above datum.

Back-sight.	Inter-mediate.	Fore-sight.	Rise.	Fall.	Reduced Level.	Distance.	Remarks.
							Levels on line (1) A to B.
1·31					105·34		On B.M. No. 5.
	6·65			5·34	100·00		At peg 10.
		12·84		6·19	93·81		
2·34							
	6·15			3·81	90·00		At peg 9.
		13·28		7·13	82·87		
0·97							
	3·84			2·87	80·00		At peg 8.
		13·84		10·00	70·00		At peg 7.
9·20							
	13·88			4·68	65·32		Bottom of hollow at peg M.
	9·20		4·68		70·00		At peg 7a.
	5·96		3·24		73·24		Top of rise at peg N.
	9·20			3·24	70·00		At peg 7b.
		12·92		3·72	66·28		
2·36							
	8·64			6·28	60·00		At peg 6.
		13·32		4·59	55·41		
0·82							
	6·23			5·41	50·00		At peg 5.
		13·90		7·67	42·33		
1·20							
	3·53			2·33	40·00		At peg 4.
		13·53		10·00	30·00		At peg 3.
0·76							
	10·76			10·00	20·00		At peg 2.
		13·82		3·06	16·94		
1·16							
	8·10			6·94	10·00		At peg 1.
		13·92		5·82	4·18		
3·67							
		7·85		4·18	0·00		On zero B.M.
23·79		129·13	7·92	113·26			
		23·79		7·92			
	Fall,	105·34	Fall,	105·34			

Description of the Level Book.—The level having been set up to command the bench mark, the staff reading 1·31 is obtained.

The reduced level of the bench mark being 105·34, the fall between the bench mark and the 100 feet contour line is 5·34 ; this added to the back-sight reading gives 6·65, the reading when the staff is held on the 100 feet contour. Keeping himself in the line along which the pegs must be placed, the staffman moves the staff slowly in the required direction, until its foot is in the desired position when the staff reading will be 6·65. The point found is marked with peg No. 10. Assuming that a 14 feet staff is used, it will now be necessary to take a fore-sight reading (12·84). The corresponding back-sight is 2·34, and the reduced level of the staff station 93·81. There will thus be a fall of 3·81 feet to the 90 feet contour ; this fall added to the back-sight reading gives 6·15, the staff reading on the 90 feet contour. The point in the line having this staff reading is then found and marked with peg No. 9. The work proceeds in this way until all the pegs have been put in.

It will be noted in the above level book that the 70 feet contour is intersected three times, thus indicating a sudden dip and rise in the ground. When this occurs the levels at the bottom of the hollow and the top of the rise should be obtained, and the points marked with special pegs. The positions of these points are often of service in drawing the contour lines on the plan.

If the nature of the ground is such that a system of radiating lines would not be convenient, the lines are placed, as before, in positions of greatest utility, but without reference to any particular point. The positions of the lines are determined either by their intersections with surface features shown on the plan, by traversing, or by chained triangles. The method of finding the positions of the contour points is precisely the same as that already described.

Determining Contour Points from Sections.—If, instead of finding the positions of contour points on the ground, the sections on each of the lines be determined, the positions of the contour points on the plan can be readily found by a graphical process. The levels, after being reduced to a common datum, are plotted on the plan with their respective lines of section as datum lines (Fig. 86).

Draw a line at right angles to each datum line, as shown at A B (Fig. 86), on this line plot the levels of the contours. Through these points draw lines parallel to the datum line cutting the outline of the section at the points a, b, c, etc., and from these points drop perpendiculars b_5, c_{10}, d_{15}, etc., cutting the datum line at the contour points 0, 5, 10, 15, etc. The contour lines are obtained, as before, by joining points of equal altitude, by a fair curve.

This method is quicker in operation, but is not so accurate as that previously described.
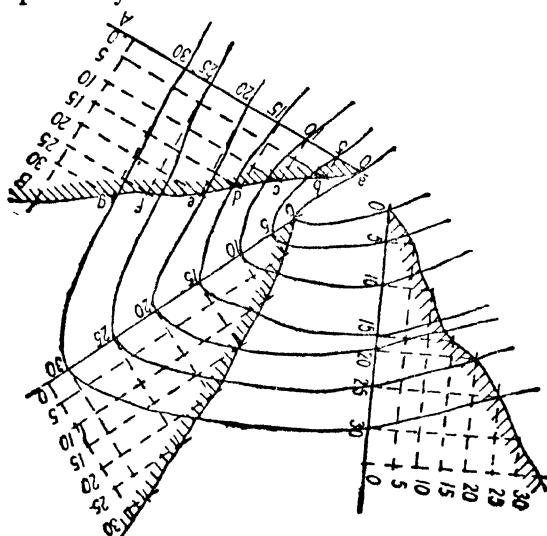
Fig. 86.

Determining Contour Points by means of the Clinometer.—The contour points along the lines of levels may be determined by means of the clinometer and chain. Since the vertical interval is a known quantity, if the slope of the ground is determined
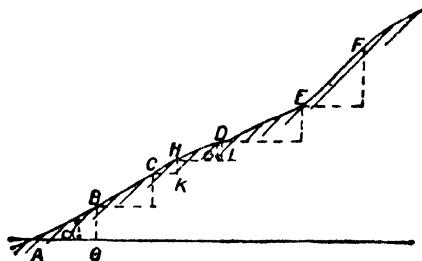
Fig. 87

the horizontal equivalent may be calculated. To take a simple illustration, suppose Fig. 87 represents the section of a hill,

and commencing at A, we are to determine points B, C, D, etc., at 5 feet vertical intervals. With the clinometer, determine the inclination (a) of the ground,

then $$\frac{A\,G}{G\,B} = \cot\,a,$$

or $$A\,G = G\,B\cot a = 5\cot a.$$
$$\text{Let } a = 5°, \text{ then}$$
$$A\,G = \cdot5 \times \cot 5°$$
$$= 5 \times 11\cdot43$$
$$= 57\cdot15 \text{ feet.}$$

The point B is at once located by chaining (horizontally) 57·15 feet in the given direction. If the slope of the ground remains constant, the horizontal equivalent of B C will also be 57·15 feet, and C is located by setting off 57·15 feet beyond B. Suppose the slope to change at the point H. Measure the horizontal equivalent of C H (say 25 feet), then,

$$H\,K = C\,K\tan 5°$$
$$= 25 \times \cdot0875$$
$$= 2\cdot187 \text{ feet}$$
$$= 2\cdot2 \text{ feet.}$$

At H again determine the slope of the ground, and suppose this to be 3·5°, and

$$H\,L = (5 - 2\cdot2)\cot 3\cdot5°$$
$$= 2\cdot8 \times 16\cdot35$$
$$= 45\cdot78 \text{ feet,}$$

this being chained from H locates the point D, 15 feet above A. The positions of the other points are determined in a similar way.

This method, even when carried out under favourable conditions, can only give approximate results. It is, however, occasionally of service on very steep lines of fairly constant slope, where ordinary levelling methods would be slow and tedious.

Spot Level Plans.—For engineering purposes spot level plans are largely superseding the ordinary methods of representing surface undulations. These plans are more quickly and cheaply prepared than contoured plans, and give all the information that is necessary to the engineer.

The ground to be dealt with is covered with a network of squares from 10 to 100 feet side, the length of the side of the squares depending on the object of the operations, the scale of representation, and the nature of the ground. The corners of the squares are marked with pegs, and the level of the ground

at each peg relative to a datum point is determined. The squares are drawn on the plan, and the level of the ground at the corner of each square is written thereon in figures ; the levels of intermediate points, when required, are interpolated.

If required, contour lines may be drawn on the plan, from the information given by the spot levels, by the graphical method given on p. 149.

Use of Contouring Rod.—In this method the individual contours are determined and marked out on the ground. The positions of the contour points, relative to the surface features, are afterwards determined by survey lines and offsets in the usual way. There should be two parties engaged on the work, one party being responsible for the levelling, and the other for the survey work.

To facilitate the levelling operations, contour points should be established along two straight lines arranged at the beginning and end of the contours. The positions of the points are determined as described on p. 149. When these points have been marked out, the levelling staff is replaced by the contouring rod. This is merely a straight rod of pine or deal about 10 feet long, 3 inches broad, and $\frac{3}{4}$ inch thick. A piece of white cardboard about 3 inches square, on which is drawn a horizontal black line about $\frac{3}{16}$ inch wide is fixed to the face of the rod in the required position by drawing pins.

The use of the rod is very simple. The level is set up to command one or more of the contour points already marked out, the contouring rod is held vertical at one of the points, and the card adjusted on the rod until the centre of the black mark is in the line of collimation. When this has been done, it is obvious that wherever the rod is held, the foot of the rod will be at the same level as the starting point if the mark on the card is in the collimation line. Hence, after the card has been adjusted in position, the assistant carries the rod forward in the direction of the contour, when he has got sufficiently far from the initial point—as determined by the scale of the plan or a change in the direction of the contour—he sets up the rod and moves it up or down as signalled by the leveller until the mark on the card is in the line of collimation. The assistant then marks the position of the foot of the rod and moves on to the next point. The work continues in this way until it is necessary to " change level." When this occurs the leveller warns his assistant, who erects the rod at the last determined contour point, and the leveller, having set up his instrument in a new position, sights

the contour rod and directs his assistant to raise or lower the card until the mark is in the new line of collimation. after which the assistant moves forward to establish the next contour point. Proceeding in this manner, the contour is followed and marked out until it closes or until it passes the limits of the ground to be contoured. The last reading should be taken on the corresponding contour point on one of the straight lines already referred to.

If the vertical interval is small, two or more contours may be established at the same time, an assistant with a contour rod being deputed to each contour line.

If the ground has already been surveyed, the survey party should begin its work of surveying the contour points as soon as the levelling party has got sufficiently far ahead, whereas if the ground has not been surveyed, this part of the work should be left until the whole of the ground has been contoured, when the positions of the pegs may be picked up as the survey work proceeds.

To prevent confusion, the tops of the pegs on each contour line should be painted a distinctive colour.

Contours by Boning Rods and Hand Level.—This method may be used where a small piece of ground is to be contoured expeditiously and extreme accuracy is not required.

Four boning rods and some form of hand level are required in this system. One of the boning rods is made shorter than the others by an amount equal to the distance between the centre of the level and its base.

For the sake of clearness, we will suppose that the boning rods are 4 feet long, and that the ground is to be contoured in 6 feet vertical intervals, from the datum point. On the stem of the short rod, nail a cross piece, so placed that when the level rests on the cross piece its line of collimation is 3 feet above the foot of the rod. This rod is now placed upright on the datum point with the level resting on the cross piece, and the ground is sighted in any convenient direction. Clearly, the point on the ground, which is apparently cut by the collimation line, is 3 feet above the datum point. The point thus found is temporarily marked, the boning rod is moved forward and set up at the mark, a forward sight with the level, by its apparent intersection with the ground, gives the position of a point 6 feet above the datum. This point is marked with a peg, and other points are determined in the same way at the given vertical interval.

If the ground is falling, one of the other boning rods is held upright by an assistant and moved down the slope until the top

of the rod is in the line of collimation. The foot of the second rod is now 1 foot below the point on which the leveller's rod stands. The leveller moves his rod forward into the position occupied by his assistant's rod, which is then carried down the slope until the next point 1 foot lower is determined. The rods are moved forward in turn, step by step—each vertical interval of 6 feet being marked with a peg—until the last contour point is attained. If much down-hill work has to be done, it is better to lengthen the forward rod 2 feet, by nailing a piece of wood to its stem. With this longer rod the 6 feet fall will be attained in two steps in place of six. The piece of wood being afterwards knocked off, the rod resumes its normal length.

When a contour point has been determined on each contour line, other points are found in the following manner :—The reflecting level is placed on the top of its rod, and an assistant carries a rod forward and adjusts it in position with its foot on the ground until the top of the rod is in the line of collimation. The leveller now lays aside the level and its rod, and uses one of the other rods. A second assistant takes the third rod, which he sets up in a forward position, being directed by the leveller to place it so that the tops of the three rods are in line. The feet of the three rods are now on the same contour line.

The positions of the two forward rods having been marked, the three men move forward, the two rear men set up their rods at the two points last determined, and the rod of the forward man is again " sighted in," to discover the position of the next contour point. This is marked and the men again move forward. In this way the contour line is rapidly marked out.

The same process is repeated on each contour line, and the positions of the lines are afterwards surveyed.

EXAMPLES.

1. The top of a mountain is known to be 50 miles from a certain station at the sea level, while a luminous point on its crest is visible just above the horizon at the station ; what is the probable height of the mountain ? Assume the diameter of the earth equal to 41,778,000 feet.   (*Ans.* 1,430 feet.)

2. To an observer at a height of 120 feet above sea level a luminous point on the top of a hill is visible just above the horizon. The top of the hill is known to be 45 miles from the observer's station. Assuming the radius of the earth to be 3,955 miles and the correction for refraction $\frac{1}{7}$ of that for curvature, find the height of the hill.   (*Ans.* 532·1 feet.)

3. Show that the difference of level of two points may be accurately determined by the operation known as reciprocal levelling.

4. Why is it necessary in levelling on a long gradient to place the level midway between the back- and fore-sights ? If this is not done, what effect would be produced on the section ?

5. If in taking levels along a line of section you encountered an obstacle in the form of (1) a lofty park wall, or (2) a tidal river so wide that the graduations on the level staff could not be distinctly seen, state how you would deal with these difficulties.

6. Draw up a level book for the following staff readings. The back-sights are shown in heavier figures.

12·84	9·32	6·54	8·39	13·20	2·81
4·26	6·72	7·32	9·85	**1·82**	4·64
7·82	9·36				

Take the reduced level of the first reading at 30 feet above datum.   (*Ans.* Fall between first and last point, 14·94 feet.)

7. Cast up the following level bookings and plot the cross-section at 150 feet. Use horizontal and vertical scales of 40 and 10 feet to one inch respectively :—

Back-sight.	Inter-mediate.	Fore-sight	Distance (Feet).			Reduced Levels.
			Left.	Centre.	Right.	
12·84				00·0		20
	8·65			50		
	3·84			75		
		1·41		110		
10·21						
	8·42			150		
	13·61		120	150		
	10·40		85	150		
	11·34		50	150		
	9·64		25	150		
	7·92			150	25	
	3·80			150	64	
	4·75			150	92	
	8·62			150	110	
	11·51			150	130	
	8·32			175		
	3·49			200		
		8·65		250		

(Total rise = 12·99 feet.)

8. Cast up the given level book. You may use either the rise and fall or the collimation system of casting. The reduced level of the first point on the section is 300·5 feet above ordnance datum.

Back-sight.	Inter-mediate.	Fore-sight	Distance.	Remarks.
13·45				On Ordnance B.M. 278·06.
		2·40		
12·89				
		1·50		
10·64			000	Commencement of section at P on plan.
	8·42		050	
	4·64		100	
		12·42	140	
2·65				
	10·86		200	
	3·62		300	At foot of high wall.
		—6·45	300	Staff inverted. Foot of staff flush with top of wall on near side.
—7·62			302	Staff inverted. Foot of staff flush with top of wall on far side.
	4·42		302	At foot of wall.
	8·69		400	
	10·62		450	
		5·30	500	End of section, at Q on plan.

(Total rise = 16·84 feet.)

9. Describe fully the process of taking a series of cross-sections with the level.

10. Write a short essay on surface contours, and the various methods of obtaining them.

11. What advantages are obtained by fitting stadia wires to a spirit level? How would you run a line of levels with an instrument fitted with these wires, without using a chain on the section?

12. It is proposed to construct an ornamental pond or lake of about 8 acres in extent. The maximum depth of water is known, and the ground is such that a dam is not required. State clearly how you would proceed to carry out the necessary surveying and levelling operations, from which the plan of the lake could be prepared.

13. Describe how you would prepare a spot level plan of the head of a valley, which it is proposed to utilise for the construction of an impounding reservoir. Show by a sketch how—from the information given by the spot level plan—the volume of the impounded water may be calculated.

14. Explain how you would proceed to contour the head of a valley, using a clinometer and three boning rods.

15. Describe how you would contour a piece of ground with a water level and a contouring rod.

CHAPTER VII.

CUTTINGS AND EMBANKMENTS.

General Remarks.—When the section along the centre line of a proposed line of communication has been prepared, the proposed foundation (or formation) level is shown on the section by straight lines, drawn in such positions that (*a*) the earth removed from cuttings may—as nearly as possible—be equal to that required for embankments, and (*b*) the gradients are not greater than a predetermined maximum.

The gradient of a road is given by the tangent of the angle that the road surface makes with the horizontal, and is generally expressed by a fraction, such as $\frac{1}{25}$, $\frac{1}{420}$, etc., meaning thereby that the road surface rises or falls uniformly 1 foot for each 25 or 420 feet of horizontal distance.

The formation level is generally 2 feet below the finished road surface.

The reduced level of the first point on a given gradient and the amount of the gradient being known, the reduced level of any other point on the formation is found by multiplying the distance of the point from the beginning of the gradient by the fraction representing the gradient, and adding (algebraically) the quantity thus obtained to the reduced level of the first point in the gradient. Thus, if the reduced level of the first point is 125·34, and the gradient $\frac{1}{120}$ up, the reduced level of a point on the formation 250 feet from the beginning of the gradient is $\frac{250}{120}$ + 125·34 = 127·42 feet. If the gradient is $\frac{1}{400}$ down, the reduced level of a point at a distance of 500 feet from its commencement at level 185·34 is, − $\frac{500}{400}$ + 185·34 = 184·09 feet.

Obviously, if the formation is lower than the surface of the ground at any point, a cutting will be necessary, and if higher the ground must be filled in to the required level.

Height of Embankment or Depth of Cutting.—The depth of a cutting or height of a bank at any point is obtained by taking the difference of the reduced level of the surface and formation at the point, a negative result indicating a bank.

Marking out Embankments and Cuttings.—Before the construction of the road is commenced, it is the duty of the engineer to mark out on the ground the toes of embankments and the tops of cuttings.

This is usually done by laying off half-breadths on each side of the centre line at each cross-section. These half-breadths are determined by measurement from a drawing of the cross-section, by calculation, or by a method of trial and error at each cross-section.

The sum of the half-breadths plus an allowance for fencing gives the whole width of land required at each cross-section, and from these whole widths the area of land required may be calculated.

The width of land required will depend on (*a*) the width of road surface, (*b*) the depth of cutting or height of bank necessary, (*c*) the side slope adopted, and (*d*) the allowance for fences.

The width of the road is settled by the purpose for which it is intended.

The side slope is the cotangent of the inclination of the side of the cutting or bank, and is usually expressed by a fraction, as $\frac{1\frac{1}{2}}{1}, \frac{2}{1}, \frac{3}{1}$, etc., meaning thereby that the side of the work rises or falls 1 foot for each $1\frac{1}{2}$, 2, 3, etc., feet of horizontal distance. The amount of the side slope will depend on and be less than the permanent slope of natural stability of the material on the site. Where the ground is expensive, as in towns, side slopes are replaced by retaining walls and banks are often replaced by viaducts.

It is obvious that it would be inexpedient to make drawings of all the cross-sections on a long road for the purpose of determining the half-breadths, nor is this necessary, since on most ground the outline of the cross-section will be more or less straight, and consequently the half-breadths and areas of the cross-sections are not difficult to calculate.

The method of calculating the half-breadths is illustrated by the following cases, in which

w = width of formation,
d = depth of cutting or height of bank,
r = the side slope,
$\frac{h}{l}$ = gradient of the surface of the ground on the cross section,
F = allowance for fences on each side.

Case I.—When $h = 0$ or the ground is level across, and $d = 0$, or the formation is in the surface of the ground.

$$\text{Each half-breadth} = \frac{w}{2} + F,$$

$$\text{and the whole width} = 2\left(\frac{w}{2} + F\right).$$

Case II.—When $h = 0$. Referring to Fig. 88, it will be seen that each half-breadth

$$= H F + F C + F$$

$$= \frac{w}{2} + r\,d + F,$$

and $$\text{whole width} = 2\left(\frac{w}{2} + r\,d + F\right).$$

Fig. 88.   Fig 89

Case III.—Where the surface is straight across, as shown in Fig. 89, and no side bank is required.

Through any point J (Fig. 89) draw a horizontal line J K, from K drop a perpendicular cutting D E in the point L. Through L draw the lines L M, L N parallel to the side slopes B D and E A respectively.

$$\text{Let J K} = l, \text{ K L} = h,$$
also $$\text{L J} = \sqrt{h^2 + l^2} = \text{S.}$$

The surface half-breadths are C D and C E, and the horizontal half-breadths are P D and E S.

Now, $\qquad\dfrac{\text{C D}}{\text{C G}} = \dfrac{\text{L J}}{\text{M J}},$

because the triangles C G D and L J M are similar.

$\therefore\qquad \text{C D} = \dfrac{\text{C G . L J}}{\text{M J}},$

$$= \dfrac{\left(\dfrac{w}{2} + r\,d\right)\text{S}}{\text{K J} - \text{K M}}$$

$$= \dfrac{\left(\dfrac{w}{2} + r\,d\right)\text{S}}{(l - r\,h)}.$$

The horizontal half-breadth

$$\text{P D} = \text{C D} \times \dfrac{\text{P D}}{\text{C D}}$$

$$= \text{C D} . \dfrac{\text{K J}}{\text{L J}}$$

$$= \dfrac{\left(\dfrac{w}{2} + r\,d\right)}{(l - r\,h)}\,l,$$

and the total horizontal half-breadth

$$= \dfrac{\left(\dfrac{w}{2} + r\,d\right)l}{(l - r\,h)} + \text{F}.$$

Considering the lower side, we have :

$$\dfrac{\text{C E}}{\text{C F}} = \dfrac{\text{L J}}{\text{N J}},$$

since the triangles C E F and J L N are similar.

$\therefore\qquad \text{C E} = \dfrac{\text{C F . L J}}{\text{N J}}$

$$= \dfrac{\left(\dfrac{w}{2} + r\,d\right)\text{S}}{\text{J K} + \text{K N}}$$

$$= \dfrac{\left(\dfrac{w}{2} + r\,d\right)\text{S}}{(l + r\,h)},$$

and the horizontal half-breadth

$$E S = C E . \frac{K J}{L J}$$

$$= \frac{\left(\frac{w}{2} + r d \right) l}{(l + r h)} \; ;$$

the total horizontal half-breadth
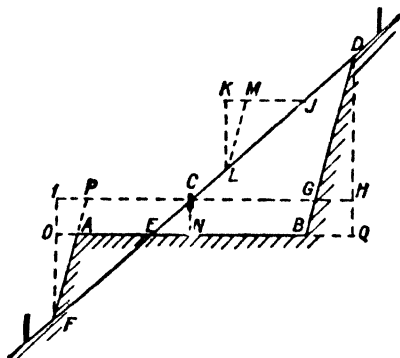
$$= \frac{\left(\frac{w}{2} + r d \right) l}{(l + r h)} + F.$$

Fig. 90.

The whole breadth at the section

$$= E S + P D + 2 F$$

$$= \frac{\left(\frac{w}{2} + r d \right) l}{(l - r h)} + \frac{\left(\frac{w}{2} + r d \right) l}{(l + r h)} + 2 F$$

$$= \frac{2 l^2 \left(\frac{w}{2} + r d \right)}{(l^2 - r^2 h^2)} + 2 F.$$

Case IV.—The finished work is partly in cutting and partly in embankment.

Referring to Fig. 90, it is evident that the surface half-breadth of the cutting

$$C D = \frac{\left(\dfrac{w}{2} + r\,d\right) S}{(l - r\,h)},$$

and the total horizontal half breadth

$$C H + F = \frac{\left(\dfrac{w}{2} + r\,d\right) l}{(l - r\,h)} + F$$

Considering the lower side, we have

$$\frac{C F}{C P} = \frac{J L}{J M}, \text{ since the triangles C P F and J M L are similar.}$$

$$\therefore \quad C F = \frac{C P \cdot J L}{J M},$$

$$= \frac{\left(w - \overline{\dfrac{w}{2} + r\,d}\right) S}{l - r\,h}$$

$$= \frac{\left(\dfrac{w}{2} - r\,d\right) S}{(l - r\,h)}.$$

The horizontal half-breadth

$$C I = C F \frac{J K}{J L},$$

$$= \frac{\left(\dfrac{w}{2} - r\,d\right) l}{(l - r\,h)}.$$

The total horizontal half-breadth

$$= \frac{\left(\dfrac{w}{2} - r\,d\right) l}{(l - r\,h)} + F,$$

and the whole width

$$= C I + C H + 2 F$$

$$= \frac{\left(\dfrac{w}{2} - r\,d\right) l}{(l - r\,h)} + \frac{\left(\dfrac{w}{2} + r\,d\right) l}{(l - r\,h)} + 2 F.$$

$$= \frac{w\,l}{(l - r\,h)} + 2 F.$$

Case V.—When the surface of the ground is irregular on the cross-section, the half-breadths cannot be directly calculated. These distances may be measured from a drawing of the cross-section, or obtained on the ground by a method of approximation. To explain this more clearly, we will assume that the width at formation level is 30 feet, depth at centre 12 feet, side slopes 2 to 1.

We first calculate the half-breadth on the assumption that the ground is level across, thus (in Fig. 91) C D = $(\frac{30}{2} + 2 \times 12)$ = 39 feet. Laying off this distance (horizontally) gives the point E vertically over D. Find the difference of level of C and E (say) 3 feet, then E F = 3 × 2 = 6 feet. Lay off 6 feet from E, and thus find the point G. Determine the difference of level of E and G (say) 1 foot, then G H = 2 feet. Find the point I by making G I = 2 feet, and let the difference of level of G and I be ·25 foot, then I J = ·5 foot. On laying off ·5 foot
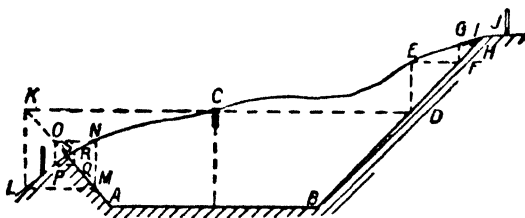
Fig. 91.

the point of intersection J of the side slope and the surface of the ground is obtained. Clearly, the half-breadth on the rising side is 39 + 6 + 2 + ·5 = 47·5 feet.

The same process is repeated on the falling side, care being taken to lay off the calculated distances towards C when the difference of level between two points is a fall, and from C when it is a rise. Thus C K = 39 feet, difference of level of C and L = − 4 feet, therefore L M = − 8 feet; difference of level of L and N = 2 feet. ∴ N O = 4 feet; difference of level of N and P = − 1 foot, ∴ P Q = − 2 feet; difference of level of Q and R = ·3 foot, ∴ R S = ·6 foot.

Hence the lower half-breadth

$$= 39 - 8 + 4 - 2 + ·6$$
$$= 33·6 \text{ feet,}$$

and the whole width $= 47·5 + 33·6 + 2 \text{ F}$
$$= 81·1 + 2 \text{ F feet.}$$

All the above formulæ apply equally well to embankments, as is at once evident if the figures in this section are reversed. The half-breadths are plotted on the plan of the proposed road, and the plotted points being joined by a fair curve, show the shape and extent of the land required.

Grade Staff Method.—This is a decided improvement on the previous method, and is the method in general use on railway construction works, in the United States, and in Canada.

In applying this method, the first duty of the engineer is to determine the staff reading when the staff is held on the formation (or " Grade ") level, at the cross-section he is about to mark out.

Having obtained the height of his instrument by levelling from the nearest bench mark, the engineer consults the longitudinal section from which he obtains the height of the grade
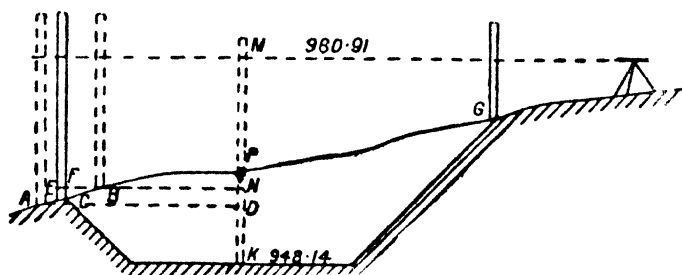
Fig. 92.

at the cross-section. The difference of these heights gives the grade staff reading.

For example, let the staff reading when held on the B.M. = 4·10, and the reduced level of the B.M. = 956·81.

Height of instrument	= 956·81 + 4·10	
	= 960·91.	
If the reduced level at formation	= 948·14,	
Grade staff reading	= 960·91 − 948·14	
	= 12·77.	

The actual staff reading (6·39) at the centre peg (P, Fig. 92) is then determined, and consequently the depth of cutting at the section is

$$12·77 - 6·39 = 6·38.$$

To locate the positions of the slope pegs at F and G (Fig. 92), the staffman judges the position of the point at which the slope meets the surface, erects the staff at the point selected, and the leveller obtains the staff reading. The leveller then calculates the " cut " (or " fill ") at the point, and from this obtains the distance of the staff from the centre peg. If this distance agrees with its measured distance the point selected is the point sought. As a rule, however, several trials are necessary before the exact spot is located.

For example, the staffman judges the point A to be the position of the required point, and the staff reading at A is (say) 9·36. The height of the cutting at A above the grade

$$= D K = M K - M D$$
$$= 12·77 - 9·36$$
$$= 3·41.$$

If the width of the road = 40', and side slopes 1½ to 1, the half-breadth corresponding to a height of

$$3·41 = 3·41 + 1·70 + 20$$
$$= 25·11$$
$$= C D.$$

Suppose the actual distance = 28·2.

The staff is too far out, and the staffman makes a second trial (say) at B. Let the staff reading at B equal 7·97. The corresponding height of cutting = 12·77 − 7·97 = 4·80, and corresponding distance from centre line

$$= 4·8 + 2·4 + 20$$
$$= 27·2,$$

but the actual distance = (say) 24·5, so the staffman must make a third trial. We will suppose that the third point selected is the point F, and the corresponding staff reading is 8·50.

Therefore, height of cutting at F $= 12·77 - 8·50$
$= 4·27,$
and distance from centre line $= 4·27 + 2·13 + 20$
$= 26·40'.$

This, agreeing with the actual distance (26·5) with a sufficient degree of closeness, the slope peg is driven at the point occupied by the staff.

The position of the point G is determined in a precisely similar way.

In this system the notes for the cross-section are entered as follows :—

L.	C.	R.
$\dfrac{+\ 4\cdot27}{26\ 5}$	$+\ 6\ 38$	$\dfrac{+\ 10\ 3}{32\ 5}$

The figures in the numerators of the fractions give the heights of the slope pegs above the grade, and the figures in the denominators the distances of the pegs from the centre, on the left and right respectively. The figures in the centre give the depth of the cutting at the centre.

In the case of an embankment, the figures denoting heights are marked with a negative sign.

Grade Pegs.—These are put in wherever an embankment ends and a cutting begins, or *vice versâ.* The point at which this occurs will be at the same height as the formation level at the section passing through it, and at a distance equal to half the width of the road from the centre line.

The position of the point will be roughly indicated by the general trend of the slope pegs already set out, but its exact position can only be determined by trial and error. The staff-man erects his staff at a point which in his estimation corresponds with the position of the point sought, and the leveller takes the staff reading. If this reading agrees with the " grade staff reading " for the cross-section on which the staff stands, and the staff is half the width of the road from the centre line, the point selected by the staffman is the point required. If not, further trials must be made until the point satisfying these conditions is found.

The position of the grade peg may be obtained by calculation if the slope of the ground is straight in the direction of the centre line.

We will illustrate this method by an example selected from the level book on pp. 178 and 179. On referring to the level book, it will be seen that a grade point occurs between 300 and 400 feet on the section. The half-width of the road is 24 feet; the reduced level at $300' = 24\cdot40$, and the reduced surface level at 24' out

$$= 24\cdot40 - \frac{24}{4} = 18\cdot40.$$

The reduced level of the formation at 300', 24' out
$$= 22 \cdot 99.$$

The reduced surface level at 400', 24' out
$$= 27 \cdot 27 - \frac{24}{8} = 24 \cdot 27,$$

and the reduced level formation at 400', 24' out
$$= 23 \cdot 49.$$

Let D = distance of grade point from the 300 feet peg.

Then
$$\frac{D}{100 - D} = \frac{22 \cdot 99 - 18 \cdot 40}{24 \cdot 27 - 23 \cdot 49}$$
$$= \frac{4 \cdot 59}{\cdot 78},$$

∴   D = 85·5 feet.

The distance of the grade peg is 385·5 feet from the commencement of the section.
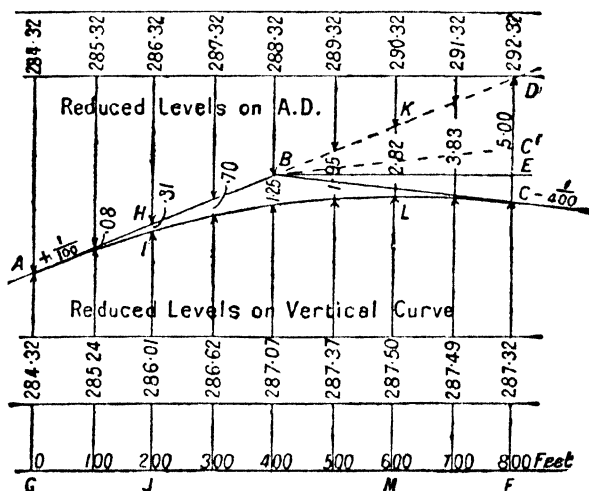
Fig. 93.

Vertical Curves.—Where a change of gradient with an algebraic difference greater than ·2 foot per 100 feet occurs in railway construction, a gradual change in direction from one gradient to the other is obtained by connecting the two gradients by a vertical curve.

These curves are usually parabolic in outline, as this form of curve, being flat near the tangent points, gives a very gradual change in direction, and the reduced levels of the points on the curve are easily worked out. The curves are usually longer at the bottoms of dips than on summits, but for convenience in setting out on a long line a standard length is adopted for each.

The curves are set out with their axes vertical, and consequently the horizontal projections of their tangents are equal.

Suppose it is required to connect two gradients of $+\frac{1}{100}$ and $-\frac{1}{400}$ by a curve 800 feet in length.

Produce either gradient (say A B) as shown in Fig. 93, and from the tangent point C draw the ordinate cutting A B produced in D. Through B draw a horizontal line cutting C D in E.

To find C D.

We have   D E = B E × gradient of A B.

           C E = B E × gradient of B C.

∴   D E + C E = B E × algebraic difference of gradients,

or $C D = \dfrac{800}{2} \left\{ \dfrac{1}{100} - \left(- \dfrac{1}{400} \right) \right\}$

           = 5 feet.

If B C had a gradient of $+ \dfrac{1}{400}$ (shown dotted), the ordinate through D would cut the gradient in C′, and D C′ would

$$= \frac{800}{2} \left(\frac{1}{100} - \frac{1}{400} \right)$$

$$= 3 \text{ feet.}$$

Ordinates to the curve are next obtained, thus :—

$$\frac{H I}{C D} = \frac{G J^2}{G F^2}$$

or $H I = \dfrac{C D . G J^2}{G F^2}.$

similarly, $K L = \dfrac{C D . G M^2}{G F^2}.$

The ordinate at 300′ $= 5 \times \dfrac{300^2}{800^2}$

$$= \cdot 70 \text{ foot.}$$

The ordinate at 650′ $= \dfrac{5 \times 650^2}{800^2}$

$$= 3 \cdot 30 \text{ feet.}$$

To obtain the reduced levels of points on the curve, the calculated ordinates are subtracted from (or added in the case of a dip curve to) the reduced levels of the corresponding points in A D. The method is clearly shown in Fig. 93.

The reduced levels of points on a dip curve are found in a similar manner, as will at once be evident if Fig. 93 is inverted.

Areas of Sections.—Before proceeding to calculate the volume of earthwork to be dealt with in the construction of a road, it is necessary to determine the areas of the cross-sections. We will consider each of the cases already given.

Case I.—If the foundation of the road is laid on the original surface of the ground the area of the cross-section is zero. When the surface soil is removed to a depth d the area of the cross-section is $w\,d$.

Case II.—Referring to Fig. 88, the area of the cross-section

$$= \left(\frac{A\,B + D\,C}{2}\right) d,$$

$$= \frac{\left\{w + 2\left(\frac{w}{2} + r\,d\right)\right\}}{2}\,d,$$

$$= (w + r\,d)\,d.$$

Case III.—Referring to Fig. 89, the area of the cross-section = area of triangle E D R — area of triangle A B R,

$$= \left(\frac{E\,S + P\,D}{2}\right).\,C\,R - \frac{A\,B\,.\,Q\,R}{2},$$

$$= \left\{\frac{\left(\frac{w}{2} + r\,d\right) l}{l + r\,h} + \frac{\left(\frac{w}{2} + r\,d\right) l}{l - r\,h}\right\}\left(d + \frac{w}{2r}\right) - \frac{w^2}{4r}$$

$$= \frac{\left(\frac{w}{2} + r\,d\right)^2 l^2}{r\,(l^2 - r^2\,h^2)} - \frac{w^2}{4r}.$$

Case IV.—Area of cutting = area of triangle E B D (Fig. 90)

$$= \frac{E\,B\,.\,Q\,D}{2},$$

$$= \frac{(E\,N + N\,B)\,(Q\,H + H\,D)}{2},$$

$$= \frac{\left(\dfrac{d\,l}{h} + \dfrac{w}{2}\right) \left\{ d + \dfrac{\left(\dfrac{w}{2} + r\,d\right) h}{(l - r\,h)} \right\}}{2}$$

$$= \frac{(w\,h + 2\,d\,l)^2}{8\,h\,(l - r\,h)}.$$

Area of bank = area of triangle A E F (Fig. 90)

$$= \frac{A\,E \cdot O\,F}{2},$$

$$= \frac{(A\,N - E\,N)\,(I\,F - I\,O)}{2},$$

$$= \frac{\left(\dfrac{w}{2} - \dfrac{d\,l}{h}\right) \left\{ \dfrac{\left(\dfrac{w}{2} - r\,d\right) \cdot h}{(l - r\,h)} - d \right\}}{2}$$

$$= \frac{(w\,h - 2\,d\,l)^2}{8\,h\,(l - r\,h)}.$$

Case V.—In this case the area must be obtained from a drawing of the cross-section by one of the methods given in Chapter VIII.

Case VI.—Assuming that the surface of the ground is straight
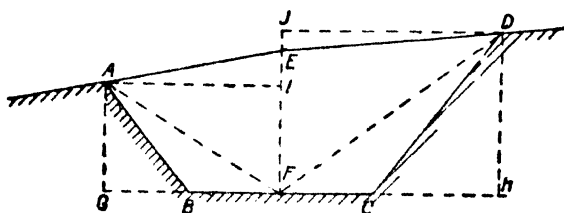
Fig. 94.

between the centre and slope pegs, and the section has been set out by the " Grade Staff " method, the area is easily calculated.

Referring to Fig. 94, it is evident that the area of the section A B C D = sum of the areas of the triangles A B F, A F E, D F E, and F C D.

Area of triangle A B F $= \dfrac{B F . A G}{2}$.

Area of triangle A F E $= \dfrac{E F . A I}{2}$,

$$= \dfrac{E F . G F}{2}.$$

Area of triangle D F E $= \dfrac{E F . J D}{2}$,

$$= \dfrac{E F . F H}{2}.$$

Area of triangle F C D $= \dfrac{F C . D H}{2}$.

$\therefore$ Area of section $= \dfrac{B F . A G}{2} + \dfrac{F C . D H}{2} + \dfrac{E F}{2} (G F + F H)$

$$= \dfrac{w}{4} (A G + D H) + \dfrac{E F}{2} (G F + F H),$$

= ¼ width of road × sum of side fills (or cuts) + ½ centre fill (or cut) × sum of distances out.

For example, let the booking at a particular section be :—

L.	C.	R.
$\dfrac{+\ 14\cdot8}{49\ 5}$	$+\ 8\cdot46$	$\dfrac{+\ 10\ 25}{38\ 5}$

and the width of the road 30 feet.

The area of the section

$$= \dfrac{30}{4} (10\cdot25 + 14\cdot8) + \dfrac{8\cdot46}{2} (49\cdot5 + 38\cdot5)$$

$$= 560\cdot1 \text{ square feet}$$

$$= 62\cdot2 \text{ square yards.}$$

Areas from Plotted Sections.—Many engineers and quantity surveyors prefer to obtain the areas of cross-sections from the plotted sections by aid of a planimeter (p. 214)   This instrument gives the actual area of paper covered by the cross-section in square inches or square feet, as the case may be.   In converting

this area to the true area, the exaggeration of outline caused by the vertical being greater than the horizontal scale must be allowed for. Let the drawn area of the cross-section as obtained with the planimeter, or in any other way, be A square inches, the vertical and horizontal scales be x and y feet to 1 inch respectively, then the true area = A x y square feet.

The quantity x y is termed the " coefficient of distortion."

If A = 2·543 square inches, x = 10 feet to 1 inch, and y = 30 feet to 1 inch, the true area of the cross-section = 2·543 × 30 × 10 = 762·9 square feet.

Volumes of Earthwork.—In computing the quantity of earth-work to be dealt with in forming a cutting or embankment, it is usual to find the volume between certain cross-sections, and the summation of these volumes gives the total volume required.

When the outline of the ground on the longitudinal section is straight and the cross-sections of a regular character, the distance between the two end sections may be considerable, but if the outline of the ground is irregular the distance between the two end sections must be short.

To facilitate the computation of earthwork quantities, Sir John Macneill, Mr. Bidder, and others have prepared and published earthwork tables. These tables give either the mean sectional area when the width at formation level, side slopes, and depths at the ends are known, or a number proportional to it. This number or mean area, multiplied by the distance between the end sections, gives the required volume.

In all formulæ for the determination of volumes some assumptions are made, which may, or may not, apply to the case under consideration. The selection of the formula to be used must be decided from the given data.

Case I.—Given two nearly equal cross-sections (areas A_0 and A_1) and their longitudinal distance apart (l), the approximate volume between the end sections is given by

$$V = l \frac{(A_0 + A_1)}{2}.$$

Case II.—When the ground is level across and the area (B) of the section midway between the end sections is assumed, the volume

$$V = \frac{l}{6}(A_0 + 4B + A_1).$$

12.

Prismoidal Formula.—This formula—known as the prismoidal formula—forms the basis of Sir John Macneill's earthwork tables, and is obtained from the following considerations :—

Let A B C D E F G II (Fig. 95) be the prismoid whose volume is required, of which C D = E H = width (w) at formation level, A B and G F are parallel to each other and parallel to D C and E H. Also the end planes A B C D and G F E J are parallel vertical planes.

Imagine a plane parallel to the base to contain A B and cut the solid in the lines B I, I J, and J A, thus dividing the prismoid into a prism A B C D E I J H, and a wedge A B I F G J. Suppose the wedge to be divided into two pyramids I F G J B and A B J G, by the plane containing B, J and G.
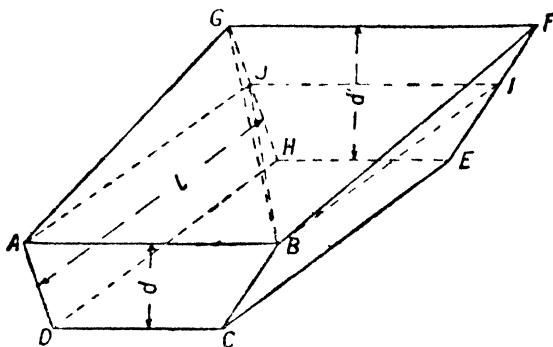
Fig. 95.

Then the volume of the prismoid = volume of prism A B C D E I J H + volume of pyramid I F G J B + volume of pyramid A B J G.

If d' and d be the greater and lesser end depths, the other ymbols being as before, we have—

$$\text{Volume of prism} = (w + r\,d)\,d\,l$$

$$\text{Volume of pyramid I F G J B} = \{(w + r\,d')\,d' - (w + r\,d)\,d\}\frac{l}{3}$$

$$\text{Volume of pyramid A B J G} = \frac{A\,B\,.\,A\,J}{2}\frac{(d' - d)}{3},$$

$$= (w + 2\,r\,d)\,(d' - d)\frac{l}{6}.$$

∴ Volume of prismoid

$$= (w + r\,d)\,d\,l + \{(w + r\,d')\,d' - (w + r\,d)\,d\}\,\frac{l}{3} + (w + 2\,r\,d)$$

$$(d' - d)\frac{l}{6}.$$

$$= \frac{l}{6}\{6\,(w + r\,d)\,d + 2\,[(w + r\,d')\,d' - (w + r\,d)\,d]$$

$$+ (w + 2\,r\,d)\,(d' - d)\},$$

which on reduction

$$= \frac{l}{6}\left\{(w + r\,d)\,d + 4\left[w + r\,\frac{(d + d')}{2}\right]\frac{(d + d')}{2} + (w + r\,d')\,d'\right\}$$

$$= \frac{l}{6}\,(A_0 + 4\,B + A_1).$$

We will take the following example to illustrate the use of the above formula :—Find the volume between two cross-sections 200 feet apart, formation breadth 30 feet, side slopes 2 to 1, depths at ends 35 and 50 feet, ground level across.

$$\begin{aligned}
\text{Area of small end} &= (w + r\,d)\,d \\
&= (30 + 2 \times 35)\,35 \\
&= 3{,}500 \text{ square feet.}
\end{aligned}$$

$$\begin{aligned}
\textbf{Area of large end} &= (w + r\,d')\,d' \\
&= (30 + 2 \times 50)\,50 \\
&= 6{,}500 \text{ square feet.}
\end{aligned}$$

$$\begin{aligned}
\textbf{Area of mid-section} &= \left(w + r\,.\,\frac{d + d'}{2}\right)\left(\frac{d + d'}{2}\right), \\
&= \left(30 + 2\,.\,\frac{35 + 50}{2}\right)\left(\frac{35 + 50}{2}\right), \\
&= 4{,}887\cdot5 \text{ square feet.}
\end{aligned}$$

Four times mid-area $= 19{,}550\cdot0.$

$$\begin{aligned}
\text{Volume} &= \frac{200}{6 \times 27}\,(3{,}500 + 19{,}550 + 6{,}500 \\
&= 36{,}481 \text{ cubic yards.}
\end{aligned}$$

By Sir John Macneill's tables, heights 35 and 50 feet, tabular number from Table XXII., $= 182\cdot41$, and volume

$$\begin{aligned}
&= 200 \times 182\cdot41 \\
&= 36{,}482 \text{ cubic yards.}
\end{aligned}$$

The above formula may be used to calculate the volume between the end sections when the ground has a sidelong slope,

but, if the end slopes are different, the slope on the assumed central section is taken as a harmonic mean between those at the ends, and the assumed depth as an arithmetic mean between the two end depths.

Case III.—Given three equidistant cross-sections (areas A_0, A_1, and A_2), and the distance (l) between the end sections, the best approximation is given by the equation

$$V = \frac{l}{6} (A_0 + 4 A_1 + A_2).$$

Case IV.—Given an even number (n) of equidistant cross-sections of areas A_0, A_1, etc., the common distance apart being d, then

$$V = d \left(\frac{A_0 + A_n}{2} + A_1 + A_2 \ldots + A_{n-1} \right).$$

Case V.—When the number of equidistant cross-sections is odd

$$V = \frac{d}{3} \{ A_0 + A_n + 2(A_2 + A_4 + A_6 \ldots + A_{n-2})$$
$$+ 4(A_1 + A_3 + A_5 \ldots + A_{n-1}) \}.$$

Quantities of earthwork in this country are usually given in cubic yards, and the data from which they are calculated is given in feet. The results given by the above formulæ must be divided by 27 (if the dimensions are in feet) to reduce them to cubic yards.

If longitudinal distances are given in Gunter's chains, and breadths and depths in feet, the calculated volumes will require to be multiplied by $\frac{66}{27}$ to give cubic yards.

Working Sections.—The level book for the working section of a proposed road is a book containing all the information necessary for the guidance of the engineer in marking out the site, the preparation of his estimates, and for the construction of the road. The number of columns and their arrangement varies in the practice of different engineers. One form of book is shown in the table on pp. 178 and 179, which gives the bookings for a proposed road, connecting two existing roads A and B. The centre line of the new road is straight in plan, and cuts the roads A and B at 70° and 82°, the acute angles being to the right and left respectively on the approaching side. The remaining data is as follows:—Width of road surface, 30 feet;

footpath on each side, 9 feet wide ; formation level, 18 inches below finished road surface, 6 inches below surface of footpath ; gradients, 1 in 200 up for the first 600 feet, and a uniform fall for the remainder of the distance ; side slopes, $1\frac{1}{2}$ to 1.

The section and plan of the road are shown in Fig. 96.

The formation level of the footpath has been worked to in determining cuttings and banks.

Plans of Cuttings and Embankments from Contours.—It is evident that if contours are drawn on the sides of all cuttings and banks, at the same levels as the contours on the surface of the ground, the points of intersection of corresponding contours will be points on the lines of intersection of the surface of the ground and the sides of the cuttings or banks. The outlines of the edges of the cuttings or banks will be obtained by joining the successive points of intersection. Thus in Fig. 97 (in which the contours on the side slopes are shown by dotted lines and those on the surface of the ground by full lines), the points of intersection of the corresponding contours at levels 0, 5, 10, etc., are indicated by the points p, q, r, etc. At v the 30 contour cuts the edge of the road at the same level, and at this point the bank ends.

To determine the contours on the side slopes, we proceed as follows :—Consider the cross-section at level 26 ; the difference of level of the edge of the road and the lowest contour is 26 feet ; assuming side slopes of $1\frac{1}{2}$ to 1, the distance of the point b (on the lowest contour) from the edge of the road is $1\frac{1}{2} \times 26 = 29$ feet. Set this distance off from a, the edge of the road, and from b set off distances $b\,c$, $c\,d$, etc., each equal to the vertical interval multiplied by the side slope, in the assumed case, $5 \times 1\frac{1}{2}$, or 7·5 feet. The points thus obtained are points on the successive contours on the side slope. On the cross-section at level 20, the distance from the edge of the road to the contour point at level 35 is $(35 - 20) \times 1\frac{1}{2} = 22·5$ feet, and the horizontal equivalent (for the same side slope) is again 7·5 feet, this stepped off (inwards) gives points on the 30 and 25 feet contours, as shown in the figure. Working in this way, contour points are found on each cross-section, and on joining points on the same level by a fair curve the required side slope contours are obtained.

When the road is straight the side slope contours will be parallel straight lines, which will be parallel to the centre line of the road, only when the road is horizontal. If the road is curved, and the curve is a circular arc, the contours on the

Back-sight.	Inter-mediate	Fore-sight.	Collima-tion Level.	Reduced Level. Surface.	Reduced Level. Forma-tion.	Distance.	Height of Bank.	Depth of Cutting	Half Left.
3·41			23·41	20·00					
	1·42			21·99	21·49	0,000		0·50	
	1·90			21·51	21·58	18	0·07		
	1·40			22·01	21·58	18		0·43	15
	1·10			22·31	21·61	24		0·70	25
	7·40			16·01	21·74	50	5·73		32·6
		13·60		9·81	21·99	100	12·18		42·3
10·62			20·43						
	14·81			5·62	22·00	103	16·38		48·6
	12·90			7·53	22·02	106	14·49		45·7
		1·25		19·18	22·49	200	3·31		38·6
13·54			32·72						(Bank)
	8·32			24·40	22·99	300		1·41	35·0
	5·45			27·27	23·49	400		3·78	25·0
	6·20			26·52	23·59	420		2·93	25·2
		2·94		29·78	23·99	500		5·79	32·7
4·69			34·47	27·14	24·49	600		2·65	32·9
	7·33								
	7·32			27·15	24·24	700		2·91	36·0
		10·92		23·55	23·99	800	0·44		(Cut) 28·2
1·25			24·80						
	6·43			18·37	23·75	900	5·38		32·1
	2·14			22·66	23·50	1,000	0·84		25·3
	0·84			23·96	23·47	1,010		0·49	24·8
	0·89			23·91	23·46	1,016		0·45	15·0
	1·34			23·46	23·46	1,016		0·00	15·0
	0·88			23·92	23·42	1,032		0·50	
		0·31		24·49					
23·51		29·02							

Breadths. Right.	Whole Widths.	Area of Section. Sq. Yds.	Quantities (Cubic Yards). Cutting.	Bank.	Remarks.
					B.M. (↑) cut in wall. P on plan.
					Spike in centre of road "A."
					In gutter.
		+ 3·34	6·7		On curb.
15	30·0				
25	50·0	+ 7·16			
			5·0		(Between 24 and 26·8). At fence, 4' high.
'32·6	65·2	− 32·7		112·6	(Between 26·8 and 50·0.) Cross-section level.
42·3	84·6	− 86·2		971·3	Edge of stream. Cross-section level.
				107·4	
48·6	97·2	−128·7			Centre of stream, cuts line at 75°. Acute angle on left.
				237·3	
45·7	91·4	−108·7			
		− 20·5		1,086·6	(Between 106 and 196). Edge of stream.
(Cut) 25·4	64·0	+ 0·2	0·1		(Between 196 and 200). Cross-section, 1 in 6. Lower side on left.
		− 5·8	235·2	438·0	
41·7	76·7	+ 21·2		55·0	(Between 300 and 385). Cross-section, 1 in 4. Lower side on left.
36·5	61·5	+ 28·3	826·0		Cross-section, 1 in 8. Lower side on left.
32·4	57·6	+ 21·2	165·0		Cross-section, 1 in 12. Lower side on left.
32·7	65·4	+ 39·7	815·0		Top of rise. Cross-section level.
24·3	57·2	+ 19·9	993·0		
		+ 23·2	212·0		(Between 600 and 632). Cross-section, 1 in 10. Lower side on right.
(Bank) 25·0	61·0	− 0·1	490·0	1·0	(Between 632 and 700). Cross-section, 1 in 7. Lower side on right.
		+ 4·3			
26·7	54·9	+ 1·8	459·3	32·6	Cross-section, 1 in 12. Lower side on right.
			10·5	27·6	(Between 800 and 822).
				456·0	(Between 822 and 900).
32·1	64·2	− 30·2			Cross-section level.
				505·0	
25·3	50·6	− 1·3			(Between 1,000 and 1,006). Cross-section level.
		+ 5·0	4·1	1·3	(Between 1,006 and 1,010). At fence, 3' 6" high.
24·8	49·6	+ 6·1			
			8·4		
15·0	30·0	+ 3·3			On curb.
15·0	30·0				In gutter.
					Spike centre of road "B."
					B.M. (↑) cut on curb at Q on plan.
Sums,			4,230·3	4,031·7	

same side slope will be concentric circles when the road is horizontal; when the road is inclined the contours will be so nearly concentric circles that they may be drawn as such without sensible error, provided the gradient is not excessive.

When the surface contours are approximately parallel to the road, as shown in the lower part of Fig. 97, the required points
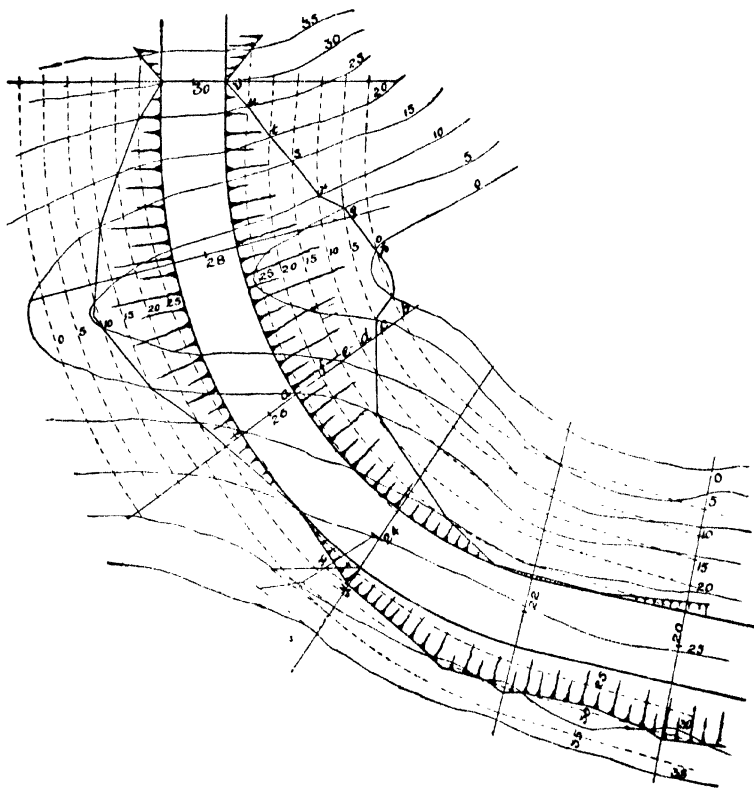
Fig. 97.

on the edge of the cutting or bank may either be obtained by the intersection of interpolated contours on both the surface of the ground and the side slopes, or a sufficient number of cross-sections may be drawn to give the required outline. In the figure, the

cross-section at level 24 is drawn to a datum of 24 feet, the point of intersection of the side slope and the surface of the ground is shown at x, and this point projected back to the plan of the cross-section, gives the point y, the point sought.

In using this method for determining the outlines of cuttings and banks, we may alter the datum level to suit the conditions of the piece of ground to be dealt with, provided we treat *all* the levels in the same way. Thus, in Fig. 97, if the actual reduced level of the zero contour is (say) 350·52, by deducting this number from all the given levels, the conditions of the problem are not altered, but the heights to be plotted are rendered more convenient by the reduction.

Great care must, however, be exercised to keep all the work to the same datum level on each part of the plan dealt with.

Many engineers prefer to use this method for obtaining the half-breadths on the cross-sections, the required half-breadths being scaled from the plan, and the slope pegs are set out on the ground to agree with the scaled distances. If the ground has not been contoured, the contour lines may be obtained from the cross-sections, as explained on p. 149.

In preparing the estimates for a proposed line of communication involving cuttings and banks, it is important that the engineer should know where the material excavated from the cuttings will be deposited to form the banks, and also the positions at which there will be an excess or defect of material. With this information at his disposal the engineer is in a position to estimate the probable cost of hauling the material from its original to its final position, and to decide when it is more economical to "waste" and "borrow," the criterion for this being, of course, that it is more economical to waste and borrow when the cost of loosening, filling, hauling, and depositing of 1 cubic yard of material is greater than the cost of the corresponding operations on 1 cubic yard obtained from a borrow pit or by widening a cutting. The problems arising in connection with the disposal of the material, in work of this kind, are best studied with the aid of a *mass diagram*. This is a diagram constructed on a distance base and such that the ordinate at any point represents the algebraic sum of the volumes up to that point, cuttings being considered plus and banks minus in the usual way. In side-hill work, where both cuttings and banks occur on the same section, their difference only is used in the summation and is entered with its appropriate sign. The ends of the ordinates are joined by a smooth curve. The scale of distance is usually

the same as that used on the longitudinal section ; the zero of the base may agree with that on the longitudinal section or with some known point on that section. The ordinates may be drawn to any scale, the plus quantities being drawn upwards and the minus quantities downwards. By drawing the mass diagram immediately below the longitudinal section references from the one to the other may be most conveniently made.

Change of Volume.—In calculating the volumes to be dealt with no allowance is made for change of volume due to disturbance. The change in volume in loosening soft earth is about + 20 per cent. and in rock about + 50 per cent. The latter is
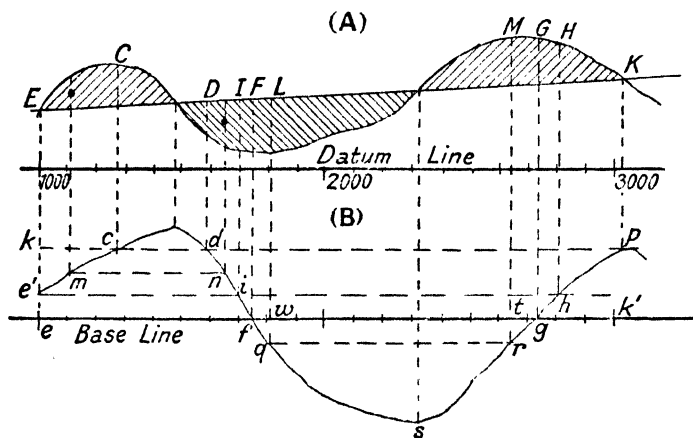
Fig. 97A.

permanent, but in the former shrinkage begins as soon as the material is deposited. Usually 10 per cent. is allowed for this on all banks, though after the lapse of one or two years the volume of the material is less than in its original *situ*, owing to better drainage and the influence of the atmosphere on the three exposed sides. If it is desired to allow for change in volume, this may be done by multiplying the successive volumes of *filling* by a factor which converts them, as near as may be, to their original volume. This factor—in default of more precise information obtained on the site—may be taken as 0·8 to 0·9 for soft earths and 0·66 for rock.

The properties of the mass diagram are best studied with the aid of a simple example. Let A, Fig. 97A, be the longitudinal

section of a proposed work and B the corresponding mass diagram. It will be observed from the figure that (*a*) an upward slope on the mass diagram indicates a cutting and a downward slope a bank ; (*b*) maxima and minima occur at change points ; (*c*) any line drawn parallel to the base line and cutting the curve, indicates points between which the volumes of the cutting and bank are equal. Thus *c d* shows that the volumes between C and D balance ; also the points *e, f* and *g* on the base line show that the material in the cutting E C will form the bank up to the point F, and that from the cutting on the right to the point G will form the remainder. Thus any line parallel to the base line is a balancing line ; (*d*) where the loop of a curve cut off by a balance line lies above that line, the material between its extremities must be moved forward, *i.e.*, in the direction of plotting, and where below the line moved backward. Thus the material at E is moved forward to F, and that at G backward to the same point ; (*e*) the intersections of a balance line and the curve indicate the maximum haul distance. Thus if *e f g* is the balance line, the maximum haul distance is from G to F, but if the balance line be raised to *e' i h*, the maximum haul will be H I ; (*f*) if we understand the term " haul " to mean the sum of the products of each load and the distance it is hauled, *i.e.*, $\Sigma v d = VD$, where V is the total volume of excavation and D is the distance between the centroids of the material *in situ* and in its deposited position, the area bounded by a loop of the curve and a balance line measures the " haul " in that section ; for example, the volume between E and C is shown by the vertical distance between *e'* and *c* ; this material is deposited to form the bank from D to I ; hence *v d* is equal to $e' k \times m n$, or equal to the area *e' c d i*, and the sum of all such areas is the area of the loop.

In computing the areas regard must be paid to the scales used. Thus, if $1'' = x$ feet, and $1'' = y$ cubic yards, then 1 square inch will represent $x y$ cubic yards feet, and n square inches $n x y$ cubic yards feet.

It is worthy of note that if we use *e f g* as a balance line, the volume between G and K must go to waste. This waste may be reduced to the volume between H and K by raising the balance line to the position *e' i h*. If the balance line be raised to the position *c d p* there is no waste on the right, but the volume between E and C must be wasted. From a knowledge of the local conditions the engineer can decide which of these arrangements is the most economical.

The balance line need not be continuous; gaps in a broken line will indicate points at which there is a defect or excess of material. If we use *e f q r g k′* as balance lines, then the gap *f q* shows there will be a shortage of material from F to L and an excess from M to G.

The conditions of a contract for the construction of the work may stipulate the price per cubic yard with or without reference to the haulage distance. In the former case the terms of the contract will include a price per cubic yard when hauled above a specified distance. This excess of distance over that included in the contract (termed " free haul distance ") is the *overhaul distance,* and the term *overhaul* is used to specify the sum of the products of the volumes dealt with and the overhaul distances.

The unit of measurement for overhaul distance is commonly 100 feet, and the overhaul is thus measured by the amount by which cubic yards $\times n . 100$ is in excess of the free haul, the factor *n* being specified in the contract.

The places on the section at which overhaul will be necessary are easily found from the mass diagram. If the free haul distance be marked on the edge of a strip of paper, and this edge be placed parallel with the base line, and so that the marks lie on a loop of the curve, as at *q r* (Fig. 97A), any material lying to the right or left of these points between *q r* and the balance line will fall into the category of overhaul. To find its amount, let *e f g k′* be the balance line ; at *q* and *r* erect perpendiculars meeting the balance line *e f g k′* at *w* and *t*, and cutting the section at L and M. The area *w q s r t* represents free haul, the excess is represented by the volume of M G or F L, and the overhaul is either of these volumes multiplied by the distance between the centroids of M G and F L minus $n . 100$ feet.

For large areas and long distances the centroids of these excess areas are found by dividing into parallel strips and taking moments about one edge, in the usual way ; for small areas and short distances the positions of the centroids are estimated by inspection.

EXAMPLES.

1. Obtain a formula by aid of which the half-breadths and whole width of land required for a cutting on sloping ground may be calculated.

2. From the following data determine the contents of the

portion of a cutting between two cross-sections 500 feet apart :—
Depth at the cross-sections, 25 and 15 feet respectively ; width
at formation level, 30 feet ; side slopes 2 to 1. Assume the
surface of the ground to be level on both cross-sections. Give
your answer in cubic yards.   (*Ans.* 26,235 cubic yards.)

3. Find the area of the cross-section of a cutting from the
following data :—Width at formation level, 30 feet ; depth at
centre, 15 feet ; side slopes, 2 to 1 ; ground falls to the left at
a gradient of 1 in 8. Give your answer in square yards.
<div align="right">(Ans. 107·5 square yards.)</div>

4. Obtain the volume of earthwork in an embankment, the
heights of consecutive sections 200 feet apart being 0, 20, 30,
10, and 0 feet. Side slopes 2 to 1. Width at formation level
30 feet, ground on cross-sections level. Give your answer in
cubic yards.   (*Ans.* 31,605 cubic yards.)

5. The surface levels of a new street about 500 yards long have
to be marked out ; the first half of the street is to have a gradient
of 1 in 50, and the remainder a gradient of 1 in 30, the former
being in embankment, and the latter in cutting. Assuming the
original surface levels known, how would you obtain and mark
out the new levels ?

6. Cast up the level book for the following staff readings :—
The back-sights are indicated by the figures in brackets, and
the reduced level of the first staff station is 40 feet above datum :
(4·53), 8·23, 9·45, 12·32, (2·54), 7·92, 3·45, 9·62, 5·34, (1·28),
6·75, 8·92, 12·46.   (*Ans.* R.L. last point = 18·23.)

7. Suppose the staff readings given in Question 6 were obtained
at consecutive points 50 feet apart on a line of section, and a
12-inch drain pipe is to be laid with a uniform gradient and
having its centre line 10 feet below the surface of the ground
at the commencement, and in the surface of the ground at the
end of the section. Find the gradient at which the pipe must
be laid, and the depth of the bottom of the trench at each 50 feet
along the section. (*Ans.* 1 in 42·48 ; 10·5, 7·98, 7·94, 6·25, 2·05,
<div align="right">7·70, 2·69, 8·15, 3·86, 2·86, 0·50.)</div>

8. The reduced levels of seven consecutive points on a section
100′ apart are as follows :—12·5, 17·5, 25·5, 23·5, 19·0, 23·6,
and 25·0 feet. The section is the longitudinal surface section
of a proposed road, the first point on the base of which is 2·5 feet
below the first point on the section. If the gradient of the road

is 1 in 30, find the reduced level of the formation at each 100 feet, and the corresponding height of embankment or depth of cutting.   (*Ans.* 10, 13·33, 16·67, 20·0, 23·33, 26·66, 30·00 ; Cutting, 2·5, 4·17, 8·83, 3·50 ; Bank, 4·33, 3·06, 5·00.)

9. Find the horizontal half-breadths of a cutting from the following data :—Width at formation level, 20 feet ; side slopes, 2 to 1 ; depth at centre, 5·64 feet ; gradient of ground on cross-section, 1 in 10.   (*Ans.* 17·73 feet ; 26·70 feet.)

10. In setting out the cross-section of an embankment by the " grade staff " method, the leveller took a "*flying level*" from the nearest B.M. (1,425·32) to the centre peg on the section, and obtained the following readings :—2·45, 10·32, 1·46, 8·41, 1·32, 10·54. If the reduced level of the " grade " (or formation level) at the cross-section is 1,409·53, what would be (*a*) the reading of the staff when "held to grade," and (*b*) the height of the bank at the centre ? Determine the complete bookings of the cross-section, assuming slopes 1½ to 1, width of road 20 feet, and gradient of ground on the cross-section 1 in 10, the lowest side being on the right.   (*Victoria University B.Sc. Tech.*, 1915.)

(*Ans.* 2·29 feet ; 8·25 feet.)

L.	C.	R.
$\dfrac{-\ 6\cdot31}{19\cdot45}$	$-\ 8\cdot25$	$\dfrac{-\ 10\cdot88}{26\cdot32}$

11. Prove the prismoidal formula which is the basis of Sir John Macneill's earthwork tables, and apply it to find the quantity of earthwork required for the embankment given by the following data :—Upward gradient of formation level, 1 in 150 ; depth of bank at far end, 10 feet ; side slopes, 2 to 1 ; distance between sections, 400 feet ; width at formation level, 30 feet. The surface of the ground slopes downwards from the near end at a gradient of 1 in 30, but is level across. Give your answer in cubic yards. (*Victoria University B.Sc. Tech.*, 1911.)
(*Ans.* 18,232 cubic yards.)

12. Determine the volume of the embankment from the bookings of the end sections given below. The distance between the sections is 100 feet, and the width of the road at formation

level is 30 feet.   Give your answer in cubic yards.   (*Victoria University B.Sc. Tech.*, 1914.)

L.	C.	R.
$\dfrac{-14{\cdot}80}{49{\cdot}50}$	$-8{\cdot}46$	$\dfrac{-10{\cdot}25}{38{\cdot}50}$
$\dfrac{-10{\cdot}80}{41{\cdot}75}$	$-5{\cdot}82$	$\dfrac{-4{\cdot}10}{23{\cdot}70}$

(*Ans.* 1,598 cubic yards.)

13. Describe, with the aid of sketches, how the plans of the outlines of cuttings and embankments may be obtained from surface contours.

CHAPTER VIII.

OFFICE WORK.

OFFICE work consists of the preparation of plans or sections from the data given by the field notes, the working out of traverses, the computation of areas, and in the case of engineering work the preparation of plans and estimates of the proposed work.

As we have already seen in the case of a survey, the plotting should immediately follow and if possible keep pace with the field work, so that any inaccuracies or omissions can be at once attended to. If the complete plan cannot be prepared on the ground, the main and filling-in lines must at least be laid down, the preparation of the final plan being afterwards carried out in the drawing office.

Office Equipment.

Drawing Table.—A firm drawing table, about 8 feet by 4 feet, with an even flat surface, is a necessity. The long edges of the table should be provided with a bull-nose about 3 or 4 inches deep, through which a slot is cut, so that the edges of the drawing paper (if the sheet is longer than the width of the table) may be passed through to prevent creasing. The top of the table should be slightly inclined towards the front edge.

Weights.—These are made of lead, $3'' \times 2'' \times 1''$, covered with leather or paper. They are useful for holding down the drawing paper while the plotting is in progress, or for holding the plotting scales in position.

Drawing Paper.—This should be well seasoned—Whatman's cold-pressed is the best—and preferably mounted on brown holland. The paper should always be stored flat in a drawer. As unmounted drawing paper is always more or less cockled, a good plan of obviating this is to stretch several sheets over each other on a drawing board. When nearly dry they are cut off the board and stored flat, the sheets being separated as required. A plan should never be plotted on paper which is stretched on

a drawing board, as the paper, when cut off the board, may contract unequally, thus altering the scale in different directions.

Before plotting a survey a scale should be drawn on the paper to which the plotting scale may be applied from time to time, in order to ascertain if the paper has been affected by changes in temperature and humidity.

T Squares.—One **T** square is required by each draughtsman, and should be of good quality, with a blade at least 42 inches long.

Steel Straight Edge.—All the main lines of a survey should be ruled-in with the aid of a steel straight edge. Two straight edges—one about 6 feet long and the other about 3 feet—should be provided.

The straight edges should be carefully wiped before and after use, and when not in use they should be kept in a specially constructed case to protect them from damp.

A·C·THORNTON MANCHESTER

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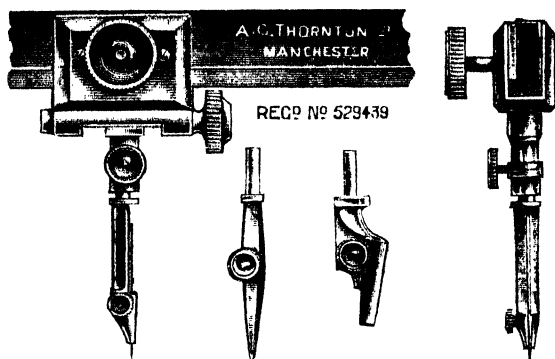
Fig. 98.

Beam Compasses or Trammels.—These are illustrated in Fig. 98, and are intended for striking arcs of larger radius than the ordinary compasses can cope with. They are especially useful in plotting the main triangles of a survey. When in use the two parts are clamped on a wooden lath, which is usually **T**-shaped in cross-section. The lath is made of this section to give it the necessary lateral stiffness. The needle trammel with its adjusting screw is clamped to the end of the lath, and the pencil trammel is slid along the lath until the distance between the needle and pencil points is approximately equal to the given

13

radius ; it is then clamped in position, and the final adjustment is made with the adjusting screw to the needle. When striking an arc with the trammels both hands should be used, the left hand steadying the fixed centre and the right hand supporting the lath and moving it round in the required direction.

Dimensions should not be taken from the plotting scales either with the dividers or the beam compasses, but they should be picked up with the instruments from a light pencil line, on which they have been laid down.

Failing a pair of beam compasses, the radius may be laid down on a pencil line drawn along the centre of a strip of drawing paper about ½ inch wide. At one end of the radius a needle (forming the fixed centre) is passed through the paper, and at the other the point of the drawing pencil. With this simple device short arcs of large radius may be struck with a fair approach to accuracy.

Parallel Rulers.—These are of two main types—(a) the ordinary and (b) the rolling parallel (Fig. 99). The former is the more

Fig. 99.

useful instrument for laying down traverses, as explained on p. 88. The latter form is not so useful for working on a cardboard protractor, but with it a parallel line is more quickly carried across the drawing paper.

The rolling parallel should be of solid gun-metal, as its weight makes it more steady when in use.

Protractors.—The construction and use of the ordinary outside and inside cardboard protractors have already been explained on pp. 87 and 88. For setting out angles more accurately than can be done with the cardboard protractors, metal protractors fitted with verniers reading to minutes are used. Two forms of protractor are illustrated in Figs. 100 and 101.

In Fig. 100 the two hinged arms are provided with needle points, which are depressed to puncture the paper when the verniers have been set to the given angle by aid of the clamp and tangent screw.

It should be premised that triangles can be more accurately set out with the beam compasses, from the calculated or measured
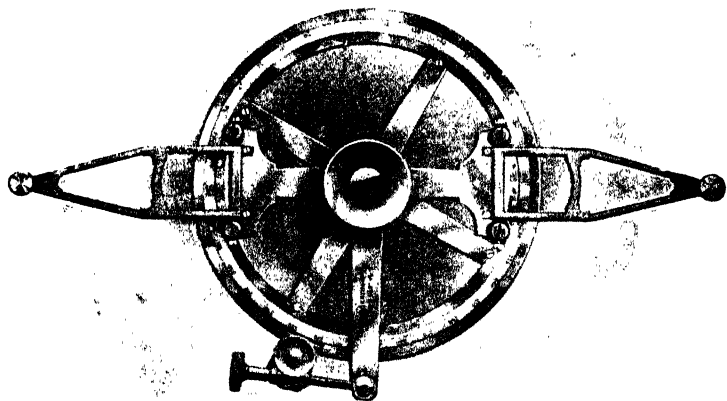
Fig. 100.

lengths of the sides, than with the protractor from measured angles.

A table of natural tangents or natural sines of angles furnishes a means of setting out angles with a high degree of accuracy. For example, let it be required to set out an angle of 54° 28′. From the table take out the value of the natural tangent of this angle, which equals 1·4002. Lay off from the point (A), at which the angle is to be set out, and along the base line a distance of 1,000 units to any scale. At this point erect a perpendicular from which cut off 1,400·2 units ; on joining the point thus obtained to A the given angle

Fig. 101.

will be set out. If the given angle is greater than 70°, the table of cotangents should be used, in which case the tabular number

Set Squares.—Two only are necessary, a 45° and a 60°; about 10 inches is a useful size.

Set of Scales.—This comprises six boxwood or ivory scales, 12 inches long, and six corresponding offset scales. The scales are one, two, three, four, five, and six chains to one inch and the corresponding feet scales. The offset scales are 2 inches long, have their edges similarly divided, and their ends cut off square with the divided edges through the zero points.

The scales are numbered 10, 20, etc., to 60, the numbers signifying the number of divisions on the scale to one inch. Hence the 1 chain to 1 inch scale is numbered 10, the 2 chains to 1 inch scale 20, and so on.

It is obvious that with this system of division the scales may be used for plotting to a scale other than those designated, by multiplying or dividing either the dimensions to be plotted or the scale divisions by a suitable factor. For example, suppose we wish to plot a plan to a scale of $2\frac{1}{2}$ inches to 1 chain— *i.e.*, 100 links to $2\frac{1}{2}$ inches. On inspecting the scales we find that if we divide the divisions on the 40 scale by 10, each division on the chain scale will represent one link, and on the feet side each division will represent one foot. Hence the 40 scale used in this way will enable us to plot the plan to the desired scale.

The feet scales are used for laying down the plans of proposed buildings or other improvements on the land plan. The chain scales are also used for plotting sections, the scales being considered as scales of 10, 20, 30, etc., feet to 1 inch.

Colours.—These should be of the best quality, preferably in cakes or sticks. They are rubbed down in water—or the colour is taken off the end of the cake with a brush—until the desired depth of tint is obtained. The following list will be found to contain the colours most used by land surveyors :—

Vandyke brown.	Sap green.
Raw sienna.	Crimson lake.
Burnt sienna.	Carmine.
Chrome yellow.	Vermilion.
Yellow ochre.	Payne's grey.
Indian yellow.	Prussian blue.
Indian red.	Ultramarine.
Emerald green.	Indigo.

A large variety of tints can be made by combining two or more of the colours, and a little practice will enable the student to mix any colour tint that he requires, but owing to the difficulty

of exactly matching a particular shade, more colour should be mixed than is required to finish the work in hand.

No hard and fast rule can be laid down regarding the colour to be used to indicate any particular surface feature, since the colours used are purely conventional, and vary in the practice of different surveyors. As a guide to the student, the following list may be of service :—

Object.	Colour.
Roads, ...	Light burnt sienna.
Footpaths of macadamised roads, ...	Darker tint of burnt sienna.
Pavements, . . .	Payne's gray.
Existing buildings, . .	Carmine, lake, gray, or black.
Proposed buildings, .	Indian red.
Water , . .	Prussian blue or ultramarine.
Pasture, . . .	Emerald green.
Private parks, . .	Sap green.
Boundaries of properties, .	A strip of any convenient colour, using a different colour for each proprietor.
Outline of section. .	A strip of Vandyke brown about ⅛ inch to ¼ inch wide along and inside the outline.
Sand pits, . . .	Yellow ochre.
Longitudinal and cross-sections. . . .	Cuttings are usually coloured pink and embankments green.

The tints used should not be too heavy, and glaring colour contrasts should be avoided.

In colouring water surfaces the edges are coloured dark, and the colour is led off towards the centre with a clean, moist brush.

Trees are sketched in ink, as indicated on p. 60, and may or may not be coloured.

Brushes should be of the best quality—preferably red sable—mounted either in a ferrule or a quill. Three brushes of different sizes will be found sufficient. Always wash and dry a brush after use ; never put it away soiled with colour.

Pencils.—For plotting purposes hard pencils, HHH or HHHH, of the best quality, should be used. The pencil should be sharpened to a long, fine point, and not to a chisel edge, as used in mechanical drawing.

Stencils are thin perforated sheets of copper, the perforations being cut to some particular design, depending on the purpose for which the stencil is intended. The object of using stencils is to save time, as they enable a draughtsman to ornament or

letter a plan quickly. Good stencil work is, however, much inferior to good hand work, and although stencils are often useful in a busy office, their use should be as restricted as possible.

In using a stencil, it is adjusted in position and held firmly in place either by weights or the fingers of the left hand, while the ink is worked through the perforations with a stiff hog-hair brush. Care must be exercised to see that (*a*) the brush carries the right amount of ink; (*b*) the end of the brush describes a small circle without the bristles scraping past the edges of the

Fig 103.

Fig 104.

Fig. 105.
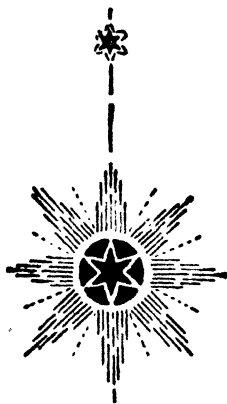
Fig. 106.

perforations; and (*c*) the stencil is lifted clean off the paper when the work is finished. There is a knack in using a stencil which can only be obtained by practice.

After use, the stencils should be carefully cleaned and stored flat.

Figs. 103, 104, 105, and 106 show various stencil forms, which illustrate a border, a corner ornament, a wood, and a north point respectively.

Plotting a Survey.—Before proceeding to plot a survey, a careful study should be made of the system of lines used on the ground, and a list containing the numbers of the lines, their lengths, and the distances of the important intermediate stations should be prepared.

An estimate of the lengths of the sides of the rectangle required to contain the plan should be made, and the rectangle lightly pencilled on the drawing paper, in such a position that the completed plan will appear to the best advantage. The position of the base line of the survey is next determined with respect to the sides of the bounding rectangle, and is drawn in its proper position. The intermediate stations on the base line are next accurately pricked off and lettered or numbered as the case may be. The triangles or other figures built up on the base line are now drawn in position, and as each figure is completed it should be fully checked by its proof lines before proceeding with the next. Continuing in this way, the whole of the ground work of the survey (with all the stations lettered and lines numbered in their proper order) is gradually reproduced on the plan.

If it is found that the proof line on any figure will not fit-in accurately, the sides of the figure should be tested for inaccuracies in plotting, as an error of a few links in laying down one of its sides may cause a considerable alteration in the shape of the figure. If the sides of the figure have been accurately plotted, an estimate must be made as to which line is most probably in error. In forming this estimate, allowance must be made for the difficulties (such as irregularities in the ground passed over, obstacles, etc.) met with in the measurement of the line. The lines on the plan are then lengthened or shortened as required, until the proof line will fit-in *with the least possible disturbance in the shape of the original figure.* If the error is greater than the greatest permissible error, the lines must be remeasured.

The main and filling-in lines are inked-in, either in faint Prussian blue or carmine, before proceeding to plot the offsets, as the draughtsman should make it a rule not to plot the offsets from a pencil line.

In plotting the offsets, commence with line 1, and deal with the others in the order in which they have been chained. Place the plotting scale with its zero division exactly at the commencement of the line, and its divided edge in coincidence with the chain line. Hold the scale firmly in position by loading it with weights, and place the offset scale with its base in contact with the divided edge of the plotting scale, as shown in Fig. 107.

Now slide the offset scale along the plotting scale until its divided edge coincides with the chainage at the first offset, the position of the object to which the offset refers is then marked on the paper from the offset scale. Continue in this way until all the offsets on the line have been laid down.

The offsets should be plotted with a fine needle, and not marked with pencil.

If offsets occur on both sides of the line, the plotting scale should be adjusted parallel to the main line, and at a distance therefrom equal to half the length of the offset scale. The centre division on the offset scale being considered zero, the offsets can now be laid down on both sides of the main line without re-adjusting the plotting scale.

When all the offsets from a main line have been laid down,
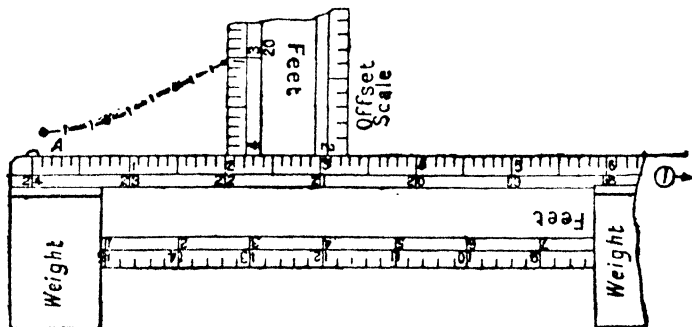
Fig. 107.

the plotted points are joined by a fair curve, the kind of line used being determined by the nature of the object represented. Having completed line 1, proceed to plot the positions of the surface features relative to line 2, lining-in the positions of the fences, buildings, etc., immediately the offsets are plotted. Continue the work in this way—taking the lines in the order in which they occur in the field book—until the whole of the survey has been laid down.

Plotting Traverses and Sections.—These have already been dealt with on p. 74, *et seq.*, to which the student must refer.

Inking-in.—After checking the whole of the plan to make certain that nothing given in the field notes has been omitted, proceed to ink-in the pencil lines, working either from left to

right or from the top of the plan downwards. Keep the drawing
pen as nearly upright as possible, rule-in all the straight fences
with the straight edge, and work-in the irregular outlines by aid
of the French curves. The left-hand and bottom edges of
buildings and the left-hand and top edges of water surfaces
should be back-lined—*i.e.*, drawn with a thicker line—as this
improves the appearance of the plan. Trees, woods, hedge-rows,
etc., are next sketched in position with a fine pointed writing pen.

The Title.—Having decided on the words of the title, arrange
the words on a spare piece of paper in two or three different
ways, in order to get best way of grouping them, either in line
or in column. Select the centre letter of each line and place
this letter on the centre line of the plan in its proper position.
The other letters are then arranged left and right of the centre
line in their proper order, due allowance being made for the
spaces between the words. In this way the title is built up in
pencil, and after final correction is inked-in.
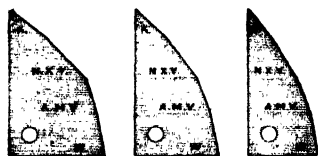
Fig. 108.

A lettering square (Fig. 108) is often useful at this stage,
as it gives the correct inclination of the sloping portions of the
letters indicated.

The descriptive writing on the different parts of the plan,
giving names of places, roads, proprietors, etc., should receive
as much attention as the main title. Nothing detracts more from
the appearance of a plan than bad writing ; more plans are spoiled
from this cause than any other.

The Scale.—This should be drawn in the centre of the plan,
either underneath the title or just inside the border at the bottom
of the drawing.

The divisions of the scale are pricked off the plotting scale,
and are carefully tested before being inked-in. Two types of
scale are shown in Fig. 109.

The North Point.—This should always point towards the top
of the plan. To determine the direction of the meridian, we
must know the bearing of one of the main lines of the survey.

To find its direction on the plan, place the protractor on the line whose bearing is known, and turn it round until the readings at the rear and forward ends of the line are equal to the fore- and back-bearings of the line. The line joining the zero and 180° divisions on the protractor is now in the direction of the
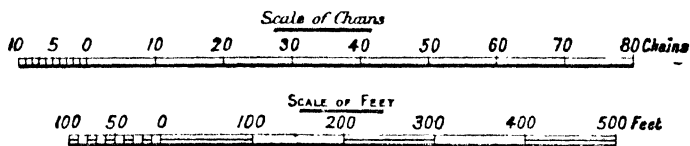
Fig. 109.

meridian, and a line is drawn parallel to this in the position in which it is intended to draw the North point.

If the magnetic bearing of the line is given, the meridian line determined from it will be the magnetic meridian, the true meridian is determined by setting off an angle equal to the declination east or west, as the case may be, of the magnetic meridian.

The shape of the north point depends on the individual taste and skill of the draughtsman; in all cases, it should be very neatly drawn. It is usual to show the direction of the magnetic meridian by a half-arrow, as indicated in Fig. 110, in which four simple north points are shown.
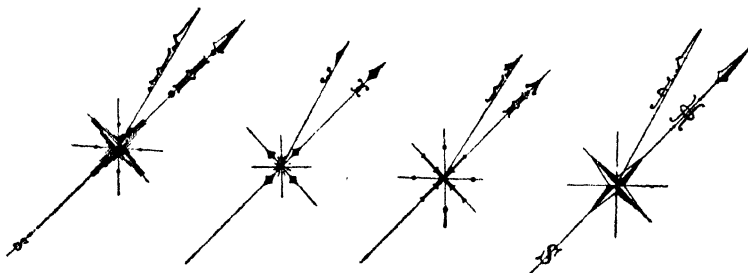
Fig. 110.

Borders.—A plan cannot be considered complete unless it has a neat border drawn round it, with a margin of 1½ to 2½ inches Borders vary from a simple line with square corners to highly complicated designs (Figs. 103 and 104), either stencilled or drawn in position. A simple border, consisting of a single strong

line or of two lines one thicker than the other, is neat and effective. If desired, a simple corner ornament may be easily drawn with the compasses or set squares.

Colouring.—Before proceeding to colour a drawing, it should be thoroughly cleaned with bread crumbs or a soft india-rubber. In putting on a flat wash the colour should be floated evenly in front of the brush, and to facilitate this the drawing board is tilted to a comfortable angle. Before colouring a large surface it hould be moistened with water to prevent the colour wash drying too quickly.

Take care to work the colour to an edge, but not over it Should the colour overstep the edge, the excess must be taken up at once either with a clean piece of blotting-paper or a clean brush. All colours should be mixed light, as they are easier to handle than dark shades ; if a tint dries out too light, it is an easy matter to darken it by putting on another wash ; but it is very difficult to evenly reduce a tint which is too dark.

When shading water surfaces, have a brush at each end of the handle, one to put the colour on, and the other—moistened with water—to lead the colour off. Shading in colour is best done from left to right.

Enlarging and Reducing Plans.—It is often necessary to enlarge or reduce the whole, or a portion, of a survey. The only satisfactory way of enlarging a plan is to replot it from the field notes to the larger scale. If a plan is enlarged by any other method the errors contained by it are enlarged at the same time, and may become appreciable ; on the other hand, in reducing a plan, the errors are correspondingly reduced.

Method of Squares.—If carefully carried out, this method is quite satisfactory.

Suppose Fig. 111 to represent a plan which is to be reduced to any given scale. Draw on the plan a network of squares of any convenient size, say 1″ to 1½″. The squares must be accurately and lightly drawn in pencil. Letter and number the rows and columns of squares as indicated. Reproduce the squares to the given scale on the sheet of drawing paper on which the new plan is to be drawn, lettering and numbering the rows and columns of squares as before (Fig. 112). To copy the plan, begin with some prominent fence line, note its points of inter section on the plan with the horizontal and vertical sides of the squares ; with the proportional compasses (set to the number representing the reduction ratio) transfer from the plan to the copy the distances of these points measured from the corners

of the squares nearest to them. On joining the plotted points by a fair curve, the plan of the fence, to the new scale, is obtained. The process is repeated, line by line, until all the detail has been reproduced on the new plan. Should any important detail fall within a square, put in an additional line (A B, Fig. 111) or lines, from which the salient points of the detail may be measured and set out.

Before inking-in the new plan, the positions of the more important points should be tested with the proportional com-
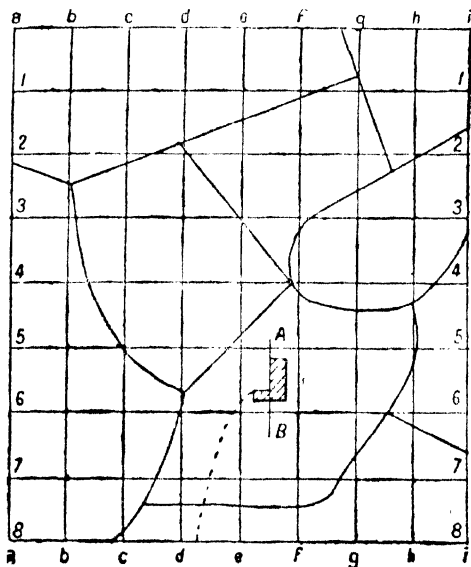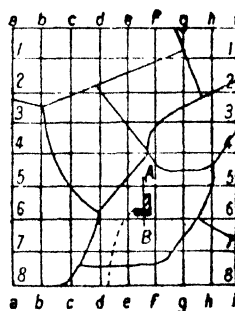
Fig. 111   Fig. 112

passes. The squares on the original and the copy are erased when all the detail has been inked-in.

To prevent injury to the original plan, the large squares may be drawn on a sheet of tracing paper pinned over it. The necessary measurements may then be made to the corners of the squares on the tracing paper, since the lines on the plan will be visible through the paper.

The Pantograph and Eidograph are instruments for enlarging and reducing plans. They are both based on the principle of

similar triangles; but their mechanical construction is quite different.

The pantograph (Fig. 113) is constructed of four tubular brass bars, square in section. Two of these bars are shorter than the others, and the four bars are hinged together to form a parallelogram having equal sides, as shown in the figure. To the extremity of one of the long bars a tracing point is fitted, the other long bar passes through a sliding frame carrying an index line and a vertical axis of rotation. This axis is fixed to a triangular weight, which keeps it firmly in position. Sliding on one of the shorter bars is a second tubular frame carrying an index line and a pencil; the pencil holder is interchangeable with the tracing point and the fixed axis. The axis and pencil
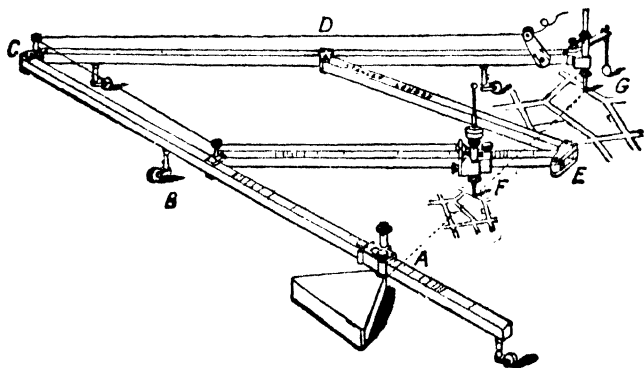
Fig. 113.

frames may be clamped at any of the divisions along their respective bars. These divisions are engraved on the bars by the maker of the instrument, and are marked $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., giving the corresponding reduction (or enlargement) ratios.

When the instrument is accurately set, the axis, pencil point, and tracing point will lie in the same straight line, and will remain so in all positions of the instrument if the jointed parallelogram is accurately constructed. Hence B F (Fig. 113) will be parallel to C G, and in consequence the triangles F B A and G C A are similar.

As this is true for all positions of the instrument, we must have

$$\frac{\text{The movement of F}}{\text{The movement of G}} = \frac{\text{A F}}{\text{A G}} = \frac{\text{B F}}{\text{C G}} = \frac{\text{A B}}{\text{A C}},$$

and both movements will be in the same direction. As arranged in the figure, the instrument can only be used for reducing (or enlarging by interchanging the pencil and tracing points), it cannot be used for copying to the same scale, since A B is always less than A C ; but by interchanging the pencil F and the axis A the movements of the pencil and tracing points will be in the ratio of A F to F G, and by setting the indexes of the axis and pencil frames to $\frac{1}{2}$ the movement of the pencil will be equal to that of the tracing point. Hence the copy will be to the same scale as the original, but as the movements of the pencil and tracing points are in opposite directions the copy will be reversed.

The instrument is supported on castors, in a position parallel to the paper, so that it can move freely over the paper in all directions. To vary the pressure of the pencil on the paper, the pencil holder is loaded with small weights. The fine thread passing round the top of the instrument enables the draughtsman to raise the pencil from the paper while he passes the tracing point from one part of the original plan to another, thereby obviating false lines on his copy.

In using the instrument, the indexes are first set to the ratio representing the required degree of reduction, and it is then placed with the tracing and pencil points respectively on the original plan and the sheet of drawing paper on which the copy is to be made. The tracer is then carefully drawn over every line of the original, and a true copy—reduced to the required scale—will be drawn by the pencil on the paper beneath it.

The pantograph is not a satisfactory instrument for enlarging a copy, as it possesses too many joints and points of support. When used for enlarging any irregularity in the movement of any part of the instrument will be magnified at the pencil.

The Eidograph.—This instrument—the invention of Professor Wallace in 1821—has only one point of support on the paper and three joints against several points of support, and five joints in the pantograph, and consequently its action is much more smooth and regular than is the case in the latter instrument.

The eidograph consists essentially of three tubular brass bars, square in section, whereof one bar C B (Fig. 114) is supported on a fixed centre of rotation E, and the other bars A B and C D are connected to its extremities in such a way that they are parallel to each other in all positions of the component bars. A tracing point is placed at D and a pencil at A. The three bars are each provided with a scale of equal parts numbered each

way from the centre to 100, a reading to one-tenth of a division
being obtained by means of a vernier engraved on the tubular
slide, through which the bar passes, and to which it may be
clamped.

The tubular slide on the bar C B is connected to a vertical
axis formed on the upper surface of the circular supporting
weight on which the whole instrument rotates. The slides on
the parallel bars are fixed to the under surfaces of two exactly
equal pulleys turning in bearings formed in the ends of the bar
C B. The pulleys are connected by a thin steel band, the tension
on which may be regulated by the adjusting screws S S'. All
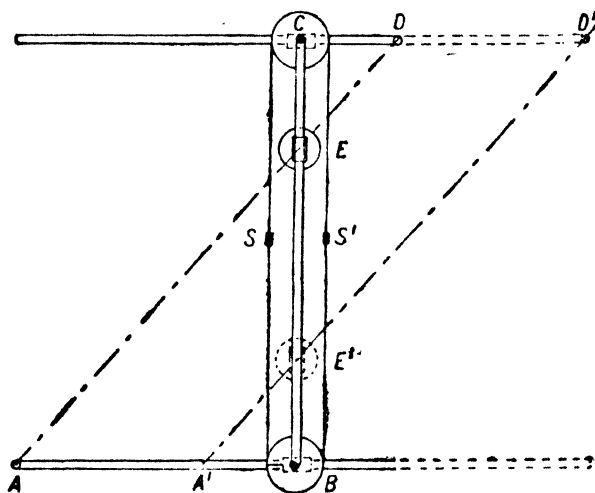
Fig. 114.

the bearing surfaces are accurately ground, and may be adjusted
for accuracy and wear. The methods of loading and raising
the pencil holder are the same as those adopted on the panto-
graph.

When the instrument is accurately adjusted, the three points
A, E, and D are always in the same straight line, and as the two
pulleys C and B are exactly the same size, any motion given
to one bar is freely communicated to the other, and consequently
the two bars will be constantly parallel. Hence the two triangles
E C D and E B A are similar; therefore,

$$\frac{\text{The motion of D}}{\text{The motion of A}} = \frac{D\,E}{E\,A} = \frac{C\,E}{E\,B} = \frac{C\,D}{A\,B}$$

$$= \frac{\text{The scale of the plan}}{\text{The scale of the copy}}.$$

Let C and P = scale of the copy and plan respectively.

Then

$$\frac{C}{P} = \frac{B\,E}{C\,E}$$

$$= \frac{100 + x}{100 - x}, \text{ where } x \text{ is the vernier reading on the bars.}$$

$$\therefore x = \frac{100\,(C - P)}{(C + P)}.$$

If the copy is to be twice the size of the original, then

$$x = \frac{100\,(2 - 1)}{(2 + 1)} = 33\cdot3,$$

and the three verniers must be set to this number to copy the plan to double its original size. If it is required to copy the plan to a scale of $\frac{1}{2}$, the verniers are set to the same number (33·3), but on the opposite side of their respective zeros. In this case the pencil, axis, and tracing point would occupy the positions shown by A', E', and D' respectively in Fig. 114.

Before setting the verniers to the number deduced from the ratio of the two scales, the bars A B and C D should be set quite parallel to each other. To do this, set the three verniers accurately to zero, and make marks on the drawing paper with the pencil and tracing-points; then swing the tracing-point round until it coincides with the mark made by the pencil, and if the pencil coincides with the mark made by the tracing-point the bars are parallel; but if not make a second mark with the pencil, and bisect the distance between the two marks.

By turning the adjusting screws S S' on the steel band, bring the pencil point exactly to the bisection point. The two bars should now be parallel, and will remain so if the steel band does not slip and the zeros of the scales have been accurately placed by the maker of the instrument.

A lead weight is usually supplied with the instrument, and is placed where required on the connecting bar to keep it in balance.

The actual process of copying a plan is the same in both the eidograph and pantograph.

14

Copying a Plan.—There are several methods of doing this. If a number of copies are required, the usual method is to make a copy of the plan on stout tracing paper or tracing cloth, and to reproduce the copy by one of the many photographic or litho• graphic processes.

When only one copy is required, lay a sheet of transfer paper over the paper on which it is intended to draw the copy ; on the transfer paper lay the original plan, the whole being held firmly in position by weights or drawing pins. A hard steel or agate tracing point is then passed over all the lines on the original, and an exact copy will be marked on the paper beneath. The copy is afterwards inked-in and finished like the original.

If much copying work has to be done, a copying glass should be used. This is a large sheet of plate glass, on which the original plan is laid (facing upwards), over this is placed the sheet of drawing paper intended to receive the copy, the two being firmly held in contact by weights. The copying glass is supported at a convenient angle, and a strong light is thrown through it from behind. This renders the lines on the original plan visible through the paper intended to receive the copy, which may be inked-in at once, without any danger of injuring the original.

Determination of Areas.

A plan cannot be considered complete until the areas of all its enclosures have been determined and written thereon. In this country the areas are stated in acres, roods, and perches, except in the case of small building plots, which are given in square yards.

The area of a plane figure may be determined by (*a*) calculacion ; (*b*) drawing, measurement, and calculation ; and (*c*) by using an instrument specially designed for the purpose, such as the computing scale or the planimeter. The method adopted in each case will be governed by the given conditions. The results obtained by calculation are the most accurate, provided the assumptions used are fulfilled. In many cases these assumptions presuppose straight line boundaries which are not commonly found in practice. In determining the area of an enclosure by calculation, the areas included by the figures formed by the survey lines are first determined, and to these are added (algebraically) the areas of the irregular strips between the main lines and the boundaries, or the irregular boundaries are averaged by an average or casting line, so as to form triangles or other rectilinear figures whose areas may be readily calculated.

The following table will be found useful for reference :—

A TABLE OF SQUARE MEASURE.

Sq. Links.	Sq. Feet.	Sq. Yards.	Sq. Poles or Perches.	Sq. Chains.	Roods.	Acres.	Sq. Mile.
625	272¼	30¼	1				
10,000	4,356	484	16	1			
25,000	10,890	1,210	40	2½	1		
100,000	43,560	4,840	160	10	4	1	
64,000,000	27,878,400	3,097,600	102,400	6,400	2,560	640	1

The student should remember that a strip of land 10 Gunter chains long and 1 chain broad contains 1 acre, and that this area is equal to 4,840 square yards, or 43,560 square feet.

Areas by Calculation.—The figures occurring in surveying whose areas may be calculated directly comprise the triangle, quadrilateral, and polygon. These figures only are dealt with in this section.

For proofs of the following rules, the student is referred to any standard book on trigonometry, or on mensuration.

The Triangle.—(1) Given the base (b) and the height (h).

$$\text{The area} = \frac{b \cdot h}{2}.$$

(2) Given two sides (a and b) and the angle (θ) between them,

$$\text{The area} = \frac{a\,b\sin\theta}{2}.$$

(3) Given the base (b) and two base angles θ and φ,

$$\text{The area of the triangle} = \frac{b^2}{2}\frac{\sin\theta\sin\varphi}{\sin(\theta+\varphi)}.$$

(4) Given the three sides a, b, and c.

$$\text{The area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

where s = the semi-perimeter of the triangle = $\dfrac{a+b+c}{2}$.

The Parallelogram.—(5) Given the two sides (a and b) and the angle (θ) between them,

$$\text{The area of the parallelogram} = a \cdot b\sin\theta.$$

(6) When the figure is a rectangle, $\theta = 90°$ and $\sin\theta = 1$

$$\therefore \text{area of a rectangle} = a \times b.$$

The Trapezoid.—Given the lengths (a and c) of the parallel sides and the perpendicular distance (b) between them,

$$\text{The area of the figure} = \left(\frac{a + c}{2}\right) b.$$

The Polygon.—The area of a polygon cannot as a rule be directly calculated. In dealing with an irregular figure having more than three sides, the figure is divided into triangles or parallelograms, which may be dealt with by the above rules, the sum of the contained figures giving the total area. The polygon may be reduced to an equal triangle by the method given on p. 207, and its area calculated from data furnished by the triangle.

Fig. 115.

Area of a Polygon formed by a Closed Traverse.—When the relative positions of the stations of a closed traverse have been determined by their " latitudes " and " departures," the area included by the main lines of the traverse may be calculated without preparing a plan of the figure. Let the traverse lines be represented by the figure A B C D (Fig. 115), and the " departures " and " latitudes " of the stations A, B, C, and D be $x\,y$, $x_1\,y_1$, $x_2\,y_2$ and $x_3\,y_3$ respectively.
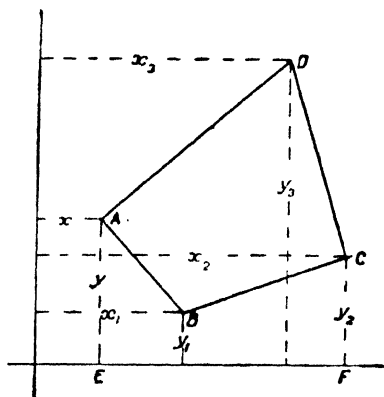
The area of the figure A B C D = the area of the figure A E F C D A — the area of the figure A E F C B A

$$= \left(\frac{y + y_3}{2}\right)(x_3 - x) + \left(\frac{y_3 + y_2}{2}\right)(x_2 - x_3)$$

$$- \left\{ \left(\frac{y + y_1}{2}\right)(x_1 - x) + \left(\frac{y_1 + y_2}{2}\right)(x_2 - x_1) \right\}$$

$$= \tfrac{1}{2}\{y(x_3 - x_1) + y_1(x - x_2) + y_2(x_1 - x_3) + y_3(x_2 - x)\}$$

In this expression we see that each product is made up of two factors, one of which is the total " latitude " of a station and the other is the difference of the total " departures " of the preceding and following stations.

Line	Station	Reduced Bearing.	Distance (Feet).	Consecutive Co-ordinates.				Independent Co-ordinates.				Double Areas in Square Feet.
				Lat.		Dep.		Lat.		Dep.		
				N.	S.	E.	W.	N.	S.	E.	W.	
..	A	..	..	0	0	0	0	0	0	0	0	0
A	B	S. 45° W.	7,500	..	5,302·5	..	5,302·5	..	−5,302·5	..	5,302·5	44,686,819
B	C	N. 30° W.	6,250	5,412·5	..	..	3,125·0	110·0	..	..	8,427·5	489,775
C	D	N. 60° E.	8,750	4,375·0	..	7,577·5	..	4,485·0	..	..	850·0	37,707,337
D	A	S. 10° 43' E.	4,565	..	4,485·0	850·0	..	0	0	0	0	0
	Sums,			9,787·5	9,787·5	8,427·5	8,427·5	..	..	..	..	82,973,931

This holds no matter how many stations there are in the traverse. Hence the rule for determining the area of a traverse polygon is :—Multiply the total latitude of each of the stations by the difference of the total departures of the preceding and following stations ; half the algebraical sum of the products is the area required.

The table given on the preceding page illustrates the method of computation.

The double areas are obtained as follows :—

The total latitude of A is zero.

∴ Area calculated for station

$$A = 0.$$
$$B = -\ 5,302 \cdot 5\ (0 - 8,427 \cdot 5)$$
$$=\ 44,686,819 \text{ square feet.}$$
$$C = 110\ (5,302 \cdot 5 - 850)$$
$$=\ 489,775 \text{ square feet.}$$
$$D = 4,485\ (8,427 \cdot 5 - 0)$$
$$=\ 37,797,337 \text{ square feet.}$$

Total double area $= 44,686\ 819 + 489,775 + 37,797,337$ sq. ft
$$=\ 82,973,931$$
$$\text{Area} =\ 41,486,965 \text{ square feet}$$
$$=\ \frac{41,486,965}{43,560} \text{ acres}$$
$$=\ 952 \cdot 4069 \text{ acres.}$$

Areas by Drawing, Measurement, and Calculation.—Graphical methods applied to the determination of areas are often very useful. They furnish a ready and accurate means of reducing complicated to more simple figures, from which the necessary mensuration data can be readily obtained by measurement. Their chief use lies in the determination of average or casting lines—*i.e.*, straight lines replacing irregular boundaries without altering the included area.

Casting lines are frequently placed in position by aid of a tightly stretched fine silk thread, which is held down to the paper at one end by a weight. The other end is moved over the line representing the boundary until the area on each side of the thread—as determined by estimation—is the same. The position of the thread is pricked off, and the marks are joined by a fine pencil line, from which the desired measurements can be made.

If the boundaries of an enclosure form a rectilinear polygon, the

necessary casting lines to convert the figure into a triangle can readily be drawn by a geometrical process. For example, let it be required to reduce the polygon A B C D E F G (Fig. 116) to an equivalent triangle having its base on A B or A B pro duced, and vertex at E.

The construction is as follows :—Join B D, and through the point C draw C 1 parallel to B D, meeting the base A B produced at the point 1. Join D 1. The line D 1 is a casting line for the sides B C and C D. Since the triangles D C B and D 1 B are on the same base D B and between the same parallels D B and C 1, they are equal to each other. Hence the line D 1 alters the shape, but not the area of the polygon. Join E to 1, and through D draw D 2 parallel to E 1 meeting the base produced at the point 2. Draw the casting line E 2 for the right-hand side of the figure.

The casting line E 4 for the left-hand side of the figure is
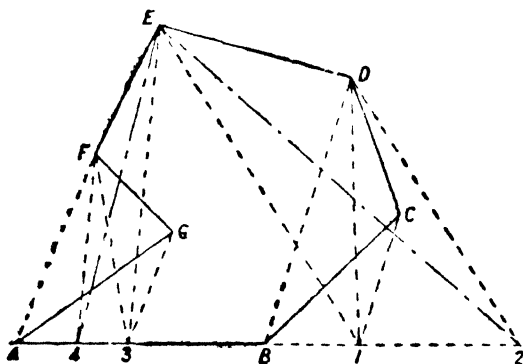
Fig. 116.

obtained in a similar way. As the triangle E 4 2 is equal in area to the polygon A B C D E F G, the half-product of the base and height of the triangle gives the area of the polygon.

As a further example of the application of this method we will consider the following example :—A and B (Fig. 117) are two fields separated by an irregular fence C D E F G. The owner of the fields decides to replace the irregular fence by a straight one starting from E, without altering their respective areas. Find the point H at which the new fence will cut the fence G I.

The construction is as follows :—Join C E, and through D

draw D 1 parallel to C E, meeting E F in the point 1. The line C 1 equalises the fences C D, D E, and E 1.

Join C F, and through 1 draw the line 1 2 parallel to C F, meeting F G at the point 2. Join C G, and through the point 2
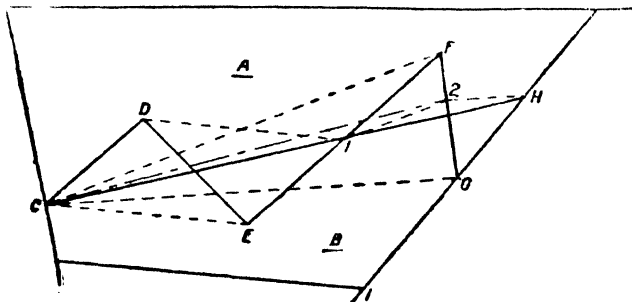
Fig. 117.

draw 2 H parallel to C G, meeting the fence I G at H. Then H is the point sought, and the line C H is the plan of the new fence.

This method may also be applied to figures bounded by straight and curved lines.

As an illustration of the method of dealing with a curved boundary, we will consider the problem of reducing the figure A B C D (Fig. 118) to an equivalent right-angled triangle, having A B as base.

At A erect a perpendicular A E; divide the curved outline
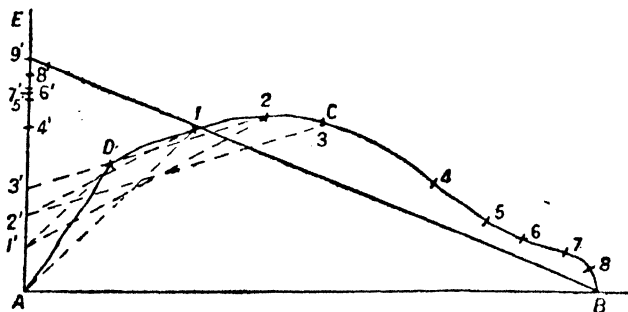
Fig. 118.

D C B into portions which are approximately straight, as shown by the points 1, 2, 3, etc. Join A to 1, and through D draw a

line parallel to A 1, meeting the perpendicular A E at 1′. Join
1′ to 2, and through the point 1 draw the line 1 2′ parallel to
1′ 2, meeting A E at 2′. Join 2′ to 3, and through 2 draw a
parallel to 2′ 3, meeting A E at 3′. Repeating this process,
taking the numbered points in order, we finally obtain the point
9′, which, joined to B, completes the required triangle. After
obtaining the point 3 in Fig. 118 the construction lines have
been omitted for the sake of clearness.

Irregular Figures.—These are bounded either by straight lines,
irregular curves, or combinations of the two. We have already
seen how both these cases may be dealt with, but in the case of
figures bounded by curves the method most in use consists in
converting the irregular figure into an equivalent rectangle,
the base of which is some fixed dimension (generally the longest)
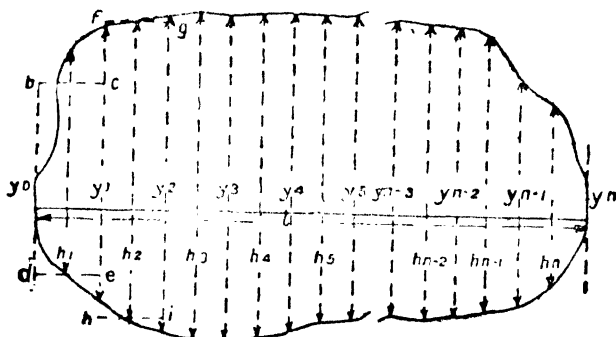
Fig. 119.

of the figure, and height the mean or average height of the figure.
To determine the mean height, the figure is divided into strips
of equal width by ordinates drawn at right angles to its base,
or at right angles to a line drawn across the figure, and so placed
that perpendiculars at its extremities are tangent to the figure.
The number of strips (n) depends on the form of the outline
of the figure, more strips being necessary when the outline is
very irregular. A convenient number is 10. There are various
rules for finding the mean ordinate from these strips, the more
important of which will now be given.

Mid-Ordinate Method.—Draw a second series of ordinates
h_1, h_2, h_3 . . . h_n midway between the ordinates of the first
series, and measure their lengths.

The mean ordinate $= \dfrac{h_1 + h_2 + h_3 + \; . \; . \; . \; + h_n}{n}$,

and the area of the figure $= l \dfrac{(h_1 + h_2 + h_3 + \; . \; . \; . \; + h_n)}{n}$,   (1)

where $l =$ length of figure measured at right angles to the ordinates (Fig. 119).

Ordinary Rule.—A second method giving a closer approximation than the above is obtained as follows :—Let the lengths of the *bounding ordinates* of the strips be $y_0, y_1, y_2, \; . \; . \; . \; y_n$, and their common distance apart be a. Suppose each strip to be converted into an equal rectangle by the lines $b\,c,\ d\,e,\ f\,g,\ h\,i$, etc. (Fig. 119), the area of the figure

$$= a\left(\frac{y_0+y_1}{2}\right) + a\left(\frac{y_1+y_2}{2}\right) + a\left(\frac{y_2+y_3}{2}\right) + \; . \; . \; . \; + a\left(\frac{y_{n-1}+y_n}{2}\right)$$

$$= a\left(\frac{y_0 + y_n}{2} + y_1 + y_2 + y_3 + \; . \; . \; . \; + y_{n-1}\right)$$

$$= n\,a\,\frac{\left(\dfrac{y_0 + y_n}{2} + y_1 + y_2 + y_3 + \; . \; . \; . \; + y_{-1n}\right)}{n}$$

$$= l\,\frac{\left(\dfrac{y_0 + y_n}{2} + y_1 + y_2 + y_3 \; . \; . \; . \; + y_{n-1}\right)}{n}. \quad . \quad . \quad . \quad (2)$$

Simpson's Rules.—In both the above rules it is assumed that the curve joining the ends of the strips is a straight line ; in Simpson's rules these curves are assumed to be short portions of parabolas.

First Rule.—When n is a multiple of 2.

The mean ordinate Y

$$= \frac{1}{3n}\;\{y_0 + y_n + 2\,(y_2 + y_4 + y_6 + \; . \; . \; . \; + y_{n-2})$$
$$+ 4\,(y_1 + y_3 + y_5 + \; . \; . \; . \; + y_{n-1})\} \quad . \quad . \quad (3)$$

Second Rule.—When n is a multiple of 3.

The mean ordinate Y

$$= \frac{3}{8n}\;\{y_0 + y_n + 2\,(y_3 + y_6 + \; . \; . \; . \; + y_{n-3})$$
$$+ 3\,(y_1 + y_2 + y_4 + y_5 + \; . \; . \; . \; + y_{n-2} + y_{n-1})\} \quad . \quad . \quad (4)$$

The area in both cases $= l\,Y$.

It must be remembered that the area of an irregular figure cannot be exactly determined, consequently all rules for determining the area of such a figure can only give an approximation to the truth ; of these, Simpson's rules give the closest degree of approximation.

Method of Squares.—This method if carefully carried out gives a satisfactory result, which is arrived at with a fair degree of rapidity.

In applying this method, the plan is covered with a network of accurately drawn squares ; the squares being lightly pencilled on the plan or drawn on a superimposed sheet of tracing paper. The length of the side of the square adopted depends on the scale
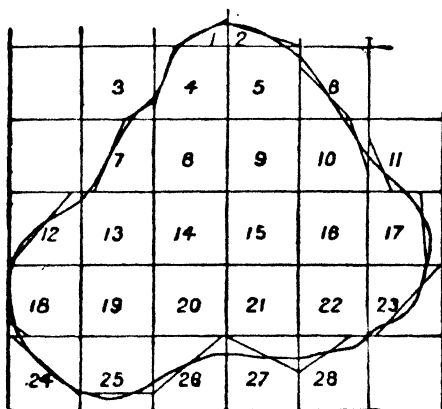
Fig. 120.

of the plan, being made smaller when the scale is small, but it need not be less than half an inch ; in all cases, however, it should represent a whole number of chains. The squares are numbered as in Fig. 120, and the area of the portion of the enclosure within each square is determined in square chains, the sum of these areas divided by 10 gives the area of the enclosure in acres. Several of the squares will lie wholly within the enclosure, and are easily dealt with ; others will lie partly within and partly without the enclosure. The area of the portion of a square falling within the enclosure is calculated from the data obtained by measuring the sides of the right-angled triangle or trapezium, into which the figure is converted, by drawing a

casting line through the part of the boundary falling within
the square, as shown in Fig. 120.

Mechanical methods are those in which areas are obtained
by the use of some instrument, such as the computing scale or
planimeter.

Computing Scale.—The construction of this instrument is
based on the fact that a rectangle 10 chains long by 1 chain
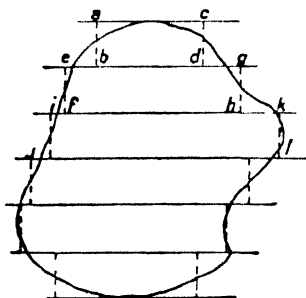
Fig. 121.

wide contains 1 acre; hence to a
scale of 1 chain to 1 inch, an acre
will be contained by a rectangle 10
inches long by 1 inch wide. If we
wish to make use of this fact in
determining the area of an enclo-
sure, we proceed to draw on the
plan a series of parallel lines 1
chain apart, and so placed that
two of the lines are tangent to the
boundary of the enclosure. The
portion of the enclosure lying
between each pair of parallels is
converted into an equal rectangle
by casting lines, as shown by $a\,b$, $c\,d$, $e\,f$, $g\,h$, etc., in Fig. 121.
Measure the bases of these rectangles in chains, then, since
the height of all the rectangles is 1 chain, the sum of
the bases of all the rectangles divided by 10 will give
the area of the enclosure in acres. In the computing scale
(Fig. 122) the casting line is a fine needle or hair line carried
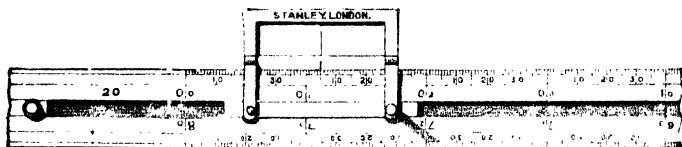
Fig. 122.

by a cursor sliding on a scale, on which the bases of the rectangles
are summed automatically. The parallel lines are not usually
drawn on the plan, but are drawn on a sheet of horn paper or
on thin celluloid, which is adjusted in position on the plan until
the boundary of the enclosure is tangent to two of the lines.
The sheet is then kept in position by weights.

When using the computing scale to determine an area, the

cursor is set to zero, the scale is placed on the horn paper so that
the line on the cursor takes up the position of the casting line
to the left of the first strip, and the scale lies parallel to the
lines on the horn paper. The scale being held firmly in position,
the cursor is moved along it until the line on the cursor occupies
the position of the casting line on the right of the strip. The
reading on the scale will be equal to the area of the first strip.
Without disturbing the position of the cursor on the scale, the
latter is set in position over the second strip, with the line on
the cursor occupying the position of the casting line on the left
of this strip, the cursor is then moved along the scale until it
occupies a similar position on the right. This process is repeated
until all the strips have been dealt with, the final reading of the
scale giving the total area of the enclosure. If the cursor reaches
the end of the scale before all the strips have been dealt with,
mark the position of the hair line with pencil on the horn paper,
slide the cursor back to zero, and recommence from the pencil
line, the area up to this line being noted, and added to the final
scale reading.

The size of the divisions on the computing scale depends on
the scale to which it is constructed. When the scale is 1 chain
to 1 inch, a length of 10 inches on the computing scale represents
(and is marked) 1 acre ; this length is subdivided into four
equal parts, marked 1, 2, 3, 4 roods, and each of these parts is
subdivided into 40 equal parts or poles. If the computing scale
is constructed to a scale of 4 chains to 1 inch, an acre is repre-
sented by a length of $2\frac{1}{2}$ inches, which is subdivided to read
roods and poles. In this case the distance between the parallel
lines on the horn paper is $\frac{1}{4}$ inch.

Two scales are usually engraved on the instrument, a chain
scale and a corresponding feet scale ; but instruments of a uni-
versal character may be obtained in which the scale is engraved
on a slip of boxwood, which may be fitted into a groove made
to receive it, in the lath on which the cursor slides. Generally
six scales of 1 to 6 chains to 1 inch are supplied with a com-
puting scale of this description.

For general office use, a scale of 1 chain to 1 inch is the most
convenient.

Conversion of Areas from the Wrong to True Scale.—While
it is desirable that the scale to which the computing scale is
constructed should be the same as that of the plan on which
it is used, it will frequently happen that the scales are different.
In such a case it is necessary to convert the areas from one scale

to the other.   There is no difficulty in doing this, if we remember that the areas of similar figures are to each other as the squares of their similar linear dimensions.

Thus $$\frac{\text{the true area}}{\text{area as given by computing scale}} = \frac{a^2}{b^2},$$

where a = number of chains to 1 inch on the plan.

b = number of chains to 1 inch on the computing scale.

Example.—The area of a field as given by a computing scale of 1 chain to 1 inch is 4 A . 3 R . 20 P. If the scale of the plan is 4 chains to 1 inch, find the true area.

$$\frac{\text{True area}}{\text{Computing scale area}} = \frac{4^2}{1^2}.$$
$$\text{True area} = 16 \,(4\, \text{A} . 3\, \text{R} . 20\, \text{P})$$
$$= 78 \cdot 0 \text{ acres.}$$

The accepted area determined with the computing scale should be the mean of those obtained with the horn paper arranged in two or three different positions relative to the plan.   The average of several trials made with the horn paper in the same position is not so satisfactory, as, owing to personal bias, errors of judgment are apt to be repeated in going over the same figures in the same order.

The Planimeter.—This instrument—the invention of J. Amsler, Professor of Mathematics at Schaffhausen—is now very largely used for the determination of areas and of the average ordinates of irregular figures.   It consists essentially of two bars of different lengths ; sliding on the longer bar is a frame to which the shorter bar is jointed, so that the bars may freely move relative to each other in one plane ; a wheel attached to the frame and rolling on the paper records by its revolutions the area traced out by a point at the end of the long bar, while a corresponding point at the end of the short arm serves as a fixed centre of rotation for the whole instrument.   The frame carrying the recording mechanism and the jointed end of the short bar may be clamped with its index mark in coincidence with any of the marks engraved on the long bar, the mark selected depending on the units it is intended to work in.   The axis of the recording wheel drives a small dial, which registers its revolutions.   The decimal mode of division is used throughout, the dial having 10 divisions, the wheel 100, each of which may be further subdivided to one-tenth by means of a vernier.   Thus, counting the divisions on

the dial as units, the readings on the wheel give tenths and hundredths, and those on the vernier thousandths of the unit.

The instrument is shown in Fig. 123.

To use the instrument, the index mark on the frame is set to the mark on the long bar denoting the desired unit; the fixed centre is placed at some convenient point on the paper, and held down by a small weight; the tracing point is brought to some marked point on the outline of the figure about to be measured.

After noting the reading given by the recording mechanism, the outline of the figure is followed clockwise with the tracing point until the starting point is again reached, when the reading is again noted. The required area is obtained from these readings, combined with the constant engraved on the instrument. Let the first and second readings be 2·897 and 4·864 respectively,
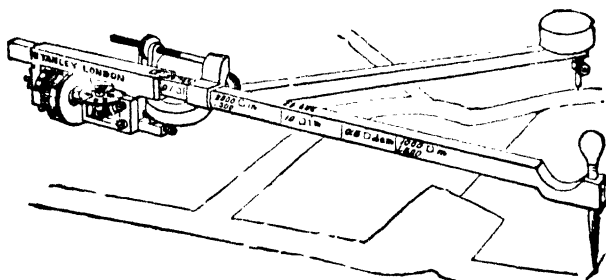
Fig. 123.

and the constant on the instrument 10 square inches. There are two cases to be considered.

(1) If the fixed centre is *outside* the figure, subtract the first reading from the second, the remainder multiplied by 10 gives the area of the figure. Thus, in the assumed case, the area = (4·864 — 2·897) 10 = 19·67 square inches.

(2) When the fixed centre is *inside* the figure, add the difference of the two readings to the number engraved on the top of the long bar above the corresponding units line before multiplying by the constant. If the number is 22·065 and the difference of the two readings 11·967, the area

$$= (22·065 + 11·967) 10 = 340·32 \text{ square inches.}$$

When using this instrument, care must be exercised to keep the motion of the tracing point as smooth as possible; moving

the tracing point with a jerky motion will cause the rolling wheel to slip, thereby giving erroneous readings; a like result will follow if the rolling wheel moves over a surface which is not flat. The latter difficulty is eliminated in some instruments by causing the rolling wheel to move over the flat surface of a disc, which forms part of the instrument. The type illustrated in Fig. 123 is, however, that most in general use.

Most planimeters read to square feet, square inches, and square millimetres. To convert square inches to acres, we use the proportion :—

$$\frac{\text{Area in acres}}{\text{Area in square inches}} = \frac{N^2}{\dfrac{10}{1^2}} = \frac{N^2}{10},$$

or, area in acres = area in square inches $\times \dfrac{N^2}{10}$

where N = the number of Gunter chains to 1 inch, to which the plan is plotted.

If the plan is plotted to 100 N feet to 1 inch, the rule becomes :—

$$\frac{\text{Area in acres}}{\text{Area in square inches}} = \frac{100^2 \, N^2}{\dfrac{43,560}{1^2}}$$

or, area in acres = $\dfrac{N^2 \times \text{area in square inches}}{4 \cdot 356}$

When the area is obtained in square feet, we have the proportion :—

$$\frac{\text{Area in acres}}{\text{Area in square feet}} = \frac{144 \, N^2}{4 \cdot 356},$$

$\therefore$ Area in acres = $\dfrac{144 \, N^2 \times \text{area in square feet}}{4 \cdot 356}$

Area of Survey made with Incorrect Chain.—In the case of a survey executed with an incorrect chain, all chain measurements will appear too short if the chain is too long, and *vice versâ*, while the offsets may be the correct length, since they are not measured with the chain. The calculated areas bounded by the chain measurements may be corrected by the proportion :—

$$\frac{\text{Correct area}}{\text{Calculated area}} = \frac{(\text{incorrect length of chain})^2}{(\text{correct length of chain})^2}.$$

The areas of the strips between the main lines and the boundaries

whose positions are settled by offsets (assumed correct) are corrected by the proportion :—

$$\frac{\text{Correct area}}{\text{Calculated area}} = \frac{\text{incorrect length of chain}}{\text{correct length of chain}}.$$

The cubical contents of impounding reservoirs may be readily calculated from the data given by their contoured plans. The space occupied by the water closely resembles the frustrum of an inverted cone on an irregular base, and for our purpose we may consider the cone to be cut into slices by planes parallel to its base, which is represented by the free surface of the water. The distance between the assumed horizontal sections is equal to the vertical interval between the consecutive contour lines, and the outlines of the contours give the shapes of the sections.

The cubical contents of the reservoir are determined from the areas of the contour sections and their vertical distance apart.

We assume that the contours (being parallel sections of a conical surface) are similar figures, an assumption approximately fulfilled in practice. In determining the required volume, we calculate the volume impounded between each pair of contour lines, and the sum of these quantities gives the total volume.
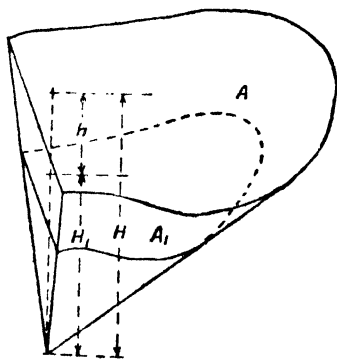
Fig. 124.

Referring to Fig. 124, let A and A_1 be the respective areas of two consecutive contours, and h the vertical distance between them. We may consider the solid, whose volume is required to be a frustrum of a cone. Let the heights of the cones, the areas of whose bases are A and A_1, be H and H_1 respectively.

Then
$$H = H_1 + h,$$

and
$$\frac{H}{H_1} = \frac{\sqrt{A}}{\sqrt{A_1}},$$

or
$$\frac{H_1 + h}{H_1} = \frac{\sqrt{A}}{\sqrt{A_1}}$$

∴
$$H_1 = \frac{h\sqrt{A_1}}{\sqrt{A} - \sqrt{A_1}}.$$

15

The required volume = the difference of the volumes of the two cones.

$$= \tfrac{1}{3} A (H_1 + h) - \tfrac{1}{3} A_1 H_1$$
$$= \tfrac{1}{3} h (A + \sqrt{A\,A_1} + A_1).$$

Example.—Find the volume of water between two contours whose areas are 6 A . 3 R . 20 P, and 5 A . 1 R . 4 P respectively, the vertical interval being 3 feet.

Area A = 6 A . 3 R . 20 P   = 299,475 square feet.
Area A_1 = 5 A . 1 R . 4 P   = 229,779 square feet.
$\sqrt{A\,A_1} = \sqrt{299,475 \times 229,779} = 262,318$

$$\text{Sum} = \overline{791,572}$$

$$\text{Volume} = \frac{3}{3} \times 791,572 \text{ cubic feet}$$

$$= 791,572 \times 6{\cdot}25$$
$$= 4,947,325 \text{ gallons.}$$

Scales.—On the next page is given a list of scales commonly used in Britain, together with a brief statement of the purpose to which each scale is best adapted.

Scales of 8, 6, 5, 4, 3, 2, and 1 foot to 1 inch are used for details of engineering works, and scales of 1, $\tfrac{7}{8}$, $\tfrac{3}{4}$, $\tfrac{1}{2}$, $\tfrac{3}{8}$, $\tfrac{1}{4}$, and $\tfrac{1}{8}$ inch to 1 foot for details of buildings.

EXAMPLES.

1. The base of a triangle is 20·47 chains and height 11·34 chains. Find its area.   (*Ans.* 11 A . 2 R . 17·6 P.)

2. Two sides of a triangle are 18·42 and 12·67 chains respectively, and the angle between them 40° 32′. Find its area.
(*Ans.* 7 A . 2 R . 13·44 P.)

3. Find the area of a triangle whose base is 16·42 chains and base angles 50° 45′ and 60° 31′.   (*Ans.* 9 A . 3 R . 0 P.)

4. A main line (A B) on a survey passes over a pond around which a triangle B D E is thrown. A B produced cuts the side D E at C. If D B = 650, B E = 740, C E = 320, and C D = 410 links, find, by calculation, the length of B C, and the area of the triangle B D E.

(*Ans.* B C = 601·3 links ; area = 2 A . 0 R . 22·72 P.)

5. Find the area of a rhomboid, sides 22·42 and 12 81 chains, and included angle 36° 41′.   (*Ans.* 17 A . 0 R . 25 6 P.)

Representative Fraction.	Usual Designation of Scale.	Purpose.
$\frac{1}{63,360}$	1 mile to 1 inch.	Scale of the smaller ordnance maps of Britain. Well adapted for exploration maps.
$\frac{1}{15,840}$	¼ mile to 1 inch.	Minimum scale for deposited plans of proposed works.
$\frac{1}{10,560}$	6 inches to 1 mile.	Intermediate scale for ordnance maps. Common scale for Parliamentary plans and preliminary estimates.
$\frac{1}{10,000}$	6·336 inches to 1 mile.	Decimal scale having the same advantages.
$\frac{1}{6,336}$	10 inches to 1 mile. ⎫ 8 chains to 1 inch. ⎭	Convenient intermediate scale for proposed public works.
$\frac{1}{4,800}$	400 feet to 1 inch.	Smallest scale permitted for deposited plans of " enlarged plans of buildings " and land within the curtilage of buildings.
$\frac{1}{4,752}$	6 chains to 1 inch.	General scale for Parliamentary plans.
$\frac{1}{4,000}$	15·84 inches to 1 mile.	Working plans and sections of great engineering works.
$\frac{1}{3,960}$	5 chains to 1 inch.	Scale for deposited plans of proposed railways showing alterations of roads.
$\frac{1}{2,500}$	25·344 inches to 1 mile.	Scale of part of the Ordnance Survey of Britain. Well adapted for plans of estates and proposed engineering works.
$\frac{1}{2,400}$	200 feet to 1 inch.	Smallest legal scale for land or contract plans in Ireland.
$\frac{1}{2,376}$	3 chains to 1 inch.	Tithe maps and estate plans.
$\frac{1}{1,584}$	2 chains to 1 inch.	Building ground. Working contract plans.
$\frac{1}{1,200}$	100 feet to 1 inch.	Scale suited for plans of less intricate towns.
$\frac{1}{1,056}$	88 feet to 1 inch. ⎫ 60 inches to 1 mile. ⎭	Ordnance maps of ordinary towns.
$\frac{1}{528}$	44 feet to 1 inch. ⎫ 120 inches to 1 mile. ⎭	Ordnance maps of the more intricate towns.
$\frac{1}{500}$	126·72 inches to 1 mile.	Decimal scale for the same purpose.
$\frac{1}{480}$	40 feet to 1 inch.	⎫
$\frac{1}{360}$	30 feet to 1 inch.	⎪
$\frac{1}{240}$	20 feet to 1 inch.	Enlarged working plans and sections of proposed engineering works.
$\frac{1}{120}$	10 feet to 1 inch	⎭

6. A trapezoidal plot of ground has one side (or base) 14·32 chains long; the perpendicular sides meeting the base at its extremities are 5·82 and 9·34 chains respectively. Find the area of the plot.   (*Ans.* 10 A . 3 R . 16·0 P.)

7.	0	(955)	B
╱	91	785	
╲	57	634	
╱	88	510	
╲	70	340	
╱	84	220	
╲	62	45	
╲	0	(000)	A

Draw the plan and find the contents of the piece of ground, the survey notes of which are given. Scale, 2 chains to 1 inch.

A B coincides with one fence and the other fence is straight between the offsets. Distances and offsets are in links.

Bearing of |A B 90°

    (*Ans.* 0 A . 2 R . 23 P . 15⅛ square yards.)

8. The bearings of two straight fences meeting at the corner of a field are 25° 30′ and 55° 30′. An area of 2½ acres is to be cut off the field by a straight fence which cuts the existing fences at points which are equidistant from their intersection. Determine the length of the new fence, and describe how you would mark out its position.

    Ans. 5·17 chains, and cuts existing fences at 10·00 chains from their intersection.)

9.	C	240	1090	
			(1040)	B
	D	73	844	
	E	250	600	
	F	140	300	
			(100)	A
	G	200	000	

Find the distance of the point H from B, at which a casting line drawn from G through the irregular fence G F E D C cuts the fence B C. Scale, 2 chains to 1 inch.

    (*Ans.* 141 links.)

10.			(440)	C
	D	178	350	
			312	150 B
	E	100	240	
	F	208	82	
			(000)	A

Plot the given polygon to a scale of 1 chain to 1 inch, and reduce it to an equal triangle having its vertex at F and base in the side B C produced.

Determine the area of the polygon.

    (*Ans.* 0 A . 3 R . 21·58 P.)

11.

0	(500)	B
75	450	
90	400	
95	350	
122	300	
145	250	
151	200	
144	150	
133	100	
97	50	
0	(000)	A

Reduce the figure (given by distances and offsets) to an equal right-angled triangle having A B as base, and write down its area. Scale 1 inch = 1 chain.

(*Ans.* 0 A . 2 R . 4·8 P.)

12. Determine the area of the figure given in Question 11 by (*a*) the mid-ordinate method; (*b*) the ordinary rule; and (*c*) Simpson's rule.

(*Ans.* (*a*) 0 A . 2 R . 7·68 P; (*b*) 0 A . 2 R . 4·16 P; (*c*) 0 A . 2 R . 5·76 P.)

13. The bearings and lengths of two sides of a triangular traverse (A B C) are as follows:—A B bearing 275°, length 850 links; B C bearing 35°, length 690 links. Find the bearing and length of the closing side C A. Find also the area of the traverse. (*Ans.* 144° 48′ ; 782·3 ; 2 A . 2 R . 6·38 P.)

14. Sketch and describe the computing scale. If the computing scale is constructed to a scale of 1 chain to 1 inch, with what factor must the results obtained by it be multiplied, if they are obtained from a plan whose scale is 2½ inches to 1 chain ?

(*Ans.* ·16.)

15. The area of a plot of land as measured on a plan is 14·32 square inches, the scale of the plan is 2 chains to 1 inch. Find the area of the plot in acres, roods, and perches.

(*Ans.* 5 A . 2 R . 36·48 P.)

PART II.

CHAPTER IX.

CONSTRUCTION AND ADJUSTMENTS OF ANGLE MEASURING INSTRUMENTS.

General Remarks.—Surveying instruments for measuring angles fall into the following three main types or their combinations :—(1) The theodolite type, for measuring angles in horizontal and vertical planes ; (2) reflecting instruments, for measuring angles in any desired plane ; (3) magnetic instruments, for obtaining bearings and angles dependent thereon.

All angle-measuring instruments possess one or more main scales, divided to read degrees and some fraction of a degree, dependent on the size and purpose of the instrument. In the smaller instruments, the main or primary scale is divided to degrees and half-degrees ; the larger and more delicate instruments read to degrees and thirds or degrees and sixths of a degree. To obtain a finer reading the unit on the primary scale is subdivided either by a small subsidiary scale, called the vernier scale, or by a micrometer reading microscope provided with a pair of movable cobwebs, whose motion is governed by a finely-pitched screw.

Instruments provided with verniers do not as a rule read finer than 20 seconds, but those fitted with micrometers read to 5 or 10 seconds of arc.

The Vernier is a short scale having the same curvature as the main scale, and is fixed with its divided edge in sliding contact with the divisions on the main scale. The vernier has one division more or one division less than is contained in an equal length of the primary scale. Thus, if the vernier scale contains 30 divisions the main scale will contain 29 or 31 divisions in the same length. In the former case the vernier divisions will be shorter and in the latter longer than the divisions on the primary scale ; but

ın both cases the reading given by the vernier will be one·thirtieth of the least division on the primary scale. The truth of this may be demonstrated as follows :—

Let N = the number of divisions on the vernier scale,

V = length of the vernier unit,

S = length of primary scale unit.

Then, the length of the vernier = $N\,V$, and the corresponding length on the primary scale (when S is greater than V) = $(N-1)S$.

Therefore, $N\,V = (N-1)\,S$

and $$V = \frac{(N-1)\,S}{N}$$

The least reading given by the vernier = $S - V$

$$= S - \frac{(N-1)\,S}{N},$$

$$= \frac{S}{N}.$$

When S is less than V, we have $N\,V = (N+1)\,S$

and $$V = \frac{(N+1)\,S}{N}$$

As before, the least reading given by the vernier

$$= S - V$$

$$= S - \frac{(N+1)\,S}{N}$$

$$= -\frac{S}{N}.$$

From this, we observe that the numerical result is the same whether S is greater or less than V. The former mode of division is, however, that most generally adopted, since the vernier and the primary scale both read in the same direction : in the latter they read in opposite directions, as shown by the negative sign. As illustrations of the use of the above formula, we will consider the following examples :—(1) How many divisions must be engraved on a vernier in order to read single minutes if the primary scale reads to (a) half degrees and (b) third of a degree ? Give the length of the arc subtended by the vernier in each case.

(a) We have, $1 = \dfrac{60}{2\,N}$,

or $N = 30$ divisions.

(b) In this case, $1 = \dfrac{60}{3\,N}$,

and $N = 20$ divisions.

Length of vernier (a) $= 29 \times \frac{1}{2}^{\circ} = 14^{\circ}\,30'$,
        ,,   (b) $= 19 \times \frac{1}{3}^{\circ} = 6^{\circ}\,20'$.

(2) What is the least reading given by a vernier containing 60 divisions, if the primary scale reads to degrees and thirds of a degree ?

The least reading $= \dfrac{60 \times 60''}{60 \times 3}$
                  $= 20$ seconds,

and the vernier would subtend $(60 - 1) \times \frac{1}{3}$ degrees, or $19^{\circ}\,40'$ on the primary scale.

Vernier scales may be constructed to give any desired fraction of the main scale unit ; they are easily read, and are not liable to get out of order.

Since the vernier subtends a certain number of divisions on the primary scale, two divisions on the vernier can only be in coincidence with two divisions on the primary scale when the vernier reading is zero ; in all other positions one only of the vernier divisions can coincide with a division on the primary scale, and the number of this division (read on the vernier scale) gives the vernier reading. The total angle is given in all cases by the sum of the primary and vernier scale readings. For example, let the reading be $23^{\circ}\,12'$, the main scale reading to half-degrees and the vernier to minutes. In this case, the zero of the vernier will be beyond the division on the main scale corresponding to 23°, and coincidence on the vernier will take place at the twelfth division, giving the vernier reading of 12 minutes. The setting of the vernier and scale is shown in Fig. 125. If the given reading is $23^{\circ}\,47'$, the reading on the main scale will be $23\frac{1}{2}^{\circ}$ or $23^{\circ}\,30'$, and the vernier reading $17'$, thus giving the total reading of $23^{\circ}\,47'$. The relative positions of the vernier and scale are shown in Fig. 126.

The Micrometer Microscope.—In this contrivance a rectangular metal frame carrying two parallel cobwebs is moved by a finely-pitched screw, in the focal plane of the eyepiece of a reading

microscope. Attached to the end of the screw and turning with it is a small drum having a scale of equal parts engraved on its curved surface; an index line engraved on the outer case of the microscope forms a fixed point of departure for this scale. It is evident that if the scale on the drum consists of N divisions, and the screw be rotated through one of these divisions, the movement of the cobwebs will be $\frac{1}{N}$th of the pitch of the screw. If the pitch of the screw is made equal to the smallest division on the primary scale, as seen in the microscope, then the value of each division on the divided drum will be $\frac{1}{N}$th of the value of this division. For example, if the main scale reads to sixths
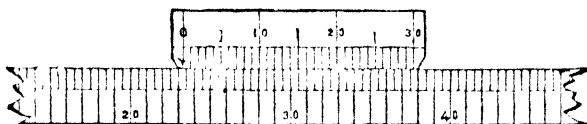
Fig. 125.
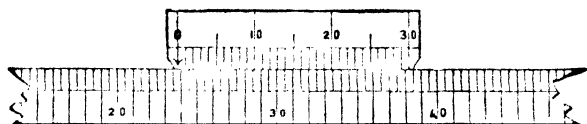
Fig 126.

of a degree, and there are 60 divisions on the drum, the value of each division $= \dfrac{60 \times 60}{6 \times 60} = 10$ seconds.

If there are 120 divisions on the drum, the instrument will read to $\dfrac{60 \times 60}{6 \times 120} = 5$ seconds.

With a primary scale reading to 10 minutes, the scale on the drum of the microscope is divided into 10 main divisions, and each division is subdivided into six equal parts. The main divisions are numbered 1, 2, etc., to 10, and the value of each division is 1 minute. Consequently, the value of each sub-division is 10 seconds.

Two micrometer microscopes are fitted to each divided circle, and are placed 180° apart. The mean of the readings given by the two microscopes is taken as the correct reading.

When set to zero, the interval between the cobwebs (Fig. 127) should be bisected by the vertex of the V-shaped incision, which is a fixed point of departure for the cobwebs, corresponding to the zero point on a vernier.

To determine the angle between two lines which intersect at a given station, the instrument is set up over the station, the main scale and micrometer microscope are set to zero, or their readings are noted when the telescope is set to bisect the left-hand signal. The telescope is then turned to bisect the right-hand signal, and the reading is again noted. The difference of the two readings is the angle required.

Let the reading in the first case be $9° 20' + x$. To determine x, turn the divided drum (C, Fig. 127), thus moving the parallel cobwebs, until the divisions denoting the number of minutes (20) in the above angle is exactly midway between the two webs. Now, suppose the conditions to be as shown in the figure, evidently
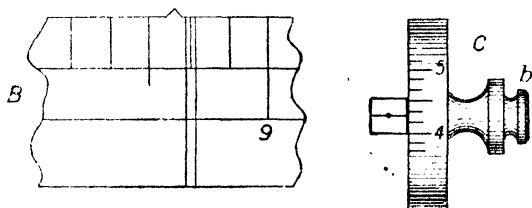
Fig. 127

x is rather more than $4' 15''$, say, by estimation, $4' 17''$, and the first angle is $9° 20' + 4' 17''$ or $9° 24' 17''$. After rotating the telescope, let the angle (obtained in a similar way) be $59° 31' 26''$. Obviously the angle between the two lines is $59° 31' 26'' - 9° 24' 17''$ or $50° 7' 9''$.

If the micrometer microscope does not read zero when the vertex of the V-shaped point of departure accurately bisects the interval between the cobwebs, the drum may be shifted relative to the screw by holding the drum in one hand and turning the milled head b with the other, the drum being attached to the screw by a friction grip only. A finer adjustment is obtained by setting the drum to zero in the ordinary way, and shifting the V notch to the desired position by a fine adjustment screw (not shown in the figure) to the left of the microscope.

An instrument fitted with micrometer microscopes is shown in Fig. 129.

Adjustment of Micrometer Microscope.— It is an essential part of the principle of the micrometer microscope that the cobwebs should move exactly over one apparent division of the scale for each revolution of the divided head. Now the cobwebs are magnified by the eyepiece of the microscope only, but the divisions on the scale are magnified both by the object glass and eyepiece. Occasionally the distance between the object glass and the plane of the cobwebs is disturbed by the continued use of the instrument, thus causing a slight alteration in the apparent distance between the scale divisions.

Taking Runs.—The method of testing the microscope is known as " taking runs," and is carried out as follows :—Clamp the divided circle, set the cobwebs to some division on the scale, record the reading given by the micrometer head, which is then turned until the cobwebs have exactly passed over (say) four divisions on the main scale. Again record the reading given by the micrometer head. Repeat these operations several times in order to get a good mean value. If the mean obtained from the reading microscope agrees with its proper value within 2 or 3 seconds the error may be neglected, if the instrument is intended for topographical work. If the error is larger than this it becomes necessary to alter the magnifying power of the microscope. Suppose, for example, that the mean result given by the micrometer head over a run of 40′ is 40′ 10″. This gives an error of 10″ in excess, showing that the power of the microscope is too great. To reduce the magnifying power, release the ring clamping the object glass cell and screw the cell slightly *inwards*, then reclamp the cell. The exact amount of adjustment of the object glass necessary to take up the error can only be obtained by trial, hence the operations of taking runs and adjusting the cell must be repeated until the microscope reads correctly.

After each adjustment of the object glass, a slight adjustment of the whole microscope for focus will be necessary. This is done by releasing the body of the microscope in its socket and sliding it towards or away from the main scale, as may be necessary.

The Theodolite.

This instrument is designed for the purpose of measuring angles in horizontal and vertical planes. The former are frequently spoken of as *Azimuths* or *Azimuthal angles*, and the latter as angles of elevation or depression.

Azimuths.—The term azimuth, when used without qualification, means the angle between two great circles of the earth which intersect at the observer's station, and passing respectively through the Poles and the observer's referring object. Obviously, the azimuth of a line is its true bearing.

Azimuthal angles are always measured from the North Pole clockwise, thus in Fig. 128 the azimuth of A B is the angle N A B, and the azimuth A C is the angle N A C.

Relative Azimuths.—In most surveying operations, however, angles between lines which intersect at the observer's station (or relative azimuths) are chiefly required. The term *Relative Azimuth* means the angle that a great circle passing through the observer's station and one object deviates to the right of a great circle passing through the observer's station and the other object. In Fig. 128 the relative azimuth of A B and A C is the angle B A C.
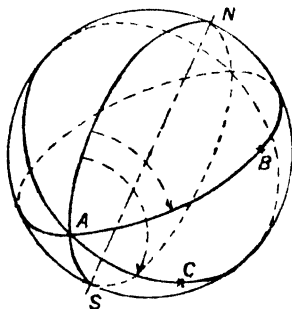
Magnetic Azimuth.—When magnetic azimuth (or bearing) is specified, the angular deviation is measured from the magnetic meridian and not from the true meridian, the direction of measurement is, however, clockwise in all cases.

Whole Circle Bearings.—All bearings obtained in this way, whether true bearings or magnetic, are termed *whole circle bearings*.

Fig. 128.

Angles of Elevation or Depression.—These are the angles between imaginary lines joining the observer's eye to distant objects and a horizontal plane through the point of sight. The angles thus observed are angles of elevation or depression, according as they are above or below the horizontal plane. The direction of the horizontal plane of reference is denoted by the position of the bubble in the principal spirit level of the instrument. All vertical angles are acute, when read on the four-quadrant system.

"Transit" and "Y" or "Plain" Theodolites.—Theodolites vary very much in their construction ; but they all contain certain essential parts which are similarly constructed. The chief difference in the construction of the various forms lies in the manner in which the telescope is fitted to the horizontal axis. This gives rise to two main types of the instrument—viz., the "Transit"

and the " Y " or " Plain " theodolite. In the first the telescope is rigidly fixed at right angles to the horizontal axis, and the " A " frames supporting the axis are tall enough to allow the telescope to completely rotate. In the second type the telescope is fitted with bearing collars, which fit into Y-shaped bearing supports, as in the " Y " level, and the A frames are low, only permitting the telescope to rotate through a limited angle.

The transit type is that in most common use.

The Tripod.—The construction of tripods for surveying instruments has been fully dealt with on p. 96, *et seq.*, both types there described are in use for the support of theodolites. The only difference between a tripod intended for use with a level and that for use with a theodolite is in the provision of some means for centering the instrument over a station. In the smaller and less delicate instruments, a hook is fitted in the axis of the tripod head on its underside, and from this the centering plumb-bob is suspended. In other cases, the hook is screwed into the lower end of the vertical axis of the instrument, the head of the tripod having an axial hole through which the plumb line passes.

The Parallel Plates.—The instrument is attached to the tripod by means of an internal screw formed in the boss on the lower side of the tribrach plate or the lower parallel plate. This screw is cut to fit the external screw on the tripod head. Both three- and four-screw instruments are made, the construction of the tribrach plate and the parallel plates is similar to that in the corresponding type of level. A description of these details is given on p. 97.

Movable Substage.—In some instruments, however, an arrangement is provided whereby the whole instrument may be moved through a short horizontal distance (in any direction) relative to the tripod. This movement (which is provided to facilitate the operation of centering the instrument over a station) is obtained by forming the tribrach plate and the boss carrying the internal screw fitting the tripod head in two separate parts. The boss is hollow, and is prolonged through the tribrach plate, which rests on an external collar or flange formed on the boss near its lower extremity. At the upper extremity of the boss an external screw is cut on which a ring nut provided with finger grips (G, Fig. 129) is fitted. Immediately below the ring nut is a loose plate, which rests on a circular facing surrounding the large axial hole in the tribrach plate. This loose plate transmits the pressure exerted by the ring nut through the tribrach
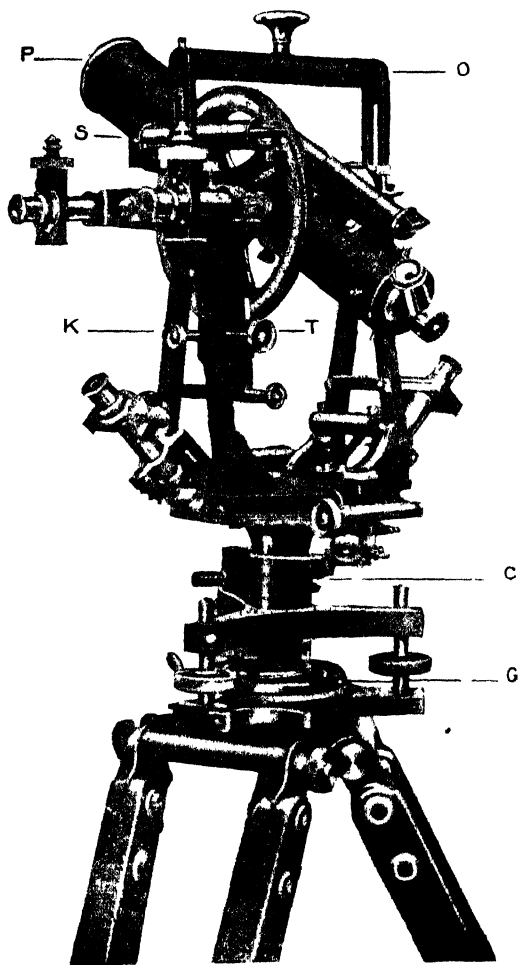
Fig. 129.

plate to the flange on which the latter rests, thereby fixing the instrument to the tripod.

When centering an instrument, provided with this device, over a station, the instrument is first roughly centred by moving the legs of the tripod until the plummet is nearly in coincidence with the mark on the ground, the ring nut is turned back, and the instrument is moved in the required direction until the plummet exactly coincides with the centre of the station mark; the ring nut is then screwed up, thus fixing the instrument in the desired position.

In another arrangement, for effecting the same purpose, the upper part of the instrument is carried on a sort of compound slide-rest, in which the movement of the instrument relative to the tripod head is controlled by two horizontal screws, placed at right angles to each other. This arrangement gives a very delicate adjustment to the position of the vertical axis, but it contains more loose parts than the arrangement previously described, and consequently is not so much in favour.

The Vertical Axis.—At the centre of the upper parallel plate (or of the tribrach in a three-screw instrument) is a hollow cylindrical boss—sometimes called the body piece— on the lower extremity of which is the ball of the ball-and-socket joint connection between the parallel plates in a four-screw instrument. The interior of the body piece is accurately ground conical to fit the hollow (or outer) vertical axis of the instrument. The interior of the hollow axis is also ground conical to fit the inner (or solid) axis. It is essential that the inner and outer axes be co-axial, the common axis forming the vertical axis of the instrument. The two axes are kept in position by a screw and washer fitted to the lower extremity of the inner axis.

The Horizontal Circle.— The outer axis is fixed to the horizontal circle, which has its edge or limb turned to a conical surface and covered with silver, on which the graduations of the circle are engraved. The size of this circle and the minuteness of its graduations depend on the nature of the work for which the instrument is intended. The ordinary limits of size vary from 4 to 8 inches, though a few have been made of larger diameter, the largest being the 36-inch instruments used in the Trigonometrical Surveys of this Country, and of India. The 5-inch and 6-inch instruments are most commonly used for general survey and engineering work; the former having its circles divided to degrees and half-degrees, reading to single minutes by verniers, while the latter is divided to degrees and thirds and reading to 20 seconds by its verniers. For extended surveys, where greater accuracy is required, 5-inch or 6-inch instruments graduated

to degrees and sixths of a degree are used. These instruments are provided with micrometer microscopes reading to 5 or 10 seconds (see Fig. 129).

The graduations on the horizontal circle are from 0° to 360°, and are numbered continuously clockwise.

Clamp and Tangent Screw.—The outer axis of the instrument (and with it the horizontal circle) may be quite free to rotate or be fixed to the upper parallel plate (or tribrach) at will, by means of a clamp and tangent screw. This arrangement is spoken of as the *clamp and tangent screw to the vertical axis*. The clamp (C, Fig. 129) consists of a split collar surrounding the outer axis, the split part of the collar is drawn together by the clamp screw until it firmly grips the axis. Projecting from the clamp collar is an arm, to which is attached the nut (N, Fig. 131) of the tangent screw, the bearing of the latter being formed in a swivelling projection on the upper parallel plate (or tribrach). Thus, on tightening the clamp screw, the horizontal circle is fixed to the upper parallel plate through the medium of the tangent screw, and consequently, by turning the tangent screw, a slow motion is imparted to the horizontal circle and to the whole of the upper part of the instrument.

The Vernier Plate.—Attached to the upper extremity of the inner axis, and accurately at right angles thereto, is the vernier plate, on which is built the whole of the superstructure of the instrument. In most instruments this plate carries two verniers placed 180° apart, in others three verniers are provided, and are placed at angular intervals of 120°. The verniers are divided on silver, and form a continuation of the conical surface of the horizontal circle, the divided edges of the primary and vernier scales being continuously in contact. A clamp and tangent screw—similar in construction to that to the vertical axis—is provided for the purpose of fixing the vernier plate to the horizontal circle. The attachments of the vernier plate consist of (a) the A frames (A, Fig. 130) for supporting the telescope and its attachments ; (b) two small spirit levels (L, Figs. 130 and 131) placed at right angles to each other ; and (c) a circular compass.

The " A " Frames.—These derive their name from their shape, which is not unlike a letter A. The frames stand vertically on the upper surface of the vernier plate, to which they are attached by screws. The bearings of the horizontal axis are formed in the upper ends of the frames, the bearing surfaces—in the best instruments—being V-shaped, one bearing being cut out of the solid, the other being cut in a loose block of metal. This block

is held in position by guide plates, and may be adjusted in a vertical direction by a screw fitted with capstan nuts. Some makers cut both bearings out of the solid, but divide one of them by a vertical saw cut through the vertex of the V. The two parts are sprung towards each other by a screw passed through the frame at right angles to the saw cut, thus elevating or depressing the axis as required.

The bearings are closed at the top by cap bars held in position by milled head screws. At the centre of each cap bar is a small cell fitted with a piece of cork, which puts an elastic pressure on the telescope trunnions.

The Vertical Circle.—In the transit type of instrument, the vertical circle is fixed to the horizontal axis and the scale—divided on silver—is graduated with the same degree of minuteness as the horizontal circle. In some instruments the graduations are numbered continuously clockwise from 0^c to $360°$. In many instruments, however, the circle is divided into four quadrants, the degrees in each of which are numbered from $0°$ to $90°$ right- and left-handed from the two zeros. The latter are placed at the ends of the horizontal diameter of the circle, the line joining the zeros being parallel to the line of collimation of the telescope when it is horizontal; the line joining the two $90°$ divisions is at right angles to the former line.

Fig. 130.

The Index Bar.—The two verniers to the vertical circle are placed at the ends of the horizontal arms of a T-shaped piece of metal (the index bar), freely centred on the horizontal axis. The extremity of the vertical portion of the index bar is forked, and opposition screws (the clip screws) are fitted in the ends of the prongs. The clip screws engage with the opposite sides of a piece of metal which projects from the cross bar of the A frame, and when screwed up in opposition they rigidly fix the index

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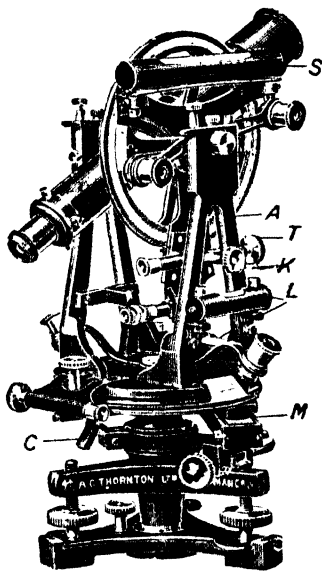
bar to the A frame. This arrangement also allows a slight
adjustment to be made in the position of the index line of the
verniers. The clamp and tangent screw to the horizontal axis
are also attached to the vertical portion of the index bar, and are
shown in Figs. 129 and 130 at K and T. As the weight of the
vertical circle and that of the index bar and its attachments
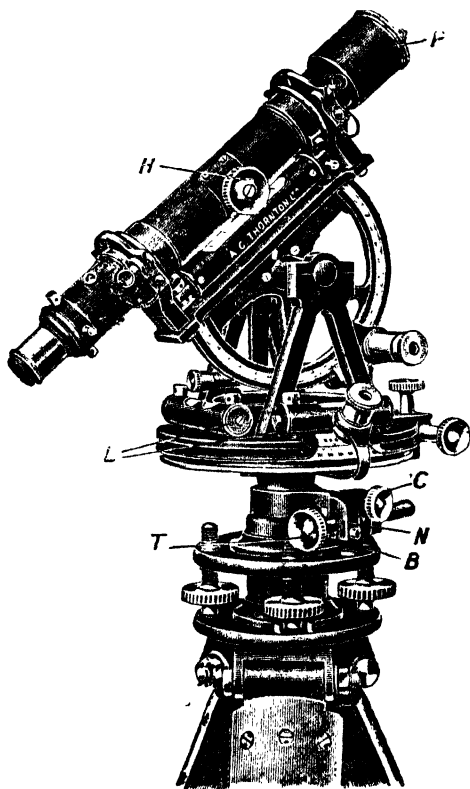
Fig. 131.

will fall very largely on the trunnion nearest to them, in the
larger instruments, to obtain an equal bearing pressure, a boss
is cast on the horizontal axis near the opposite trunnion, the
weight of the boss being carefully adjusted by the maker, until
the pressure at the two bearings is the same.

In many transit theodolites the principal spirit level is attached to the index bar, as shown at S in Fig. 129 ; in others it is attached to the telescope tube, as in Fig. 131.

In the " **Y** " or " Plain " theodolite, the vertical circle is curtailed to little more than a semicircle, and only one vernier is fitted to it, as shown in Fig. 131.

The Telescope.— The telescope consists of two tubes, one sliding within the other. At the forward end of the larger tube is the object glass, which is a compound lens, consisting of a bi-convex lens outside and a concavo-convex lens inside.

The Object Glass.—The object glass is protected from rain or spray by a short metal tube (P, Fig. 129), which may be drawn out as required ; as a further protection the tube may be closed by a pivotted metal disc. The object glass forms an inverted image of the object sighted near the rear end of the smaller tube of the telescope, and the image is magnified by a Ramsden eyepiece, which is fitted in the rear end of the smaller tube.

The Eyepiece.—The eyepiece consists of two small planoconvex lenses fitted in the opposite extremities of a short tube, with their curved surfaces turned towards each other. This eyepiece does not invert the image it magnifies, and consequently objects seen through the telescope appear inverted. " Erecting " eyepieces are usually supplied with theodolites ; these eyepieces contain two additional lenses, which cause the images of objects to appear in their natural positions. These additional lenses, however, cause a loss of light and, consequently, a reduction in the clearness of the resultant image, hence they are not often used.

The Diagonal Eyepiece.—The ordinary eyepiece is not suitable for observing objects at high altitudes, since in such cases the eyepiece would be too close to the vernier limb for an accurate observation to be made. The diagonal eyepiece is constructed to overcome this difficulty. In this device, a Ramsden eyepiece is fitted in a short tube, connected with, and at right angles to, another short tube, which forms a continuation of the inner tube of the telescope. At the junction of the two tubes a mirror is fitted at an angle of 45° with the axis of the telescope. By this arrangement objects are viewed in a direction at right angles to the axis of the telescope. A diagonal eyepiece is shown in Fig. 129.

Focussing.—The object glass is focussed on the object sighted by moving the inner tube in or out as required, the motion being actuated by a rack and pinion, turned by a milled head (H, Fig. 131). The eyepiece is focussed by sliding it inwards or

outwards relative to the tube in which it fits.   When the telescope
is correctly focussed, the foci of the eyepiece and object glass
coincide in one plane, and in this plane the " diaphgram " is
fixed.

The Diaphragm consists of a stout metal ring held in position
within the telescope by four screws (the diaphragm screws),
passing through slotted holes at the ends of the horizontal and
vertical diameters of the tube.   Crossing the centre of the ring
are the sighting webs, which may be formed of spider's lines,
lines engraved on glass, or fine iridium points, the former being
—in the author's experience—the most satisfactory.   The ar-
rangements of the sighting webs are various ; in theodolites
intended for general survey and engineering work, the lines
form an acute cross intersected by a horizontal thread (Fig. 132) ;
in other cases the lines are arranged in horizontal and vertical
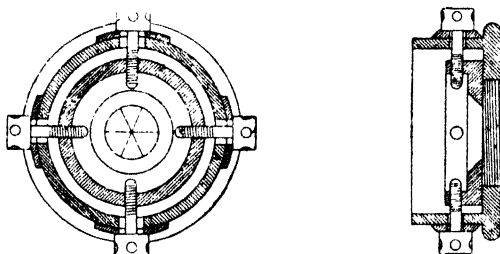
Fig. 132.

pairs, which enclose a square of 40″ to 50″ side at the centre of
the telescope.   The image of the observed object is brought
to the centre of the square or is bisected by the cross.

When the theodolite is intended for astronomical work, the
webs are either arranged in the form of a simple cross or consist
of five vertical lines crossed by one or three horizontal lines.

Additional horizontal webs are sometimes fitted for stadia
reading (see p. 393).

The Compass.—There are two forms of magnetic compass
fitted on theodolites—viz., the circular box compass and the
trough compass.   The former is placed on the vernier plate and
consists of a finely balanced edge needle enclosed in a cylindrical
box having a glass lid.   Approximate bearings are read on a
circular scale within the box, the zero of the graduations is
placed at the *North* end of the needle.   The graduations are

numbered anti-clockwise, since the divided circle turns with the telescope. In the trough compass (M, Fig. 130), the needle is enclosed in a narrow rectangular box having a glass lid. No scale is provided, but two standard marks are engraved in the interior of the box by the maker, the direction of the magnetic meridian is defined when the needle is brought to rest with its ends opposite these marks. This form of compass is usually fitted to the under-side of the horizontal circle, the line joining the standard marks in the compass box being at right angles to the line joining the zero and 180° divisions on the horizontal circle.

Of the two forms, the trough compass is the most satisfactory, since (a) the ends of the needle are more clearly visible than in the circular compass, which is partly concealed by the telescope; (b) when taking bearings, the standard line and needle in the trough compass remain stationary; in the circular compass this line rotates with the telescope, hence in the latter form any disturbance of the standard direction is not so easily detected; (c) the needle in the trough compass is longer, and consequently more delicate, than that of the circular compass on the same size of instrument.

Illuminating the Cross Wires.—When making observations at
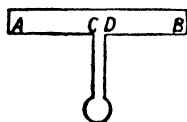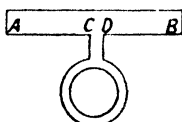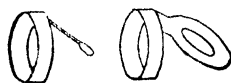
Fig. 133.   Fig. 134.   Fig. 135.

night or in tunnels, some arrangement is necessary for illuminating the cross wires. In some instruments this is accomplished by boring a hole in the horizontal axis through the trunnion opposite to the vertical circle. The exterior end of this hole is closed by a small lens, which transmits a ray of light from the lamp carried on a bracket, fixed to the A frame. The ray of light is reflected by a tiny mirror placed at the intersection of the axes of the hole and telescope, and the light from the mirror is thrown on the cross wires. The lamp is usually supported on a wooden block attached to the bracket, in order that heat from the lamp may not reach the A frame by conduction.

In instruments not provided with this arrangement, the cross wires may be illuminated by a lamp held near the object glass by an assistant. If much night work has to be done, a piece of bright tin should be cut to the shape shown in Fig. 133 or

to that shown in Fig. 134. The part A B is bent to clip on the
end of the spray shade, and the strip supporting the small disc
or circle is bent along the line C D, so that it is inclined at an
angle of 45° to the axis of the telescope, as shown in Fig. 135.
The disc is illuminated by a lamp placed on the ground, and
reflects sufficient light down the telescope tube to make the
cross wires visible.

Reading Microscopes.—These are fitted to all the verniers on
a theodolite, and are similar in construction to the eyepiece of
the instrument (see Figs. 130 and 131).

Tests and Adjustments of Theodolites.

While both types of instrument must satisfy the same tests,
the methods of adjusting the instruments are different. The
mode of procedure in each type will be dealt with separately.

The adjustments may be divided into two classes—(*a*) temporary,
and (*b*) permanent. The former have to be continually made
and unmade ; the latter remain permanent for long periods.

The temporary adjustments are as follows :—(1) To centre
the instrument over a station ; (2) parallax ; and (3) to set the
vertical axis truly vertical.

(1) *To Centre the Instrument over a Station.*—When the instru-
ment is provided with a movable substage, proceed as described
on p. 231. If the instrument is not provided with this arrange-
ment, the adjustment is made by moving the legs of the tripod
until the plummet is vertically over the station mark. When
the instrument is correctly centred, the legs of the tripod should
be firmly planted on the ground, and the axis of the tripod-head
should be approximately vertical.

(2) *The Adjustment for Parallax* is made by moving the eye-
piece in or out of its containing tube, until the cross wires are
in perfect focus (see p. 114).

(3) *To set the Vertical Axis truly vertical.*—If the spirit levels
on the instrument are in correct adjustment, this adjustment
is made with the levelling screws as in the case of the level,
for which see p. 113. If the spirit levels are out of adjustment,
they must be first adjusted as described in adjustment (4).

The permanent adjustments are as follows :—(4) To set the
spirit levels at right angles to the vertical axis ; (5) to place
the line of collimation at right angles to the horizontal axis ;
(6) to place the line of collimation parallel to the spirit level, or
to make the vertical circle read zero when the line of collimation

is horizontal ; and (7) to place the horizontal axis at right angles
to the vertical axis.

Transit Type.

(4) *To set the Spirit Levels at Right Angles to the Vertical Axis.*
—Set up the instrument on a firm foundation, clamp the vertical
circle at zero, and fix the index bar in position by the clip screws.'
Turn the telescope round until the principal spirit level (fixed on
the index bar or on the telescope) is vertically over the diagonal
line joining two foot screws (or parallel to the line joining a pair
of foot screws in a three-screw instrument), and by them bring
the bubble to the centre of its run.   Leaving the vertical circle
clamped, unclamp the vernier plate, turn the telescope round
through 180°, and if the bubble be not central after rotation,'
take up half its deviation with the same pair of foot screws, and
the remaining half by turning the clip screws. Now, turn the
telescope through 90°, and bring the bubble to the centre of its
run by turning the other pair of foot screws (or the third foot
screw). Repeat till perfect.

When the bubble remains central, in all positions of the tele-
scope, the vertical axis is truly vertical, it remains to adjust the
levels on the vernier plate by turning their adjusting screws until
the bubbles are brought to the centre of their respective runs.

(5) *To place the Line of Collimation at Right Angles to the
Horizontal Axis.*—Having set up the instrument, obtain an
accurate bisection of the image of some well-defined distant
point ; observe that the vertical axis and horizontal circle are
tightly clamped.   Reverse the telescope in its bearings (*i.e.,*
place the right-hand trunnion in the left-hand bearing, and
vice versâ), using great care not to jar the instrument while doing
this.   Replace the bearing caps, rotate the telescope on its
horizontal axis, and if the cross wires do not again bisect the
image of the object, take up half the error by the tangent screw
to the horizontal circle, and the remaining half by moving the
diaphragm with the horizontal diaphragm screws. Repeat until
perfect.

(6) *To place the Line of Collimation parallel to the Spirit Level.*
—This may be done at one station, as follows :—Bring the bubble
of the principal spirit level to the middle of its run by turning
'the clip screws.   Bisect the image of some well-defined distant
object with the horizontal wire, and read one vernier on the
vertical circle.   Reverse the telescope in its bearings and relevel
the principal spirit level by the clip screws ; again bisect the

image of the object and read the same vernier. Set the vernier to read the mean of the observed angles, then by turning the clip screws bisect the image of the object a third time, the bubble —which will have moved out of the centre—must be brought back to the middle of its run by the capstan screws, fixing it to the index bar or the telescope as the case may be. Repeat until perfect.

This adjustment may be made in a manner similar to that described on p. 115 for adjusting the line of collimation of a dumpy level. In this method the instrument is set up exactly midway between two levelling staves, the vertical circle is clamped at zero, and the bubble of the principal spirit level is brought to the centre of its run by the clip screws. The levelling staves are then read and the difference of the readings noted. The instrument is then set up beyond one of the staves, re-levelled, and the staves again read. If the second difference of the staff readings does not agree with the first, the direction of the telescope is altered by turning the clip screws until the difference of the staff readings is equal to that obtained in the first position of the instrument. The telescope and index line will now be horizontal, and it remains to bring the bubble back to the middle of its run by adjusting the capstan screws fixing the level to index bar or the telescope. The operations should be repeated until the instrument satisfies the test.

(7) *To place the Horizontal Axis at Right Angles to the Vertical Axis.*—If one of the trunnion bearings is not adjustable, this adjustment must be left to the maker of the instrument; otherwise the adjustment is made as follows :—Bisect the image of some well-defined distant object, which should be at a considerable altitude. Leaving the vertical axis clamped, rotate the telescope and vernier plate exactly through 180°. Transit the telescope, and if the image of the object cannot be again bisected (by turning the telescope on its horizontal axis), take up *half* the error by the tangent screw to the horizontal circle, and the remaining *half* by the screws fitted to the adjustable bearing. Repeat till perfect.

This adjustment may be made by means of the striding level, an example of which is shown at O, Fig. 129.

The Striding Level consists of two vertical arms of brass joined at their upper extremities by a delicate spirit level. The lower ends of the arms are provided with V-shaped bearing notches, which rest on the ends of the trunnions of the horizontal axis. The arms are long enough to permit the spirit level to pass over the vertical circle of the instrument.

When adjusting the instrument, the striding level is placed in position with its bearing surfaces resting on the ends of the axis trunnions, and the bubble of its spirit level is brought to the middle of its run by the foot screws. The ends in contact with the bearing trunnions are then reversed, and if the bubble be not central after reversal, half its deviation is taken up by the foot screws, and the other half by the screws adjusting the position of the movable bearing block. Repeat till perfect.

The Adjustments of the " Y " or " Plain " Theodolite.

(1) *The adjustment for parallax* is made as in the transit instrument.

(2) *To set the Spirit Levels at Right Angles to the Vertical Axis.*— Place the telescope over a pair of diagonally opposite foot screws (or parallel to a pair of screws in a threw-screw instrument) and bring the bubble of the principal spirit level to the middle of its run by the tangent screw to the vertical circle. Clamp the vertical axis and turn the vernier plate exactly through 180°. The telescope will now be over the same pair of foot screws, and if the bubble is not central, make it so by taking up *half* its deviation from the centre by the foot screws over which the telescope stands, and the *remaining half* by the tangent screw to the vertical circle. Now, turn the telescope through 90°, and bring the bubble to the middle of its run by the second pair of foot screws (or the third screw). The vertical axis will now be nearly vertical, and the operations must be repeated until the adjustment is perfect.

The levels on the vernier plate are then to be adjusted entirely by the screws fixing them in position.

(3) *To make the Vernier to the Vertical Circle read Zero when the Bubble of the Principal Spirit Level is in the Centre of its Run.*—After completing adjustment (2), observe the reading at the vernier to the vertical circle ; if the reading be not zero, make it so by moving the vernier under the screws fixing it in position.

Instead of moving the vernier, the index error may be noted and its amount applied—as an index correction—to all vertical angles read on the instrument.

(4) *To place the Point of Intersection of the Cross Wires in the Axis of the Telescope*—i.e., *to adjust the Instrument for Collima-tion.*—Set the telescope with one pair of diaphragm screws vertical, and bisect the image of some well-defined distant point. Observe that all the clamps on the instrument are tight, then

slowly turn the telescope half-round in its Y's, and if the image moves vertically relative to the cross wires, correct *half* the apparent displacement by the vertical pair of diaphragm screws, and the *other half* by the tangent screw to the vertical circle. Now, rotate the telescope (in its Y's) through 90°, and repeat the process with the other pair of diaphragm screws. When perfectly adjusted, the image of the distant point will remain at the point of intersection of the cross wires during a complete rotation of the telescope.

This adjustment places the line of collimation in the axis of the bearing collars, the adjustment of this line in coincidence with the optic axis of the telescope must be left to the maker of the instrument.

(5) *To set the Principal Spirit Level parallel to the Line of Collimation.*—Bring the bubble on the principal spirit level to the middle of its run by the foot screws, and observe that all the clamps are tight. Take the telescope out of its Y's, and reverse it end for end ; replace and pin down the bearing clips so that there may be the same bearing pressure at the two bearings. If, after reversal, the bubble be not central, take up *half* its deviation from the centre by the *vertical* adjusting screws to the spirit level, and the remaining half by the tangent screw to the vertical circle. Repeat until perfect.

When the adjustment has been made, rotate the telescope *slightly* in its Y's, and if the bubble moves, bring it back to the centre by the *horizontal* adjusting screws, at the other end of the spirit level.

The Box Sextant.

The box sextant (Fig. 136) is a handy and compact instrument for reading and setting-out angles in the field. Angles may be read by this instrument to single minutes with a fair degree of accuracy, and since a tripod is not required, the instrument is very useful in town work where angles have often to be read in streets carrying a heavy traffic.

Figs. 137 and 138 show the lower and upper faces of the instrument respectively. The letters used in the following description refer to these figures.

The case which surrounds the working parts of the instrument is in the form of a cylindrical box about 3″ diameter and 1″ deep. The curved surface of the case is perforated by several openings—viz., (1) a round hole about $\frac{1}{2}$″ diameter ; (2) a small

square window diametrically opposite the round hole; (3) a large window placed on the right of the line joining the centre of the hole and the small window; and (4) a small round hole,
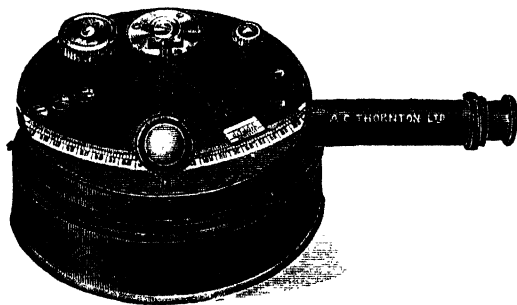
Fig. 136.

through which the key may be passed in order to turn an adjusting screw to the horizon glass.

For long sights, a small telescope T may be fixed to the top of the case by means of a screw, so that the axis of the telescope coincides with the centre of the round hole; but, for general use, the telescope is replaced by a small sight-hole cut
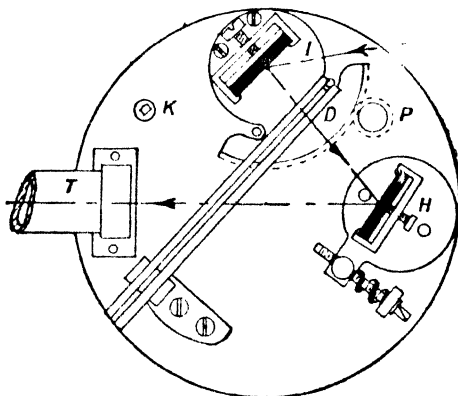
Fig. 137.

in a segmental slide, which may be brought into position as required.

Fixed to the underside of the upper flat end of the case is a

half-mirror—or horizon glass—H, which is placed with its plane
perpendicular to the end of the case, and inclined at an angle
of about 120° to the line joining the sight-hole and the small
window. The upper half only of the horizon glass is silvered,
the remainder being plain glass. A second mirror, I, called the
index glass, is fixed to a circular rack, which is capable of rotation
about an axis perpendicular to the flat end of the case. The
plane of the index glass is perpendicular to the plane of the
rack, and contains its axis of rotation. The rotation of the rack,
and consequently that of the index glass, is controlled by a small
pinion P which gears with the teeth on its rim, the pinion being
turned by a small milled finger grip M placed on the top of
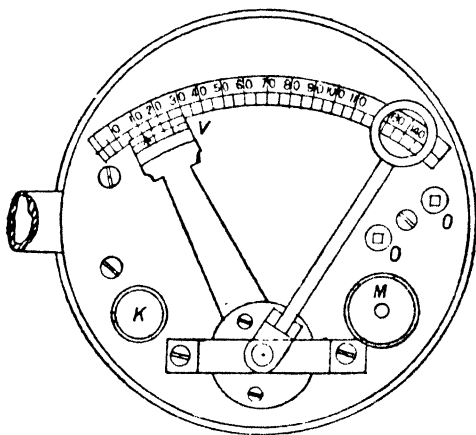
Fig 138.

the case. The axis of the rack is connected to a vernier arm
placed on the top of the instrument ; the vernier V (which is
divided on silver) moves over a scale engraved on a silver arc.
The scale is usually divided to read degrees and half-degrees,
but the degrees, as figured on the scale, are double of their real
value, since the instrument is intended to measure the angle
turned through by the ray reflected from the index glass, which
is double the angle turned through by the latter. The vernier
is divided to read to single minutes.

   The upper end of the case is perforated by two small circular
openings O, through which the adjusting key K may be passed
in order to turn the upper adjusting screws of the horizon glass.

When taking observations on the sun, two darkening glasses D may be brought into the line of sight, in order to protect the eye ; at other times, these glasses are deflected out of the way through a rectangular slot cut in the bottom of the case. The slot is closed by a slide when the instrument is not in use.

A thin cylindrical cover may be screwed into place in order to protect the working parts of the instrument when it is not in use. When about to be used, the cover is taken off and screwed on the lower part of the case, where it forms a convenient handle for holding the instrument.

The principle underlying the action of the box sextant is the same as that which underlies the action of the optical square, and is fully dealt with on pp. 35 and 36.

Tests and Adjustments.—If in correct adjustment, the instrument should satisfy the following conditions :—(1) When the vernier is set to read zero, the two mirrors should be parallel; (2) the horizon glass should be perpendicular to the plane of the instrument.

To test the instrument, set the vernier exactly to the zero of the main scale, then, holding the instrument in the hand, sight some well-defined distant object, such as a telegraph pole about half-a-mile away. If the instrument is in adjustment, the part of the object as seen by reflection will be exactly in line with the part seen by direct vision. If the two parts are not in line, apply the adjusting key to the screw at the *side* of the instrument, and turn the screw until an exact coincidence is obtained.

The second condition is tested by holding the instrument on its side so that its plane is vertical and sighting some well-defined horizontal line, such as the horizon line, or some horizontal line on a distant building. If the instrument is not in adjustment, the line will appear broken ; to correct this, the screws on the *top* of the instrument are turned (by the key) until the reflected and direct images appear as one unbroken line.

Index Error.—The adjustments must be taken in turn until the instrument satisfies both. The first adjustment may be allowed for by noting the reading at the vernier when the direct and reflected images of a distant object appear as one. This reading should be noted as the index error, and must be allowed for on every angle read with the instrument.

Parallax Error.—If the instrument is in perfect adjustment, and any object be viewed by it after reflection at both mirrors, the reading given by the vernier is the total deviation of the

rays of light passing to the eye from the object, and· is equal
to the angle R P R′ (Fig. 139). If E is the place of the eye,
the angle R P R′ is the angle between the lines joining the centre
of the index glass (I) and the observed object, and that joining
the reflected image of the object in the horizon glass (H) and the
eye, the two directions meeting at P. In taking an observation,
the angle required is the angle R E R′, and this angle is greater
than that given by the vernier by the small angle E R P, which
is the angle subtended at the object by the line joining the eye
to the centre of the index glass. The angle E R P is called the
Parallax of the instrument. Clearly, the parallax varies with
the distances I E and E R, and vanishes when the centre of the
index glass is in a direct line with the object and the eye. The
distance I E is constant for the same instrument, and varies
from about 1″ to 1½″ in different instruments. If we take the
greater value, the parallax
angle at 100 feet will be
about 4 minutes, while at a
quarter of a mile its value
will be nearly 2 seconds.
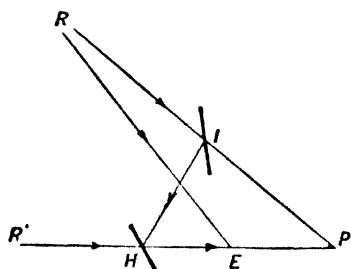
Fig. 139.

The existence of parallax
must be kept in mind when
using this instrument. If in
observing the angle between
two signals one be much
nearer than the other, the
more distant signal should be
seen by reflection ; if both
signals are relatively near, more distant objects in line with the
signals should be sought, and their angular deviations observed.

Angles greater than 120° cannot be observed with the box
sextant, as the angle of incidence becomes so great that the rays
of light become lost by refraction, in the thickness of the glass.

The Sextant.—This instrument (Fig. 140) is based on the same
principle as the box sextant, and the two instruments are very
similar in construction. The sextant is, however, a much larger
and more powerful instrument than the box sextant, the radius
of the divided arc varies from about 6 to 8 inches, and the
divisions are subdivided by the vernier read to 15 seconds. As
in the box sextant, the index glass is connected to the vernier
arm and rotates with it, the horizon glass is fixed, and both
glasses are provided with darkening glasses for reducing the
intensity of the illumination when sighting the sun.

The sextant is rarely used by the land surveyor for terrestrial observations, its use being chiefly confined to the determination of the altitudes of heavenly bodies for time or latitude, for which purpose it is used with the artificial horizon (Figs. 143 and 144). It is the chief instrument used by marine surveyors for measuring angles from stations afloat, and in other positions where the theodolite cannot be used.

The tests and adjustments of this instrument are the same as in the box sextant, and are made as in that instrument.
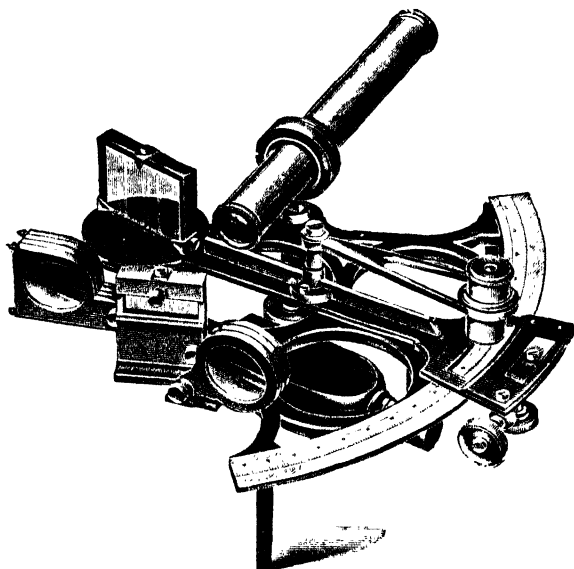
Fig. 140

The Semi-Circumferenter.—This instrument (Fig. 141) is intended for measuring and setting-out angles where great accuracy is not required. It is useful in chain surveying, where the measurement of a few angles would obviate chaining troublesome tie or check lines ; also for setting-out roads and building foundations in the development of estates.

The instrument consists of a divided semicircle connected by a ball and socket joint to a hollow vertical stem, which fits on a projecting wooden peg on the tripod head. The two pairs of sights are of the slit and window pattern arranged for right-

and left-hand reading; one pair (the " fixed sights ") is attached to the semicircle with their sighting line in the plane containing the zeros of the divided scale. The second pair (the " movable sights ") is attached to the vernier, which turns on an axis containing the centre of the divided scale, the zeros of the verniers and the sighting line being in the same plane. The instrument is provided with a compass having its North and South line parallel to the zero line on the scale. A spirit level (not shown on the figure) is usually attached to the vernier bar, and the instrument is levelled by swivelling the divided semicircle on the ball and socket joint. The plane of the semicircle may be
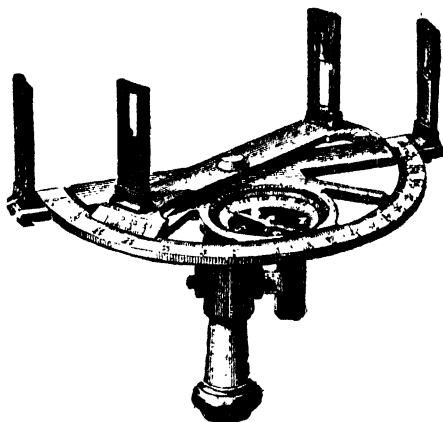
Fig. 141.

placed in a vertical position for the purpose of determining angles of elevation and depression.

Usually the verniers are divided to read to single minutes, but this degree of accuracy can only exist in the happy optimism of the maker, as it is impossible of attainment in any instrument fitted with sights of the slit and window type. The adjustments of this instrument must be left to the maker.

The Prismatic Compass.—This instrument has been fully dealt with (*vide* p. 44).

EXAMPLES.

1. On a certain instrument, the primary scale is divided to read degrees. and the vernier has 20 divisions. What is the least

reading given by the vernier, and what length will it subtend on the primary scale ? (*Ans.* 3 minutes ; 19°.)

2. On a certain theodolite, the main scale is read by a micrometer microscope, in which the reading webs are moved by a screw actuated by a milled drum head. The main scale is divided to degrees and sixths of a degree, the drum has a scale of sixty equal divisions, and the pitch of the screw is equal to the least division on the main scale. What is the value of each division on the divided drum ? If the reading webs cut the primary scale between the second and third divisions beyond 64°, and after moving the webs to bisect the second division (after 64°) on the primary scale the drum reads 55·5, what is the reading given by the instrument ?

(*Ans.* 10 seconds ; 64° 29′ 15″.)

3. Sketch and describe the construction of the axis clamp and tangent screw of a 5″ theodolite.

4. How would you test a theodolite to see if the instrument is suitable for setting out a long straight line ?

5. The horizontal axis of a theodolite is believed to be slightly bent. How would you test the instrument for the purpose of detecting this defect ?

6. It is known that all the spirit levels on a theodolite are out of adjustment. Describe how you would set the vertical axis of the instrument truly vertical.

7. Some important vertical angles have to be determined with a theodolite. How would you test the instrument to see if it is in a suitable condition for this purpose ?

8. Why is it that a box sextant will not measure angles on short lines so accurately as those on long lines ?

9. How would you proceed to measure an angle of about 160° with a box sextant ? If in reading an angle with a box sextant the rod seen by reflection is 200 feet from the observer, and the distance between the sight-hole and the centre of the index glass is 1″, what instrumental error would occur in the observed angle ? (*Ans.* 1′ 26″.)

10. How would you test a theodolite for collimation error ?

11. Describe the striding level, and state the purpose for which it is used.

12. It is intended to set-out a straight line over level ground by projecting points forward with a theodolite, but it is found that the points lie on a curve. What would you consider the source of error ? How would you test and correct the instrument ? (*B.Sc., London*, 1908.)

CHAPTER X.

USE OF INSTRUMENTS.

The Theodolite.

To Centre the Instrument over a Station.—For instruments pro·
vided with a movable substage, proceed as described on p. 231.
In other cases, the position of the tripod must be adjusted by
trial until the point of the plumb-bob is exactly over the station
mark.

If the ground at the station is of a spongy nature, a firm
foundation for the feet of the tripod may be obtained by placing
them on the ends of stout stakes, which have been firmly driven
into the ground in the required positions. The tops of the stakes
are sawn off, 2 or 3 inches above the ground.

Theodolite station marks are of various kinds, the form given
to the mark depending on (1) the importance of the station
and (2) whether the station is a temporary or permanent one.
Hence, we find station marks vary from solid constructions in
stone, brick, or concrete, carrying the mark in the form of a
fine cross, cut in a slab of metal, to a simple peg, driven into the
ground. Where the instrument has to be set over the hole left
by the removal of a ranging rod, a false picket is very useful.
The picket consists of a conical piece of wood about 8 inches
long and 1¼ inches diameter, the centre of the picket being marked
by the point of intersection of two saw cuts at right angles to
each other, made in its base. The picket is placed in the hole
made by the foot of the rod, and the instrument is centred over
it. When the station is of a semi-permanent character, its
position may be marked by a centre punch in the head of a
brass nail driven into a stout wooden peg, or by a corresponding
mark in the top of an iron spike driven between the sets of a
road.

To Determine the Angle between Two Intersecting Lines.—
Let the lines intersect at A and be marked out by signals at
B and C. Set up the instrument over A, clamp the divided circle

at zero, and sight the left-hand signal (B), obtaining an accurate bisection of the signal by the tangent screw to the vertical axis. Leaving the vertical axis clamped, unclamp the vernier limb, turn the telescope clockwise, and sight the right-hand signal (C). When the image of C is nearly bisected, clamp the vernier limb, and obtain an accurate bisection by the tangent screw to the horizontal circle. The mean of the angles given by the verniers (after deducting 180° from the reading at the right-hand vernier), is the required angle B A C.

Unfortunately, angles read on a theodolite are subject to many sources of error, and an angle obtained by this mode of procedure will rarely be quite accurate.

Sources of Error in Observing.—The chief sources of error in theodolite observations are as follows :—(1) Inaccurate centering of the instrument over the station ; (2) error due to dislevelment ; (3) imperfect adjustment of the instrument ; (4) structural defects ; (5) personal errors ; (6) error due to displacement of signal ; (7) bad atmospheric conditions.

1. The error due to inaccurate centering of the instrument at the station of observation may with care in setting-up the instrument be reduced to a negligible quantity.

2. The error due to dislevelment may also be reduced to a negligible quantity, by care in levelling the instrument, using the principal level rather than the small levels on the vernier limb. As a result of this error angles are read in a plane slightly inclined to the horizon, and not in a horizontal plane.

3. **Errors due to Imperfect Adjustment of the Instrument.**— The adjustments of a theodolite are so delicate, and depend on so many factors that perfect adjustment in a delicate instrument is practically unattainable. A little consideration will show that (*a*) if the line of collimation is not at right angles to the horizontal axis, it will trace out a conical surface as the telescope revolves, on the horizontal axis ; hence, the vertical plane containing a distant signal and parallel to the vertical plane containing the horizontal axis of the instrument, would intersect the locus of the line of collimation in a hyperbolic curve, and not in a vertical line ; (*b*) if the horizontal axis is not at right angles to the vertical axis, the line of collimation will sweep out a plane slightly inclined to the vertical, and the error caused thereby will vary with the altitude of the distant signal ; (*c*) if the line of collimation is not parallel to the principal spirit level, the zero line of the verniers to the vertical circle is not a true line of reference, and consequently vertical angles read on

the instrument will be too great or too small, by the angle between the line of collimation and the zero line.

"**Face Left**" and "**Face Right**" **Observations.**—The errors due to imperfect adjustment of the instrument may be entirely eliminated by face left and face right reiterations of the required angle. An observation of an angle (whether horizontal or vertical) made with the " face " of the vertical circle on the left of the observer is called a face left observation; similarly, a face right observation is made when the " face " of the vertical circle is on the observer's right. In this mode of reiteration, the required angle is read as described on p. 251, with the vertical circle on the left, after which the telescope is transited (or in a " Y " theodolite reversed end for end in its Y's), the vernier limb is rotated through 180°, thus placing the vertical circle on the right of the observer, and the angle is again read. The mean of the angles obtained in this way is freed from errors of adjustment, since it is clear that by reversing the positions of the trunnions, whatever bias the instrument had towards the left, on reversal it has an equal bias to the right, and consequently, on taking the mean value, the errors cancel each other.

4. The chief structural defects are as follows :—(a) Eccentricity of the inner and outer vertical axes, thereby causing the verniers to read at greater and less distances from the axis of rotation ; (b) irregularity in the length of the graduations on the divided circle ; usually the irregularity, when present, consists of a gradual increase in the size of the divisions, followed by a gradual decrease ; if the faulty divisions are irregularly spaced on the circle, the instrument should be rejected ; (c) eccentricity of the vertical circle, this has a similar effect to the eccentricity of the vertical axis, but is more important on ordinary instruments as the verniers ride in plane contact with the vertical circle, whereas the verniers are in end contact with the divisions on the horizontal circle ; (d) lack of rigidity in the instrument may be caused by slackness of joints due to wear, or to structural weakness ; in the latter case, the instrument should be returned to the maker. This defect shows itself by the irregular settlement of the instrument, causing dislevelment.

Method of Repetition.—The errors due to the defects (a) and (b) may be reduced, but are not entirely eliminated, by repeating the measurement of an angle on different parts of the divided circle, the mean of the readings being taken as the correct value. The method of repetition is carried out as follows .—Clamp the vernier limb at zero and bisect the left-hand signal (B), the

vertical axis being clamped, unclamp the vernier limb and bisect the right-hand signal (C); leaving the vernier limb clamped, unclamp the vertical axis, turn the telescope anti-clockwise, and again bisect the signal B; the vertical axis being clamped, unclamp the vernier limb, turn the telescope clockwise, and again bisect the signal C. The process is repeated as often as desired with face left, and the whole series of observations is then repeated with face right. As a further check, face left and face right repetitions are made from a new point of departure, say 45°.

It is obvious that by this method of repeating observations the divided circle is moved backwards at each repetition by an amount equal to the required angle; hence, the final reading—after allowing for whole revolutions of the divided circle—divided by the number of repetitions, is the angle required.

5. **Personal Errors** are largely due to defective eyesight, or to want of care in focussing the instrument. Most observers have a slight personal bias in bisection, causing the centre of a signal to appear a little to the left or right of its true position. With the same observer, under the same conditions, this error is approximately constant, and consequently it cannot be elimi-nated by repetition. The error may, however, be reduced in repetition by bringing the cross wires on the object from (say) the left and the right alternately in consecutive observations. Personal bias of this description is usually quite unconscious, and is often present with apparently normal eyesight. If the vision of the observer is defective, he must use properly made spectacles.

Defective focussing may be caused by an indifferent object-glass, as well as want of care in manipulation. If the telescope is correctly focussed, the image of an object should not flicker as the eye is moved into different positions, but remain steadily bisected by the cross wires. A good object-glass should present a well-defined image of the observed object, and a very slight movement of the focussing pinion should make the image blurred and indistinct. If the object-glass is defective, the image of the observed object will remain fairly distinct through a considerable movement of the focussing pinion. This is due to different parts of the glass having different focal lengths. A faulty object-glass is a nuisance, and should be returned to the maker.

6. **The Displacement of Signals** is probably the most fruitful source of error in angular measurements. In undulating country, it will occasionally happen that the top of a signal—may be 10 or more feet high—only is visible at the observer's station.

The top of the signal may be 3 or 4 inches from its true position, and the error may easily pass unnoticed at the time. The error arising from this cause will be greatest when the signal is viewed in a direction at right angles to the plane of displacement, and cannot be eliminated by the process of repetition. At a distance of 1,000 feet, a displacement of 4 inches will produce a constant error of 1′ 8·7″, at 3,000 feet an error of 22·9″ ; the error varying inversely as the distance between the signal and the observer's station.

When bisecting a signal, the point of bisection should be as near its foot as local conditions will permit ; if it is necessary to bisect the top of the signal, its position over the station should be first tested with a plumb line, or the theodolite.

7. Observing under bad atmospheric conditions is a waste of both time and temper ; accurate work cannot be done in driving rain or in fog ; other work can easily be found to occupy the observer's time during the enforced delay.

In hot climates the atmosphere is often very unsteady at certain times of the day. This is due to the currents of rarefied air rising from the hot ground, and objects seen through these uprising currents appear to dance about and are distorted in outline. It is useless to take observations under these conditions ; nothing can be done until the air is cooler and has again become steady.

To Read a Round of Angles at a Station.—In determining the angles between pairs of lines radiating from a station, we may proceed to determine each angle separately, checking the round by the sum of the angles being equal to 360°, or we may measure the angular deviations of the lines from some referring object, after the manner of taking bearings, the required angles being obtained by difference. The former method is more laborious than the latter, as the instrument must be re-set for each angle ; also the extra manipulation required becomes an additional source of error.

To read a round of angles by the second method, the verniers are clamped approximately at zero, the referring object (any selected signal or mark) is bisected, and the verniers are then read and their readings recorded. The vertical axis being tightly clamped, the signals are bisected successively ; at each bisection the verniers are read and the readings recorded ; finally, the referring object is again bisected. This last reading should agree with the first ; if it does not agree, then the vertical axis has slipped, or the lower part of the instrument has moved under the

HORIZONTAL ANGLES READ AT STATION Q.

REFERRING OBJECT............CHURCH SPIRE BETWEEN V AND R.

Date..............   Observer...............

Signal at	First Round—Face Left.		Second Round—Face Right.		Mean Reading.	Included Angle
	Vernier A.	Vernier B.	Vernier A.	Vernier B.		
P	00° 00' 00"	0' 5"	240° 0' 0"	0' 10"	0° 0' 9"	
R	46° 50' 30"	50' 35"	286° 50' 40"	50' 30"	46° 50' 34"	76° 9' 41"
S	123° 00' 15"	0' 20"	3° 00' 10"	00' 15"	123° 0' 15"	92° 28' 26"
T	215° 28' 35"	28' 40"	95° 28' 40"	28' 50"	215° 28' 41"	79° 2' 13"
U	294° 30' 45"	30' 55"	174° 30' 50"	31' 5"	294° 30' 54"	52° 9' 18"
V	346° 40' 15"	40' 10"	226° 40' 10"	40' 15"	346° 40' 12" (V.R.)	60° 10' 22"
P	00° 00' 10"	00' 15"	240° 00' 15"	00' 20"	..	

torsion induced by the axis friction, or the referring object has moved. If the closing error does not exceed one minute, both readings should be recorded and their mean accepted as correct. If the error is more than one minute the round should be redetermined.

On completing the first round, the face of the instrument should be changed, and the round again observed, starting from a new point of departure (say 60°) on the horizontal circle. Usually two rounds obtained in this way will be sufficient; but, if not, then four or six rounds may be taken, in pairs.

If the instrument is provided with three verniers placed 120° apart, it is not necessary to move the lower limb to obtain a new point of departure on the second round, as the operation of changing the face of the instrument places the verniers automatically on a different part of the circle.

If the first round has been read in a clockwise direction, it is advisable to take the second anti-clockwise, as this tends to reduce personal bias and torsional effects.

The foregoing is a convenient form of table for entering the field notes ; the entries serve to illustrate the method of booking

To Determine an Angle of Elevation or Depression.—Having set the instrument over the observing station, clamp the verniers to the vertical circle at zero, and bring the bubble of the principal spirit level to the middle of its run by means of the clip screws. Clamp the vertical axis, unclamp the vertical circle, and bisect the elevated or depressed signal, using the tangent screws to the vertical and horizontal circles, to obtain an exact bisection. Record the readings of both verniers to the vertical circle, change the face of the instrument, and repeat the observation. The mean of the four vernier readings, thus obtained, is free from instrumental errors, except those due to dislevelment.

If the required angle is an important one, the whole process should be repeated, and the mean value of the angle used in subsequent calculations.

Altitude of a Moving Body.—When determining the altitude of a moving body, such as a star or the sun, it may happen that only one observation can be made. In such a case, circle left and circle right observations for altitude must be made to some convenient referring object, and from the mean value of its altitude the index error is obtained and applied to the observed altitude.

Correcting Altitudes for Instrument.—The following example illustrates this method of correcting an altitude for the instrument :—

Example.—A circle left altitude of a certain star was found to be 75° 25′ 32″. Later, circle left and circle right observations of a church spire were obtained, the observed angles being 6° 44′ 55″ and 6° 45′ 15″ respectively. What was the star's altitude, corrected for the instrument ?

Circle left altitude of church spire,	.	6° 44′ 55″
Circle right altitude of church spire,	.	6° 45′ 15″
Mean altitude, . . .	.	6° 45′ 5′
Less circle left altitude, .	. .	6° 44′ 55′
Index correction, circle left,	.	. + 0° 0′ 10″
Observed altitude,	.	. 75° 25′ 32″
Corrected altitude, .	. .	. 75° 25′ 42″

Correction for Level Error.—In the preceding method it is assumed that the bubble of the level on the vernier arms remains central when taking face left and face right repetitions of the required vertical angle. This, however, is rarely the case, hence a correction for bubble error is required on all vertical angles. Before we can allow for this it is necessary to determine the value, in arc, of a division of the level scale. To do this, set up the instrument, and by means of the clip screws bring one end (say the object end) of the bubble to the end division of the bubble scale ; now sight some well-defined distant object, read the vertical circle and the bubble scale. Next, by turning the clip screws, shift the same end of the bubble over as many divisions on the scale as possible, taking care that the bubble is really floating and within the scale divisions. Again sight the same referring object, note the readings of the vertical circle, and the same end of the bubble in its new position. Evidently the value of each division is obtained by dividing the difference of the vertical angles by the difference of the scale readings. Thus :—

	Vertical Angle.	Scale Reading.
First observation, .	. 8° 15′ 24″	18
Second observation,	. 8° 17′ 44″	4
Difference, . .	. 0° 2′ 20″	14

Value of each division $= \dfrac{140''}{14} = 10''$.

The whole operation should be repeated several times, in order to obtain a good average value. As the length of the bubble may alter owing to change of temperature, it is advisable to note its length in scale divisions, and if a change of length is observed, the readings at both ends of the bubble must be recorded. This must be done for each repetition of each vertical angle read with the instrument. Calling the sum of the readings of the object end of the bubble A, the sum of the readings at the eye end B, the correction to be applied to the mean value of an observed vertical angle is equal to

$$+ \frac{A - B}{\text{Number of bubble readings}} \times \text{value of 1 division of scale.}$$

As there will be two readings of the bubble for each repetition of the observed angle, the number of bubble readings will be double the number of repetitions.

If A is greater than B, the result is to be added to the mean value of the observed angle, and *vice versâ* when B is greater than A.

The method of obtaining and applying the correction is as follows :—

	Value of 1 division of Level = 15″.				
Face.	Vertical Angles.		Mean of C and D.	Level Reading.	
	Vernier C.	Vernier D.		Eye end (B).	Object end (A).
R	311° 58′ 30″	58′ 24″	48° 1′ 33″	7	6
L	228° 0′ 32″	0′ 31″	48° 0′ 31·5″	8	4
L	228° 1′ 21″	1′ 8″	48° 1′ 14·5″	8	4
R	311° 57′ 20″	56′ 58″	48° 2′ 51″	5	7

Sum 4) (4) 48° 6′ 10″   28 | 21

Mean,   48° 1′ 32·5″   21

Diff.,   7

$$\frac{\text{Level}}{\text{error}} = \frac{7 \times 15}{8} = 13\cdot1''$$

−0° 0′ 13·1″

Corrected mean = 48° 1′ 19·4″

The method of allowing for the spherical form of the earth and for refraction, in vertical measurements, is dealt with on p. 300, *et seq.*

To Determine the Magnetic Bearing of a Line.—The instrument is first centred over a station on the line, and the vernier limb is clamped at zero. The magnetic needle having been released, the whole of the upper part of the instrument is rotated until the needle comes to rest with its extremities opposite the meridian marks in the compass box, the final adjustment to position being made with the tangent screw to the vertical axis. The telescope will now be in the magnetic meridian with the object-glass towards the North, and it remains to unclamp the vernier limb, direct the telescope to and bisect the signal on the line. The mean of the angles given by the verniers is the required whole circle bearing.

Usually circle left and circle right readings of a magnetic bearing are not obtained, as the error in setting the needle to position will, in most cases, be greater than the instrumental errors ; but, if a round of bearings is to be determined, and the angles between the lines obtained by difference, then observations should be made with both faces of the instrument. Usually a sufficient check is obtained by taking fore- and back-bearings, the difference of which should be (within permissible limits) 180°.

When taking bearings with a theodolite, the telescope should always be rotated clockwise from the North, and the bearings recorded as whole circle bearings. The reduced bearings are easily obtained, if we remember that—

When the bearing is between   0° and   90° the line goes N.E.

,,	,,	90° and 180°	,,	S.E.
,,	,,	180° and 270°	,,	S.W.
,,	,,	270° and 360°	,,	N.W.

To Use the Instrument as a Level.—Although it is not advisable to use a theodolite for ordinary levelling work, it sometimes happens that a short section must be run, and a theodolite is the only instrument available. To use the instrument for this purpose, the index error must be determined and allowed for in setting the verniers to the vertical circle ; after setting the index, the clamp and tangent screw to the vertical circle are left undisturbed. The line of collimation must then be placed at right angles to the vertical axis. To do this, place the telescope over a diagonal pair of foot screws (or parallel to a pair of foot

screws in a three-screw instrument), the clip screws being tight, bring the bubble of the telescope spirit level to the middle of its run by this pair of foot screws; next, rotate the telescope through 180°; if after rotation the bubble be not central, take up half the displacement by the clip screws, and the other half by the same pair of foot screws. Repeat until the bubble remains central on reversal. The instrument may now be used as an ordinary level, the setting up being done with the foot screws alone.

Levelling on Steep Ground.—The theodolite is very useful for levelling work on very steep ground. For this purpose the instrument must be set up in the line of section and the line of collimation placed roughly parallel to the slope. The distances are chained on the slope, beginning anew at each back-sight station. Otherwise, the work proceeds as in ordinary levelling.

Plotting the Section.—This may be done in two ways—(1)
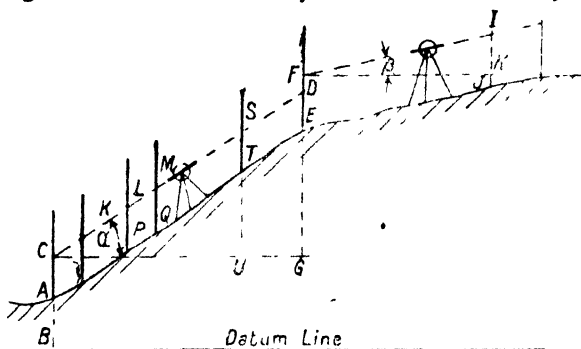
Fig. 142.

graphically from the field notes, or (2) from calculated distances and reduced levels. Referring to Fig. 142, let the section commence at the point A and A C be the first back-sight, clearly the reduced level of C is B A + A C. Plot the point C in position, and through C draw C D at the given inclination (a). Scale off chained distances along C D, thus obtaining the points K, L, M, etc. At these points drop vertical lines from which cut off K O, L P, M Q, etc.—the respective staff readings—thus giving the points O, P, Q on the section. When the position of the fore-sight station E has been obtained, make E F equal to the second back-sight, at F set off F H at the given inclination (b), and proceed as before. In this method the reduced levels and (hori-

zontal) distances must be scaled from the section. In the second method, the reduced levels and distances are calculated from the observed data, and the section is plotted as in ordinary levelling. In calculating the reduced level of any point, we consider the back-sight readings as positive, the intermediates and fore-sight readings as negative. Having found the reduced level of the point C (Fig. 142), the reduced level of any point (T) on the section is equal to reduced level of $C + CS \sin \alpha - ST$ (the staff reading).

Similarly, the reduced level of J is equal to reduced level of $F + FI \sin \beta - IJ$. The reduced level of F is equal to the reduced level of E + the back-sight reading at E.

The horizontal distance of any point T from the corresponding back-sight station equals $CS \cos \alpha$. Clearly the horizontal distances are all of the same sign, and those on the second and succeeding lines of collimation must be added to the distances of the preceding lines to give the total distance.

The Sextant.

As this instrument measures angles in the plane in which it is held, care must be exercised to keep the limb of the instrument horizontal, when reading or setting out horizontal angles.

To Read the Angle between Two Lines (A B and B C).—Take the cover from the top of the instrument, and screw it into place on the lower screw thread, deflect the dark glasses out of the line of sight, and, holding the instrument in the left hand vertically over station B, sight the nearer signal directly. Next, turn the finger grip until the signal at C (as seen by reflection) is brought into coincidence with the signal at A. The vernier reading now gives the required angle A B C.

If the signals at A and C are much above or below station B, other objects in line with A and C must be sought and observed to, the selected objects and the observer's station being approximately in the same horizontal plane.

Since this instrument is not suitable for reading angles greater than 120°, when dealing with obtuse angles, the supplementary angle (obtained by producing one of the lines backwards) should be observed, and carefully checked. The required angle is then obtained by subtracting the observed angle from 180°.

To Set Out a Given Angle.—Set the vernier to read the given angle, and holding the instrument over the station at which the angle is to be set out, sight the known station, and direct

an assistant to set up a rod in the direction given by the instru-
ment. The rod must, of course, be placed so that its reflected
image is exactly in coincidence with the rod seen directly.

To obtain an Angle of Elevation or Depression.—Hold the
instrument in the hand with its plane vertical, and sight directly
a mark which is placed at the same height as the observer's
eye, now turn the finger grip until the image of the object whose
angular elevation or depression is required is brought into co-
incidence with the image of the mark. The reading at the vernier
is the angle required.

Altitude of the Sun.—In reading the altitude of the sun, it is
usual to sight the horizon line directly, and to bring the reflected
image of the sun into contact with this line. The observed angle
must then be corrected for " dip "—*i.e.*, the angle of depression
from the observer's eye to the horizon line, which may be obtained
from tables when the observer's altitude above sea level is known
—" refraction " and sun's semi-diameter.
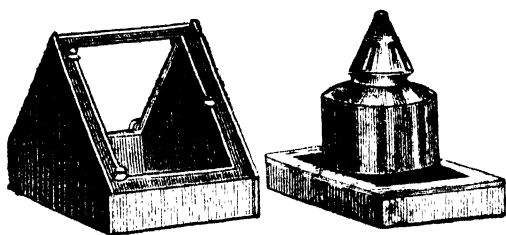
Fig. 143.

Artificial Horizon.—If the angular elevation of the sun or a
star is required at an inland station, an artificial horizon of some
kind must be used. An artificial horizon consists of a shallow
vessel containing mercury (Fig. 143), or a piece of black plate
glass suitably mounted (Fig 144), or, failing these aids, the
surface of still water in a pond may be used. The surface of the
mercury in the artificial horizon shown in Fig. 143 is protected
from atmospheric disturbance by a sloping roof of accurately
ground plate glass.

When not in use the mercury is stored in the iron bottle,
shown in the figure. In Fig. 144 the plate glass artificial horizon
is set horizontal by means of the spirit levels and levelling screws,
shown in the figure.

When using an artificial horizon, the observer places himself

in such a position that he can sight directly the reflection of
the sun or star in the surface of the instrument, the image of
the sun or star as seen by reflection in the mirrors of the sextant
is brought into coincidence with the image seen in the artificial
horizon, when the vernier reading will be double the required
altitude.

Let S S′ (Fig. 145) be the reflecting surface of the artificial
horizon, M′ A the incident ray, A E the reflected ray passing
to the instrument, and M E the direct ray from the sun or star
to the index glass. Also, let E H be a horizontal line drawn
through E. Then the angle M′ A S is equal to the angle E A S′,
by the laws of reflection. Owing to the great distance of the sun,
the ray M E is parallel to the ray M′ A; therefore, the angle
M E H is equal to the angle M′ A S, and, consequently, the angle
M E H is equal to the angle H E A; but the angle read on the
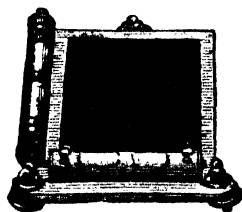
Fig. 144.
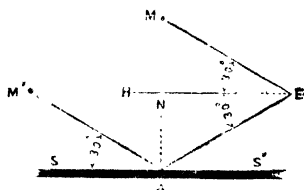
Fig. 145.

instrument is the angle M E A, which is double of the angular
altitude M E H.

**To Fix the Position of a Point by Angular Observations to
Three Known Points.**—This is an operation of frequent occur-
rence in marine surveying, and the problem is generally spoken
of as "*the three-point problem.*"

The point to be fixed may be a submerged rock, a buoy, a
point on a submerged contour, or, as in exploration work, the
position of a village relative to three or more prominent mountain
peaks. It is assumed that the positions of three points observed
to have been fixed by other operations, and are shown on the
plan.

To fix the position of a point afloat, the observer, accompanied
by his assistants, is rowed out to the point, and the boat is either
anchored or kept in position by the oars, until the necessary
observations have been made.

Suppose the point D (Fig. 147) is the point of observation, and A, B, and C are the known points on shore. The angles A D B and B D C are observed in turn with the sextant, the whole angle A D C should then be observed, as this reading checks their sum. The three angles should be read in succession as quickly as possible, in order to guard against the movement of the boat.

From the observed data the position of the point D may be plotted on the plan (1) by means of a special instrument (Fig. 146) known as a station pointer; (2) by a graphical construction; or (3) from the calculated lengths of A D, B D, and C D.

Station Pointer.—The station pointer (Fig. 146) consists of a circular protractor fitted with three radial straight edges, whereof one is fixed and the other two are movable. Each movable
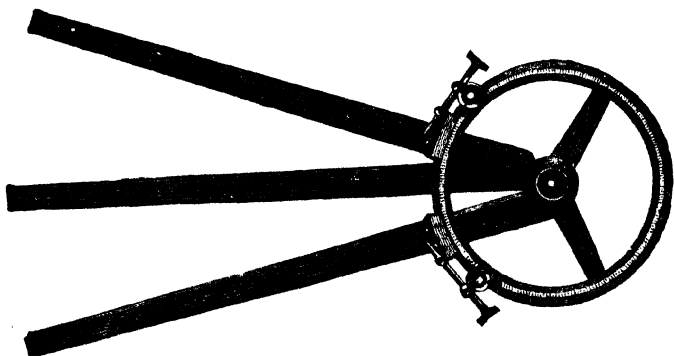
Fig. 146.

straight edge is provided with a vernier, and a clamp and tangent screw, so that it may be fixed at any given angular interval with the fiducial edge of the fixed straight edge. At the centre of the protractor is a small hole, through which the point of a needle may be passed.

To use the instrument, the verniers are set to read the respective right- and left-hand angles, the instrument is placed on the plan, and is moved about until the three fiducial edges are in contact with three fine needles, which have been passed through the plan at the points A, B, and C. The centre of the protractor now occupies the position of the point D, which may be pricked off by passing the point of the pricker through the central hole.

Graphical Method.—Referring to Figs. 147, 148, and 149, which illustrate the three cases, draw the perpendicular bisectors,

E F and G H of the lines joining A B and B C. At A or B, set off the angle 90 — α, and at B or C set off the angle 90 — β, where α and β are equal to the angles A D B and B D C respectively. Let the lines marking out these angles intersect the lines E F and G H at K and L respectively, then K is the centre
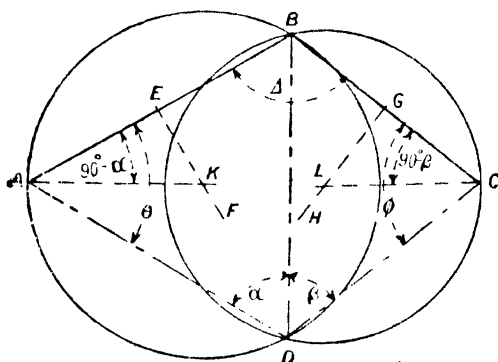
Fig. 147.
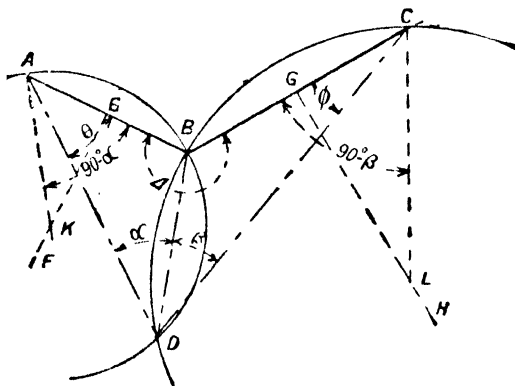
Fig. 148.

of a segment of a circle containing an angle equal to α, and passing through the points A and B. Similarly, L is the centre of the segment of a circle containing an angle β, and passing through the points B and C.

Obviously the two segments, when drawn, must intersect at

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the point B, and as they can only intersect in two points, their second point of intersection must be the point sought D.

On inspecting the figures, it will be seen that in all three cases a third circle may be drawn through A, C, and D, the angle to be set off at A or C with the chord A C in cases I. and II. being $90 - (\alpha + \beta)$, and in case III., $90 - (360 - \overline{\alpha + \beta})$. In Fig. 148 the position of the point D is ill defined, as the intersection of the circles is acute. A better intersection will be obtained by using the segments standing on the chords A B and A C, or on B C and A C. In each case the pair of segments which will cut most nearly at right angles should be used.

It is as well to note that the problem is indeterminate when the four points lie on the circumference of the *same* circle. This
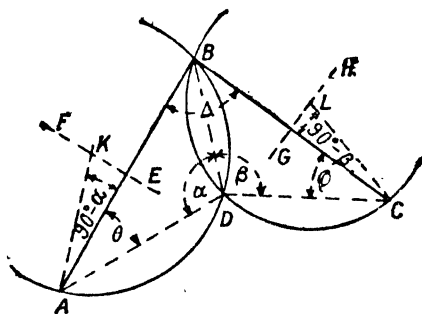
Fig. 149.

occurs when the sum of the opposite angles of the quadrilateral figure A B C D is equal to two right angles.

By Calculation.—The object of the calculation is to determine the lengths of A D, B D, and C D.

Let the angle A B C $= \Delta$,

        ,,   D A B $= \theta$,

and   ,,   B C D $= \varphi$.

Then $\theta = 360 - (\alpha + \beta + \Delta) - \varphi$

               $= Z - \varphi$,

where $Z = 360 - (\alpha + \beta + \Delta)$, a known quantity,

and $\varphi = Z - \theta$.

Also $\dfrac{A B}{\sin \alpha} = \dfrac{B D}{\sin \theta}$,

∴ $B D = \dfrac{A B \sin \theta}{\sin \alpha}$.   •   • *(a)*

Again,
$$\frac{B\,D}{\sin \varphi} = \frac{B\,C}{\sin \beta}.$$

∴
$$B\,D = \frac{B\,C \sin \varphi}{\sin \beta}. \qquad \text{.} \quad \bullet \quad \bullet \quad \bullet \quad (b)$$

But
$$B\,D = \frac{A\,B \sin \theta}{\sin \alpha},$$

∴
$$\frac{A\,B \sin \theta}{\sin \alpha} = \frac{B\,C \sin \varphi}{\sin \beta},$$

and
$$\frac{\sin \theta}{\sin \varphi} = \frac{B\,C \sin \alpha}{A\,B \sin \beta}.$$

or
$$\frac{\sin (Z - \varphi)}{\sin \varphi} = \frac{B\,C \sin \alpha}{A\,B \sin \beta}.$$

From this it follows that

$$\text{Cot } \varphi = \frac{B\,C \sin \alpha}{A\,B \sin \beta \sin Z} + \cot Z.$$

This gives the angle φ, from which the angle θ follows, since

$$\theta = Z - \varphi.$$

B D may now be calculated from both equations (a) and (b), thus checking the calculation, as the results should, of course, be identical.

Again, in the triangle A B D, the angle

$$A\,B\,D = 180 - (\alpha + \theta),$$

∴
$$\frac{A\,D}{\sin (180 - \alpha + \theta)} = \frac{A\,B}{\sin \alpha}$$

or
$$A\,D = \frac{A\,B \sin (\alpha + \theta)}{\sin \alpha}.$$

In the triangle B D C the angle

$$C\,B\,D = 180 - (\beta + \varphi),$$

∴
$$\frac{D\,C}{\sin (180 - \beta + \varphi)} = \frac{B\,C}{\sin \beta}$$

or
$$D\,C = \frac{B\,C \sin (\beta + \varphi)}{\sin \beta}.$$

The point D may be plotted from any pair of the known lengths A D, B D, and D C, the remaining length then acts as a check line.

The Semi-Circumferenter.

To Obtain the Angle between Two Lines.—Set the tripod over the point of intersection of the two lines, and bring the plane of the divided semi-circle horizontal by swivelling it about its ball and socket joint. Direct the " fixed sights " to one of the signals, then direct the movable pair of sights to the other signal. The required angle may now be read at the vernier, in contact with the scale. Before reading the angle it is advisable to make certain that both pairs of sights bisect their respective signals. To check the angle the process should be repeated in the reverse order.

To Set Out a Given Angle at a Given Point.—Set up the instrument over the given point, and set the vernier to read the given angle. Direct the " fixed sights " to the signal in the given line, the "movable sights" will now point in the required direction, which is then marked out in the usual way.

To Determine the Magnetic Bearing of a Line.—Having set up the instrument over a station in the given line, release the magnetic needle and turn the divided semi-circle round until the needle comes to rest with its ends opposite the meridian marks in the compass box. The " fixed sights " will now be in the magnetic meridian, and on sighting the forward signal on the line with the " movable sights," the angle at the vernier will give the required bearing.

As the scale on the instrument reads from 0° to 180° only, whole circle bearings cannot be obtained ; care must, therefore, be exercised to book the cardinal direction at the time the bearing is determined.

To Determine an Angle of Elevation or Depression.—Turn the plane of the semi-circle into a vertical position, set the verniers to zero, then turn the semi-circle about the ball joint until the bubble of the spirit level is in the middle of its run. Both pairs of sights are now horizontal, and it remains to direct the " movable sights" to the elevated or depressed object whose angular altitude is required. The required angle will, of course, be given by the vernier in contact with the scale.

EXAMPLES.

1. Describe how you would measure (with a theodolite) the angle between two intersecting lines.

2. Describe the setting and manipulation of the instrument in reading a round of angles at a given station.

3. What instrumental imperfections do we seek to counteract by " face left " and " face right " observations of a horizontal angle ?

4. In observing horizontal angles at a station two rounds were observed. The setting and manipulation of the instrument were as follows :—

	1st Round	2nd Round.
Zero point.	360°	45°
Face.	Left.	Right.
Wire brought on object.	Right to left.	Left to right.

Explain the objects of the different setting and manipulation.

5. The face right angular altitude of a certain star was observed to be 65° 24′ 18″. The face left and face right altitudes of the top of a flag post were 4° 55′ 29″ and 4° 55′ 37″ respectively. Correct the star's altitude for the instrument.

(*Ans.* 65° 24′ 14″.)

6. Explain clearly how you would determine the magnetic bearing of a line with a theodolite. Write down the reduced bearings of lines having the following whole circle bearings :—
45° 32′ 13″ ; 124° 35′ 46″ ; 264° 18′ 42″ ; 320° 15′ 24″.

(*Ans.* N. 45° 32′ 13″ E. ; S. 55° 24′ 14″ E.; S. 84° 18′ 42″ W. ;
N. 39° 44′ 36″ W.)

7. A theodolite has to be used for the purpose of taking levels on a section. How would you test the suitability of the instrument for this purpose ?

8. The following staff readings were taken (100 feet apart) on a section run with a theodolite ; the distances were chained on the slope and the figures in brackets are back-sights :—(8·42), 7·39, 8·64, 9·81, 10·32 ; (6·82), 4·62, 6·84, 7·91, 6·52. The inclination of the first line of collimation being + 25° 32′, and that of the second + 27° 15′, draw up a level book for the section, taking the reduced level of the first point 54·32 feet above datum.

(*Part Ans.* Total horizontal distance, 716·5 feet ;
reduced level of last point, 408·28 feet.)

9. What precautions must be taken when using a box sextant in reading an angle between two objects if (*a*) the observed

objects are fairly near to the observer, and (*b*) if the difference of level of the objects is considerable ?

10. Describe one form of artificial horizon. Explain clearly how you would proceed to determine the angular altitude of a star by aid of an artificial horizon and a sextant.

11. Describe the station pointer and explain how you would use the instrument to locate the position of a point on a plan, from given angular observations at the point to three fixed points.

12. To an observer at a station A, three posts (C, D and E) are so placed that the line joining C and D subtends an angle of 30°, and that joining D to E subtends an angle of 25° 30′. At a second station B, the corresponding lines subtend angles of 20° 30′ and 28° 15′. The three posts are in the same straight line. If C D = D E = 200 yards, find graphically and by calculation the distance from A to B. If the bearing of the line joining the posts is N. 40° 30′ E., find the bearing of A B. (*Victoria University B.Sc. Tech.*, 1915.)

(*Ans.* A B = 226·1 yards ; bearing of A B, N. 36° 10′ E.)

CHAPTER XI.

DETERMINATION OF DISTANCES, HEIGHTS, AND ANGLES.

IN survey work with the theodolite, the calculation of distances, heights, and angles forms an important part of the sequence of processes ending in the preparation of the final plan.

The ground work of all surveys of an extensive character must consist of triangles fitted together in various ways, and as these triangles are laid down from the lengths of their sides, and not from protracted angles, it is necessary to calculate the lengths of the sides of the triangles from the field data. Further, the triangles when fitted together form polygons, and as we have seen, it is impossible to measure angles without some error creeping into the work, it is necessary to examine the conditions that a perfect polygon built up of triangles must fulfil, in order to arrive at some method of dealing with the small errors in the angular measurements.

In the following problems the method of obtaining the requisite data is not given—except in special cases—as the direct measurement of distances and angles has already been fully dealt with. Where proofs of the formulæ are not given, they are either beyond the scope of this book, or may be obtained by reference to any standard text-book on plane or spherical trigonometry.

Problem I.—*Given the base and two base angles of a triangle, to find the sides.*

Let b be the given base, A and C the given angles. We are required to find the sides a and c.

Since $A + B + C = 180°$,

$$B = 180° — (A + C), \text{ which determines B.}$$

Also,
$$\frac{a}{\sin A} = \frac{b}{\sin B},$$

$$\therefore \quad a = \frac{b \sin A}{\sin B}$$

$$= \frac{b \sin A}{\sin (A + C)}.$$

Similarly,
$$c = \frac{b \sin C}{\sin (A + C)}$$

Problem II.—*Given two angles of a triangle and a side opposite to one of them* (i.e., *given* A, B, *and* a) *to find the remaining sides* b *and* c.

Here,
$$C = 180^\circ - (A + B),$$

and
$$\frac{b}{\sin B} = \frac{a}{\sin A},$$

or,
$$b = \frac{a \sin B}{\sin A}.$$

Similarly,
$$c = \frac{a \cdot \sin C}{\sin A}$$

$$= \frac{a \cdot \sin (A + B)}{\sin A}.$$

Problem III.—*Given the two sides of a triangle and an angle opposite one of them, to find the remaining side* (i.e., *given* a, b, *and* A, *to find* c, B, *and* C).

In this case we have $\sin B = \dfrac{b \cdot \sin A}{a}$, where B is an angle less than 90°.

Also,
$$C = 180^\circ - (A + B);$$

and
$$c = \frac{a \cdot \sin C}{\sin A}$$

$$= \frac{a \sin (A + B)}{\sin A}.$$

Problem IV.—*Given two sides of a triangle and the angle between them, required the remaining side and angles.* (*Given* a, b, *and* C, *to find* c, A, *and* B.)

In this case we may apply the formula

$$c = \sqrt{a^2 + b^2 - 2\,a\,b \cdot \cos C}, \text{ which gives } c.$$

This formula is, however, not adapted for logarithmic computation, and is consequently cumbersome to use. It is better to first find the angles A and B, and then to apply the sine rule to find c.

As before, $\qquad A + B = 180° - C.$

$\therefore \qquad \frac{1}{2}(A + B) = \frac{1}{2}(180° - C),$

also, $\qquad \text{Tan } \frac{1}{2}(A - B) = \dfrac{a - b}{a + b}\cot\dfrac{C}{2}.$

$\frac{1}{2}(A - B)$ being determined, and $\frac{1}{2}(A + B)$ being known, it follows that

$$A = \frac{1}{2}(A - B) + \frac{1}{2}(A + B),$$

and $\qquad B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B).$

Applying the Sine Rule, we get

$$c = \frac{a \cdot \sin C}{\sin A},$$

or, $\qquad\qquad = \dfrac{b \cdot \sin C}{\sin B}.$

Problem V.—*Given the three sides of a triangle, to find the three angles* (i.e., *given* a, b, *and* c, *to find* A, B, *and* C).

The angles may be found by one of the formulæ :—

$$\text{Sin }\frac{A}{2} = \sqrt{\frac{(s - b)(s - c)}{b\,c}}, \qquad \cos\frac{A}{2} = \sqrt{\frac{s(s - a)}{b\,c}},$$

$$\text{Tan }\frac{A}{2} = \sqrt{\frac{(s - b)(s - c)}{s(s - a)}}, \qquad \cot\frac{A}{2} = \sqrt{\frac{s(s - a)}{(s - b)(s - c)}},$$

where $s = \dfrac{a + b + c}{2}.$

If more than one angle is required, the third or fourth formula should be used, as the same logarithms will enter into the computation of all the angles.

The above problems are so well known that they do not require to be illustrated by examples.

Problem VI.—*To determine the " spherical excess."*

Since the earth is a spheroid, the lines of intersection of vertical planes containing the sides of a triangle on its surface meet in a point, the earth's centre. Consequently, the angles measured at the angular points of a triangle are the dihedral angles between the sloping faces of a pyramid, and their sum exceeds 180° by an amount which is dependent on the area of the triangle, and the assumed value of the earth's radius. The amount by which the angles of a triangle exceed 180° is called the " spherical excess."

In ordinary surveying operations, where the sides of the tri-angles employed seldom exceed 2 miles, spherical excess is ignored; but in geodetic operations, where the sides of the triangles are often greater than 30 or 40 miles, this quantity cannot be disregarded.

Approximately, the spherical excess in seconds is

$$= \frac{\text{Area of triangle in square miles}}{75 \cdot 5}.$$

The more exact formula for spherical excess is

$$E = \frac{A \times 180 \times 60^2}{\pi R^2} \text{ seconds,}$$

where A = area of the triangle (calculated as plane) in square feet,

R = mean radius of the earth in feet.

Taking the mean radius of the earth as 20,889,000 feet, then

$$\log \frac{\pi R^2}{618,000} = 9 \cdot 3254098,$$

and we have

$$\text{Log } E = \log A - 9 \cdot 3254098, \text{ seconds.}$$

If the triangle is very large, the local mean radius of curvature (R^1) must be used in computing the spherical excess, and the value of the constant log altered accordingly. The local mean radius may be calculated by aid of the formula :—

$$R^1 = \frac{\text{equatorial radius} \times (1 - Q^2)}{1 - Q^2 \sin^2 L},$$

where $Q = \dfrac{\text{equatorial radius} - \text{polar radius}}{\text{equatorial radius}}$,

and, L = mean latitude.

Colonel Clarke's computation of the dimensions of the earth give :—

Length of equatorial radius, .   20,926,202 feet.
Length of polar radius,   20,854,895 feet.

From these data $Q = \dfrac{1}{293 \cdot 466}.$

Problem VII.—*To apportion the errors of observation among the angles of a triangle.*

Having computed the spherical excess, and obtained the mean values (A, B, and C) of the observed angles, then $A + B + C$

should be equal to $180 + E$. If this equation does not hold, the small error should be divided equally among the angles if the number of observations of each is the same; but, if otherwise, the following methods are open to us :—

(*a*) The error may be apportioned among the angles inversely as the number of observations.

(*b*) It may be apportioned inversely as the square of the number of observations.

(*c*) The error may be apportioned inversely as the " weights " of the observations, as given by Gauss's rule, in the following manner :—

Let T = the total error to be apportioned,
$\qquad n$ = number of observations of angle A,
l, l_1, l_2, etc. = the number of seconds of the readings of the several observations,
$\qquad m$ = mean average of the seconds in the observations.

Then, the errors of the several observations are $(m - l)$, $(m - l_1)$, $(m - l_2)$, etc., and the " weight " of the observed

angle **A** $= \dfrac{\frac{1}{2} n^2}{(m - l)^2 + (m - l_1)^2 + (m - l_2)^2 + \text{etc.}}$

If $\qquad a$ = the relative coefficient of the angle **A**,
and $\qquad x$ = actual error in A,
then $\qquad a = \dfrac{(m - l)^2 + (m - l_1)^2 + (m - l_2)^2 + \text{etc.}}{\frac{1}{2} n^2}$.

The values of β and γ, the corresponding coefficients for the angles B and C, having been determined in the same way, if y and z are the actual errors in B and C respectively, then

$$x = T \frac{a}{a + \beta + \gamma}, \quad y = T \frac{\beta}{a + \beta + \gamma}, \quad z = T \frac{\gamma}{a + \beta + \gamma}.$$

If only one observation has been made of a single angle (B) then an arbitrary value must be assumed for β.

Problem VIII.—*To compute the sides of a large triangle.*

One side of the triangle and the three angles will usually be known in the cases occurring in survey work. The remaining sides may be computed (1) by the ordinary rules of spherical trigonometry ; (2) by Delambre's method ; or (3) by the method of Legendre.

The first method is troublesome and not more exact than the two latter. Of the three, Legendre's method is that most commonly used.

Delambre's Method.—In this method, the spherical side is first reduced to the chord joining its extremities, and the spherical angles to the plane angles between the chords. The remaining chords are then calculated by the methods of plane trigonometry, and from these chords the corresponding spherical sides are computed.

Legendre's Method.—The theorem of Legendre states that, " In any spherical triangle, the sides of which are small compared with the radius of the sphere, if each of the angles be diminished by one-third of the spherical excess, the sines of these angles will be proportional to the lengths of the opposite sides, and the triangle may, therefore, be calculated as if it were plane."

Example (1).—Solve the triangle A B C from the following data : the length of the side a is 186,540·23 feet, and the observed angles are : –

A, 55° 45′ 33·24″,	B, 72° 18′ 24·32″,	C, 51° 56′ 6·64″
30·42″	25·46″	7·72″
31·38″	23·14″	
32·19″	22·84″	
	23·31″	

	Sums, 127·23″	119·07″	14·36″
Means	55° 45′ 31·81″,	72° 18′ 23·81″,	51° 56′ 7·18″

(*a*) To compute the spherical excess.

Calculating the approximate values of b and c, thus

$b = a$ sin B cosec A
   $= 186,540·23 \times$ sin 72° 18′ 23·81″ $\times$ cosec 55° 45′ 31·81″
   $= 214,977$ feet.

$c = a$. sin C . cosec A
   $= 186,540·23 \times$ sin 51° 56′ 7·18″ $\times$ cosec 55° 45′ 31·81″
   $= 177,658$ feet.

Area $= \frac{1}{2} b . c .$ sin A
     $= \frac{1}{2} \times 214,977 \times 177,658 \times$ sin 55° 45′ 31·81″
     $= 15,786,308,600$ square feet.

Spherical excess.   Log E $=$ log 15,786,308,600 $-$ 9·3254098
                    and E $= 7·462$ seconds.

(*b*) To apportion the error among the angles.

The sum of the mean values of the angles $= 180° 0′ 2·8″$,
       the spherical excess $+ 180° = 180° 0′ 7·46″$,
       hence the total error T $= + 4·66″$.

(1) Correction by reciprocals of the number of observations.

$\frac{1}{4} = 0.25$ Correction of A $= \dfrac{.25}{.95} \times 4.66 = 1.23''$

$\frac{1}{5} = 0.20$ Correction of B $= \dfrac{.2}{.95} \times 4.66 = 0.98''$

$\frac{1}{2} = 0.50$ Correction of C $= \dfrac{.5}{.95} \times 4.66 = 2.45''$

Sum $\overline{0.95}$ $T = \overline{4.66''}$

(2) Correction by **squares of** reciprocals of the number of observations.

$\dfrac{1}{4^2} = .0625$ Correction of **A** $= \dfrac{.0625}{.3525} \times 4.66 = 0.83''$

$\dfrac{1}{5^2} = .0400$ Correction of B $= \dfrac{.04}{.3525} \times 4.66 = 0.53''$

$\dfrac{1}{2^2} = .2500$ Correction of C $= \dfrac{.25}{.3525} \times 4.66 = 3.30''$

Sum $.3525$ $T = \overline{4.66''}$

(3) Correction by Gauss's rule.

$$\alpha = \left\{ \frac{\begin{array}{c}(31.81 - 33.24)^2 + (31.81 - 30.42)^2 + (31.81 - 31.38)^2 \\ + (31.81 - 32.19)^2 \end{array}}{\frac{1}{2} 4^2} \right\}$$

$$= \frac{1.43^2 + 1.39^2 + .43^2 + .38^2}{8}$$

$$= .537.$$

Similarly, $\beta = \dfrac{4.62}{12.5}$

$= .369,$

and $\gamma = .292.$

$\alpha + \beta + \gamma = .537 + .369 + .292$

$= 1.198.$

Correction of A $= \dfrac{.537}{1.198} \times 4.66 = 2.09''$

Correction of B $= \dfrac{.369}{1.198} \times 4.66 = 1.43''$

Correction of C $= \dfrac{.292}{1.198} \times 4.66 = 1.14''$

$T = \overline{4.66''}$

By comparison, we observe that the results given by the three methods are divergent ; but it must be noted that the first two methods allow nothing for the individual errors in the series.

Accepting the latter values as correct,

$$\begin{aligned}
\text{The corrected value of A} &= 55^\circ\ 45'\ 33{\cdot}90'' \\
\text{,,} \qquad\qquad B &= 72^\circ\ 18'\ 25{\cdot}24'' \\
\text{,,} \qquad\qquad C &= 51^\circ\ 56'\ \ 8{\cdot}32'' \\[4pt]
\hline
\text{Sum} &= 180^\circ\ 00'\ \ 7{\cdot}46''
\end{aligned}$$

(c) Computation of sides (Legendre's method)

One-third spherical excess $= \dfrac{7{\cdot}462}{3} = 2{\cdot}487''$.

Diminishing each angle by $2{\cdot}49''$, we get :—

$$\begin{aligned}
A &= 55^\circ\ 45'\ 31{\cdot}41'' \\
B &= 72^\circ\ 18'\ 22{\cdot}75'' \\
C &= 51^\circ\ 56'\ \ 5{\cdot}83''
\end{aligned}$$

$b = 186{,}540{\cdot}23 \times \sin 72^\circ\ 18'\ 22{\cdot}75'' \times \operatorname{cosec} 55^\circ\ 45'\ 31{\cdot}41''$
$\quad = 214{,}976{\cdot}69$ feet.

$c = 186{,}540{\cdot}23 \times \sin 51^\circ\ 56'\ 5{\cdot}83'' \operatorname{cosec} 55^\circ\ 45'\ 31{\cdot}41''$
$\quad = 177{,}657{\cdot}74$ feet.

Problem IX.—*In a " Minor " Triangulation, to determine the equations of condition of a polygon built up of triangles.*

In a minor triangulation (*vide* p. 365), the sides of the triangles are seldom more than two miles in length, and in adjusting the errors of observation spherical excess is ignored.

Polygon built up of Triangles.—In the case of a polygon built up of triangles having a common vertex O (Fig. 151), the angular measurements must satisfy the following conditions if the polygon is perfect :—

(1) *The sum of all the angles around the common vertex must be equal to four right angles.*

(2) *The sum of the angles of each triangle must be equal to two right angles.*

A third condition is necessary, since the above conditions may be fulfilled, although the polygon may be distorted, as shown in Fig. 150. With regard to this third condition, there are two cases to be considered, depending on whether O is within or without the figure. In the former, calling all the angles

(L_1, L_2, L_3, etc.) on the left of an observer who traverses the boundary of the figure, always facing the point O, left-hand angles, and those on his right (R_1, R_2, R_3, etc.) right-hand angles, the third condition may be stated as follows:—*The sum of the log sines of the left-hand angles must be equal to the sum of the log sines of the right-hand angles.*

When the point O is without the figure, if we call all the triangles which lie partly within and partly without the polygon positive triangles, and those which lie wholly without the figure negative triangles, the third condition is *the sum of the log sines of the left-hand angles in the positive triangles increased by the sum of the log sines of the right-hand angles in the negative triangles must be equal to the sum of the log sines of the right-hand angles of the positive triangles, increased by the sum of the log sines of the left-hand angles in the negative triangles.*
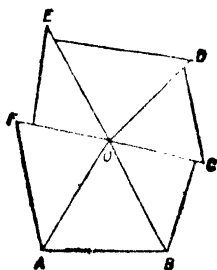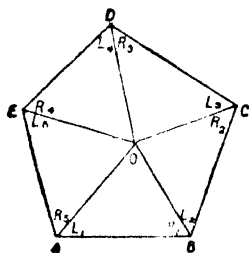
Fig. 150.   Fig. 151.

The third condition is demonstrated as follows:—

Referring to Fig. 151, and applying the sine rule, we have

$$\frac{O A}{O B} = \frac{\sin R_1}{\sin L_1}, \frac{O B}{O C} = \frac{\sin R_2}{\sin L_2}, \frac{O C}{O D} = \frac{\sin R_3}{\sin L_3}, \frac{O D}{O E} = \frac{\sin R_4}{\sin L_4}, \text{ etc.}$$

The product of the left-hand sides of these equations is equal to unity, therefore $\sin L_1 \times \sin L_2 \times \sin L_3 \times \sin L_4 \times \sin L_5$, etc., is equal to $\sin R_1 \times \sin R_2 \times \sin R_3 \times \sin R_4 \times \sin R_5$, etc., or $\log \sin L_1 + \log \sin L_2 + \log \sin L_3 + \log \sin L_4 + \log \sin L_5$, etc. $= \log \sin R_1 + \log \sin R_2 + \log \sin R_3 + \log \sin R_4 + \log \sin R_5 +$ etc.—*i.e.*, the sum of the log sines of the left-hand angles is equal to the sum of the log sines of the right-hand angles.

When the point O lies outside the polygon, as in **Fig. 152,** we have

$$\frac{O\,A}{O\,B} = \frac{\sin R_1}{\sin L_1}, \quad \frac{O\,B}{O\,C} = \frac{\sin R_2}{\sin L_2},$$

$$\frac{O\,C}{O\,D} = \frac{\sin L_3}{\sin R_3}, \quad \frac{O\,D}{O\,E} = \frac{\sin L_4}{\sin R_4},$$

$$\frac{O\,E}{O\,A} = \frac{\sin R_5}{\sin L_5},$$

$$\therefore \quad \frac{O\,A}{O\,B} \times \frac{O\,B}{O\,C} \times \frac{O\,C}{O\,D} \times \frac{O\,D}{O\,E} \times \frac{O\,E}{O\,A} = 1$$

$$= \frac{\sin R_1}{\sin L_1} \times \frac{\sin R_2}{\sin L_2} \times \frac{\sin L_3}{\sin R_3} \times \frac{\sin L_4}{\sin R_4} \times \frac{\sin R_5}{\sin L_5},$$

or $\log \sin L_1 + \log \sin L_2 + \log \sin R_3 + \log \sin R_4 + \log \sin L_5$

$= \log \sin R_1 + \log \sin R_2 + \log \sin L_3 + \log \sin L_4 + \log \sin R_5$, but L_1, L_2, and L_5 are the left-hand angles in the positive triangles, and R_3, R_4 are the right-hand angles in the negative triangles; also, R_1, R_2, and R_5 are the right-hand angles in the positive triangles, and L_3, L_4 are the left-hand angles in the negative triangles, hence the preceding rule.
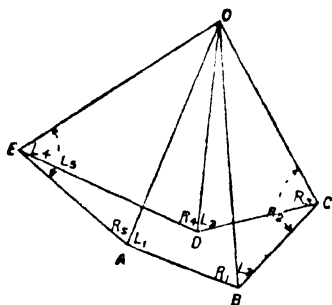
Fig. 152.

Problem XI. — *To apportion the errors of observation among the angles of a minor polygon.*

The laborious and exact methods adopted for correcting the errors of observation in a Great Trigonometrical Survey are not necessary in a minor triangulation, as an error of a few seconds in the angles would not appreciably affect the result, since the sides of the triangles are comparatively short. Hence, in correcting the angles, they are made to satisfy the above equations of condition by a judicious alteration—by trial and error—of the seconds in the observed values. This is done polygon by polygon, until the whole of the figures have been dealt with.

In applying the three equations of condition to a particular figure, the polygons are first drawn (Fig. 153) by aid of a protractor, approximately to scale, and the observed angles are

entered on the figure in their proper positions. A table is then prepared, having the central angles of each polygon with their corrections in column I.; the left-hand angles of the triangles with their log sines and corrections are placed in column II.; the right-hand angles with their log sines and corrections in column III.; sums of the observed angles and errors in column
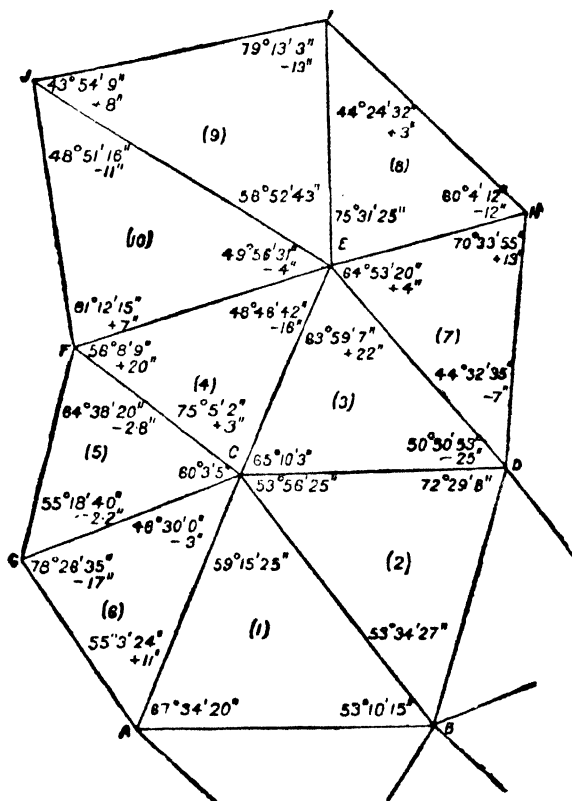
Fig. 153.

IV.; and the sum of the corrected angles of each triangle in column V.

The angles of each triangle are arranged in the same horizontal line with their final sum.

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In columns II. and III. the difference for 1″ is entered against the corresponding log, with its proper sign, noting that the sines of angles greater than 90° are decreasing, and consequently their log differences are negative.

No complete rule can be given for determining the correction to be applied to the angles, as the number of solutions is infinite ; but, in deciding on the proper correction to be applied, the following guiding principles should be kept in view :—(*a*) The corrections which are applied to any triangle hold good for all the figures of which it forms a part ; (*b*) all angles are liable to error, and though the sum of the central angles of any polygon may be 360°, as will generally be the case owing to the method of measurement, they are none the less open to correction ; in applying a correction to an error of summation in log sines, correct small angles rather than large, as this produces a smaller change in the observed angle, since the sines of small angles increase at a greater rate than the sines of large angles ; (*d*) in correcting an error in summation of angles, correct large angles rather than small, as the difference of the log sines of large angles is small ; (*e*) in commencing the correction, divide the summation error of each triangle by three, and apply the quotient algebraically to each angle of the triangle. This correction multiplied by the corresponding log difference for 1″ is applied in its proper place and the results are summed, thus giving a first approximation to the desired result. This may or may not improve matters ; in any case, the final correction must be made by slightly increasing some angles and decreasing others until the equations of condition are satisfied.

It may happen that the correction to be applied to some angles is greater than the permissible error of observation ; if this occurs the incorrect angles must be re-observed.

The remarks in the field book as to the conditions under which the angles were observed will often be of assistance in apportioning the corrections.

The whole process is exemplified in Table B and Fig. 153. The data of the triangles A C B and B C D is not to be altered, as these triangles are assumed to have been corrected in the previous polygon, of which they form a part.

Problem XII.—*To compute the sides of the triangles in a minor polygon.*

The computation of the sides of the triangles in a minor polygon is carried out by the repeated application of the sine rule, using the corrected values of the angles in the computation.

Referring to Fig. 153, let A B be the known side, then we have

$$A C = \frac{A B \cdot \sin 53^\circ 10' 15''}{\sin 59^\circ 15' 25''},$$

∴ log A C = log A B + log sin 53° 10′ 15″ — log sin 59° 15′ 25″

Similarly,

log B C = log A B + log sin 67° 34′ 20″ — log sin 59° 15′ 25″.

In both cases we note that, to obtain the log of the required side, we add to the log of the known side the log difference of the angles opposite the unknown and known sides respectively. Bearing this in mind, and arranging the work in a methodical way, the labour of computation is much reduced.

The process is illustrated in Table C, which is similar to that used in the Topographical Survey of India. The triangles dealt with in the table are those illustrated in Fig. 153. By reference to the table, it will be seen that in each case the known side of each triangle is arranged in the centre, and in column VII. the log differences are obtained by subtracting the central log sine from the upper and lower log sines. These differences, added to the log of the known side, give the respective logs of the unknown sides.

On completing the round of any polygon, at least one side will be computed twice [*e.g.*, A C in triangle (1) is again computed in triangle (6), similarly E F is computed in triangles (4) and (10)], and the two results should agree very closely, otherwise an arithmetical error has been made.

Problem XIII.—*To compute the horizontal angle subtended at a known station by two elevated signals, the angular elevations (which must be small) of the signals and the plane angle they subtend at the given station being known.*

This problem does not arise when the angles are observed with a theodolite, but sometimes occurs when using the sextant. The reduction of the angle may be obviated by sighting two objects at the same level as the observer's station, the selected objects being in the vertical planes containing the observer's station and the distant signals.

Referring to Fig. 154, let A and B be the distant signals, O the observer's station, C E D his horizon, Z his zenith, ∠ A O B = θ, ∠ D O E = $\theta + x$, h and h' the altitudes of A and B respectively ; then from the triangle A Z B we have

$$\text{Cos D Z E} = \cos(\theta + x)$$

$$= \frac{\cos\theta - \cos Z A \cdot \cos Z B}{\sin Z A \quad \sin Z B}$$

$$= \frac{\cos\theta - \sin h \cdot \sin h'}{\cos h \cdot \cos h'}.$$

Since h and h' are small angles, we may write $\sin h = h$, $\cos h = 1 - \dfrac{h^2}{2} +$, and neglecting higher powers of h than the second. we obtain by substitution :—
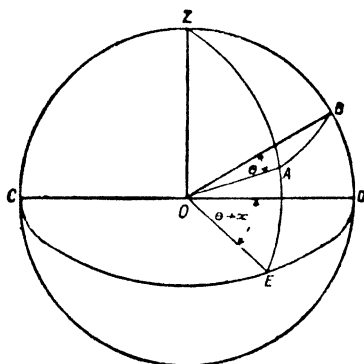
Fig. 154.

$$\text{Cos}(\theta + x) = \frac{\cos\theta - h\,h'}{\left(1 - \dfrac{h^2}{2} + \cdot\,\cdot\right)\left(1 - \dfrac{h'^2}{2} + \cdot\,\cdot\,\cdot\right)}$$

$$= \frac{\cos\theta - h\,h'}{1 - \left(\dfrac{h^2 + h'^2}{2}\right) + \text{etc.}}$$

$$= (\cos\theta - h\,h')\left\{1 + \tfrac{1}{2}(h^2 + h'^2)\right\}\text{ nearly,}$$

and as $\cos(\theta + x) = \cos\theta \cdot \cos x - \sin\theta \cdot \sin x$

$$= \cos\theta - x \cdot \sin\theta \text{ very nearly,}$$

hence, $x \cdot \sin\theta = \cos\theta - (\cos\theta - h\,h')\left\{1 + \tfrac{1}{2}(h^2 + h'^2)\right\}$,

$$= h\,h' - \tfrac{1}{2}(h^2 + h'^2)\cos\theta,$$

$$= \tfrac{1}{4}\left\{(h + h')^2 - (h - h')^2 - [(h + h')^2 + (h - h')^2]\cos\theta\right\}$$

$$\therefore \quad x = \frac{1}{4 \sin \theta}\{(h + h')^2 (1 - \cos \theta) - (h - h')^2 (1 + \cos \theta)\}$$

$$= \frac{1}{4}\left\{ (h + h')^2 \tan \frac{\theta}{2} - (h - h')^2 \cot \frac{\theta}{2} \right\}$$

when x is in circular measure.

If h and h' are expressed in seconds, then x in seconds

$$= \frac{1}{4 \sin 1''}\left\{ (h + h')^2 \sin^2 1'' \tan \frac{\theta}{2} - (h - h')^2 \sin^2 1'' \cot \frac{\theta}{2} \right\}$$

$$= \frac{\sin 1''}{4}\left\{ (h + h')^2 \tan \frac{\theta}{2} - (h - h')^2 \cot \frac{\theta}{2} \right\},$$

Example 2.—Let $h = 1°\ 10'$, $h' = 1°\ 30'$, and $\theta = 60°$, find x. Here $h = 4,200$ seconds and $h' = 5,400$ seconds.

$$(h + h') = 9,600'', \quad (h - h') = -1,200'', \quad \frac{\theta}{2} = 30°.$$

Log 9,600	=	3·98227
Do. to square		3·98227
Log tan 30°	=	9·76144

Sum, − 10	=	7·72598	= log 53,208,000

Log 1,200	=	3·07918
Do. to square		3·07918
Log cot 30°	=	10·23856

Sum, − 10	=	6·39692	= log 2,494,100

Diff. **=** 50,713,900

Log	= 7·70513
Log sin 1''	= 4·68557

Sum, − 10	= 2·39070
Log 4	= 0·60206

Log 61·47 = 1·78864

$$x = 61·47,$$

and $\qquad \theta + x = 60°\ 01'\ 1.47''.$

Problem XIV.—*To reduce an angle measured at a satellite station, to the centre.*

It occasionally happens that angles are measured to a station over which the instrument cannot afterwards be placed, in order to check the sum of the three angles of the triangle having the station as vertex. When this occurs, the instrument is placed at a convenient distance from the principal station, the distance between the two stations is determined, and the angular deviations of the distant stations from the line joining the nearby principal station and the new station are measured. From this data the angle subtended by the distant stations at the principal station is calculated, thus checking the value obtained by difference.

The station over which the instrument is placed is called a "satellite" or "supplementary" station.
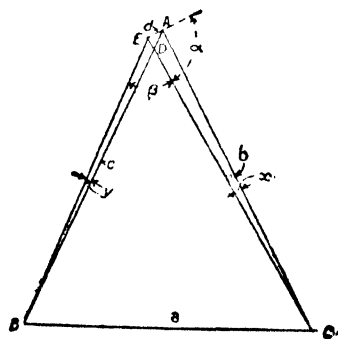
Fig. 155.

Let A (Fig. 155) be the position of the principal station observed from B and C, E the position of the satellite station,

$$x = \angle \text{E C A}, \quad y = \angle \text{E B A}, \alpha = \angle \text{A E C},$$
$$\beta = \angle \text{B E C}, \text{A E} = d.$$

Then $A = 180 - (B + C)$, and on this assumption the sides b and c may be calculated, since the side a is known.

Again, $A = \angle \text{B D C} - x,$

but, $\angle \text{B D C} = \beta + y,$

∴. $A = \beta + y - x.$

As both the angles x and y are very small, x and y in circular measure are

$$x = \sin x = \frac{d}{b} \sin \alpha, \text{ and } x \text{ in seconds} = \frac{d \sin \alpha}{b \sin 1'''}.$$

Similarly, $y = \sin y = \dfrac{d \sin (\beta + \alpha)}{c \sin 1''}$, seconds.

$$\therefore \qquad A = \beta - \frac{d}{\sin 1''} \left(\frac{\sin \alpha}{b} - \frac{\sin (\beta + \alpha)}{c} \right).$$

Example 3.—From the following data determine the angle at A (Fig. 155):—B C = 5,280 feet, angle A B C 67° 46', angle A C B 55° 24', A E = 20 feet, angle A E C 62° 50', angle B E C 56° 49'.

$$A = 180° - (67° 46' + 55° 24')$$
$$= 56° 50'.$$

From this we have,

$$b = 5,280 \times \sin 67° 46' \times \operatorname{cosec} 56° 50'$$
$$= 5,838 \cdot 65 \text{ feet}.$$
$$c = 5,280 \times \sin 55° 24' \times \operatorname{cosec} 56° 50'$$
$$= 5,192 \cdot 03 \text{ feet}.$$

$$y'' = \frac{20 \sin (180 - 56° 49' + 62° 50') \operatorname{cosec} 1''}{5,192 \cdot 03}.$$

$$x'' = \frac{20 \sin 62° 50' \operatorname{cosec} 1''}{5,838 \cdot 65}.$$

Log 20,	1·3010300	Log 20,	1·3010300
L sin 60° 21',	9·9390515	L sin 62° 50',	9·9492349
L cosec 1″,	5·3144251	L cosec 1″,	5·3144251
	10 + 6·5545066		10 + 6·5646900
Log 5192·03,	3·7153373	Log 5838·65,	3·7663126
690·51 log,	2·8391693	628·60 log,	2·7983774

$$y = 11' \, 30 \cdot 51''. \qquad\qquad x = 10' \, 28 \cdot 6''.$$
$$A = 56° \, 49' + 11' \, 30 \cdot 51'' - 10' \, 28 \cdot 6''$$
$$= 56° \, 50' \, 1 \cdot 91'',$$

thus checking the value of A obtained by difference.

Problem XV.—*To determine the distance between two points from angular observations at the extremities of a measured base line.*

This problem is of frequent occurrence in surveying; for example, A B and C D (Fig. 156) may be traverse lines on the

opposite sides of a river or ravine, the lines from A and B to
C and D tie the two lines in position; or, again, C and D may be
on the opposite sides of a river, and consequently the line C D
cannot be measured directly.

If C D be measured directly, the agreement of the calculated
and measured lengths gives a complete check on the work. If
C D is not measured, the assumed data is not sufficient to do
more than fix the length and position of the line. A complete
check, however, may be obtained by angular observations from
any intermediate point (E) in the base line, as shown by the dotted
lines in Fig. 156.

Let A B be the measured base and α, β, θ, and φ the angles
measured at its extremities.
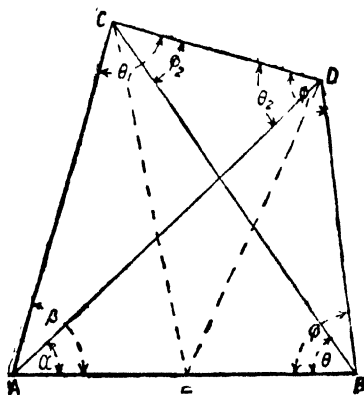
Fig. 156.

In solving the problem, the lengths of the lines A C and A D
are calculated by the sine rule, C D then follows by the method
of Problem IV., since the angle C A D is known. A check on the
calculation is obtained by calculating C D from the triangle C B D.

From the triangle A C B we have :—

$$A C = \frac{A B \cdot \sin \theta}{\sin (\beta + \theta)}.$$

Similarly, from triangle A D B we get :—

$$A D = \frac{A B \cdot \sin \varphi}{\sin (\alpha + \varphi)}.$$

Also, $$C D = \sqrt{A C^2 + A D^2 - 2 A C \cdot A D \cdot \cos (\beta - \alpha)}.$$

In a similar way we get :—

$$CD = \sqrt{BC^2 + BD^2 - 2BC \cdot BD \cdot \cos(\varphi - \theta)}.$$

Or, alternatively, we have $\frac{1}{2}(\theta_1 + \theta_2) = \frac{1}{2}(180 - \beta - \alpha)$, and

$$\tan \tfrac{1}{2}(\theta_1 - \theta_2) = \frac{AD - AC}{AD + AC} \cdot \cot \frac{(\beta - \alpha)}{2}.$$

Hence it follows that C D is known, since

$$CD = \frac{AD \cdot \sin(\beta - \alpha)}{\sin \theta_1},$$

or, alternatively, $\qquad CD = \dfrac{AC \cdot \sin(\beta - \alpha)}{\sin \theta_2}.$

Similarly from triangle C B D we get :—

$$CD = \frac{BD \cdot \sin(\varphi - \theta)}{\sin \varphi_2},$$

or $\qquad\qquad = \dfrac{CB \cdot \sin(\varphi - \theta)}{\sin \varphi_1}.$

Problem XVI.—*To determine the positions of two unknown relative to two known points by angular observations from the unknown points to the known, and to each other.*

This problem occurs in fixing the position of secondary points in a survey, the known points being at the extremities of any line fixed by the primary operations.

Let A and B (Fig. 157) be the known points, C and D the unknown points, and the observed angles α, β, θ, and φ. Our object is to determine the lengths of the lines A C, C B, B D, D A, and C D. As the point of intersection of A B and C D is not known, we cannot calculate the required lengths directly from the known length (of A B) and the known angles. To solve the problem, we assume some convenient length, such as 1 or 1,000, for the length of C D, and on this assumption, calculate the required lengths, finally calculating A B.

Since the contracted figure formed by assuming the length of C D as unity will be similar to the real figure, the sides computed on this assumption must be corrected by the proportion :—

True length of C D : assumed length of C D :: true length of A B : computed length of A B—*i.e.*,

$$\text{True length of } CD = \frac{1 \times \text{true length of A B}}{\text{computed length of A B}}.$$

The other sides are treated in the same way.

The steps of the computation are as follows :—
From the triangle D B C we have :—

$$D B = \frac{C D . \sin \beta}{\sin (\beta + \varphi)},$$

since C D is assumed to be unity,

$$D B = \frac{\sin \beta}{\sin (\beta + \varphi)}.$$

Similarly, from the triangle A D C we get :—

$$A D = \frac{\sin \alpha}{\sin (\alpha + \theta)}.$$
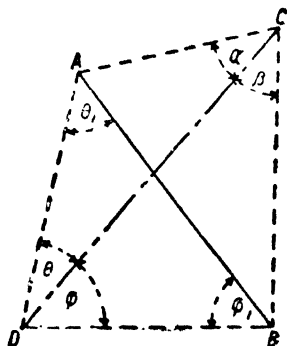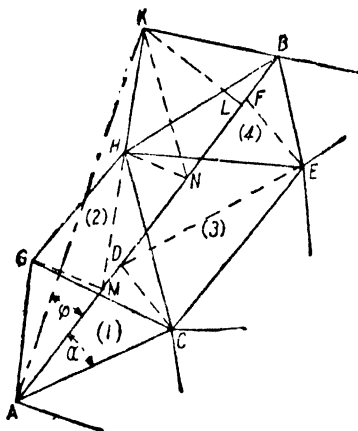
Fig. 157.   Fig. 158.

In triangle A D B, $\frac{1}{2} (\theta_1 + \varphi_1) = \frac{1}{2} (180 - \overline{\theta + \varphi})$, and

$$\text{Tan } \tfrac{1}{2} (\theta_1 - \varphi_1) = \frac{D B - D A}{D B + D A} \cot \tfrac{1}{2} (\theta + \varphi).$$

From the last two equations we obtain θ_1 and φ_1, and from the triangle A D B we have :—

$$\text{Computed value of A B} = \frac{D B . \sin (\theta + \varphi)}{\sin \theta_1},$$

$$= \frac{D A . \sin (\theta + \varphi)}{\sin \varphi_1}.$$

A C and C B are next calculated from the triangles A C D and C B D, and before proceeding to the correction of the sides, the computed length of A B must be checked from the triangle A C B

Problem XVII.—*To determine the direct distance between two distant stations of a triangulation.*

Let A and B (Fig. 158) be the two given stations, and let the line A B cut the triangles 1, 2, 3, and 4 as indicated. It is assumed that the sides and angles of all the triangles are known, and also the angle that some line such as A C makes with A B.

On A B cut off a length A D equal to A C; join D C. Then, in the triangle D A C, A D = A C, and the angle D A C is known; hence, D C and the angle A C D may be calculated after the manner of Problem IV. Join D to any convenient station E. Since the angles G C H, A C D, and H C E are known, the angle D C E is also known; hence, knowing C D, C E and the angle D C E, the side D E and its adjacent angles may be computed. Next, lay off D F equal to D E along A B, and calculate F E and its adjacent angles in the triangle D E F. This may be done, since we know the two sides D F and D E and the angle (180° − ∠ E D C + ∠ A D C) between them. Lastly, from the triangle F E B, knowing F E, E B and the angle F E B, F B is calculated. As a check, the distance is calculated from an entirely different set of stations of the triangulation.

This problem occurs in surveying by a system of small or minor triangles, a country in which a Grand Triangulation exists, A and B being trigonometrical stations. The sides of the small triangles will be known with a less degree of accuracy than those of the Grand Triangulation, since the instruments employed will be of a lower order of precision. To bring the base line measurements of the lesser into conformity with the major operations, one of the sides of the chain of triangles is measured as accurately as the instruments available will permit. Starting from this as a known side, the chain of triangles is worked out, and the distance from A to B calculated in terms of the measured base. This distance is, of course, known from the main triangulation, and the comparison of the true and calculated lengths enables us to adjust the base line measurement, since *the true length of the measured base is to its measured length, as the true length of* A B, *is to the calculated length of* A B, and all the calculated sides of the minor triangles must be corrected in the same proportion. As we always use logarithms in solving a triangle, the adjustment of the sides of triangles is made by adding to the logarithmic value of their computed lengths the constant

$$\log \left(\frac{\text{true length of A B}}{\text{calculated length of A B}} \right).$$

This problem is also useful in determining the " latitude,"

" departure," and azimuth of a station relative to another
station, and the meridian passing through it. For example,
let A B (Fig. 158) be the meridian through A, and K the station
whose " latitude," " departure," and azimuth are required.
The distances A M and M N are computed as described above,
and from the triangle H N K, N K, and the angle K N H are
obtained. The angle K N B is then known, and the required
" departure " = N K . sin . K N B.

The corresponding "latitude" = A M + M N + N K . cos K N B

also $$\text{Tan K A B} = \frac{K L}{A L}.$$

$$= \frac{\text{" departure "}}{\text{" latitude "}} \cdot$$

Problem XVIII.—*A base line is measured at a known height
to determine its equivalent length when reduced to mean sea level.*

In trigonometrical surveying, over an extensive tract of
country, a line is measured with great care on the most level
tract of ground that the country affords ; from this line as a
base, a network of triangles commences which covers the whole
of the ground to be surveyed. The sides of all the triangles are
computed in terms of the measured base from the known angles
of the triangles. The stations of the triangulation will be at
many different levels, but the sides of the triangles, computed
from the length of the measured base line, are the lengths of the
sides projected on the spheroidal surface containing the base.

" Bases of Verification."—To check the work as it proceeds,
new bases are measured at considerable distances apart, and
the triangles in their vicinity are caused to close upon them.
Thus, in the trigonometrical survey of this country, six principal
bases were used, commencing with the Hounslow Heath base,
measured in 1791, and ending with the Lough Foyle base,
measured in 1827. In the survey of India ten bases were used.
As all these bases are measured with the same care, the triangu-
lation may commence at any one or several of them, the remainder
then form bases of verification. Since these bases are measured
at different altitudes, in order to have a true comparison of the
computed and measured lengths, it is necessary to project the
whole of the triangulation on some spheroidal datum surface.
The datum selected is that of the mean sea level (M.S.L.).

This projection of a line to M.S.L. or its converse is also neces-
sary in " trigonometrical levelling," as we shall see later.

To reduce a triangulation to M.S.L.—assuming the angles have been corrected—it is necessary and sufficient to reduce the base line to this datum before proceeding to the computation of the sides of the triangles.

"**Geodetic Lines and Distances.**"—To reduce a base line to M.S.L., we proceed as follows :—Let A B (Fig. 159) be the " horizontal length " of the base line, which has been measured in short horizontal lengths as indicated in the figure, and $a\,b$ the radial projection of A B on a spheroidal surface at M.S.L. If l = length of measuring bar used, h = its height above mean sea level (Fig. 160), x = its radial projection on a spheroidal surface at M.S.L., and r = radius of the earth at the place.
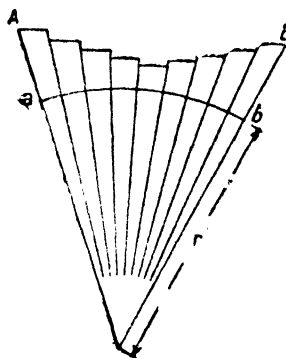
Fig. 159.   Fig 160.

then

$$\frac{x}{l} = \frac{r}{r+h},$$

$$\text{or, } x = \frac{r\,.\,l}{r+h};$$

$$= \frac{r\,.\,l + l\,.\,h - l\,.\,h}{r+h}$$

$$= l - \frac{l\,.\,h}{r+h};$$

Since h is very small compared with r, we may neglect this

quantity in the denominator of the second term, and we get $x = l - \dfrac{l \cdot h}{r}$, and for the whole base we have

$$\Sigma x = \Sigma \left(l - \frac{l \cdot h}{r} \right)$$
$$= \Sigma l - \Sigma \left(\frac{n \cdot l \cdot h}{r \cdot n} \right)$$
$$= L - \frac{L}{r} \cdot \frac{\Sigma h}{n},$$

where L = total length of the base, and n = number of times l is contained in L, also

$$\Sigma \frac{h}{n} = \text{average height of base.}$$

$\therefore$ length of base reduced to mean sea level $= L - \dfrac{L}{r} \times$ average height of base.

If we know the horizontal distance (A B) between two stations, and know the height of station A only, we may use the proportion

$$\frac{X}{A \ B} = \frac{r}{r + h}.$$

Taking the mean radius of the earth at 20,889,000 feet, and $h = 1,000$ feet, then

$$\frac{X}{A \ B} = \frac{20,889,000}{20,890,000}$$
$$= \frac{99,995}{100,000},$$

or, by neglecting to reduce a line at the given height to mean sea level, an error of 5 parts in 100,000 is introduced. As this is less than the probable instrumental and other errors in measurement, in ordinary surveys with the theodolite covering a few square miles of country, this refinement is not necessary.

The converse problem of determining a horizontal distance, from its given geodetic distance, is solved as follows :—

Let r = mean radius of the earth,
d = required horizontal distance,
g = given geodetic distance,
h = height of station above M.S.L.
Then $\dfrac{d}{g} = \dfrac{r + h}{r}$,

or, $\log d = \log g + \{\log (r + h) - \log r\}$.
$\log g$ will usually be given from the triangulation.

The expression $\{\log (r + h) - \log r\}$ may be simplified by the following artifice :—

Let $h = 10,000$ feet, and $r = 20,889,000$ feet.

Then $\log (r + h) = \log 20,899,000 = 7\cdot3201255$

$\qquad\quad \log r = \log 20,889,000 = 7\cdot3199176$

$$\text{Diff.} = 0\cdot0002079$$

Or the increase in $\log r$ for an increase of $10,000$ feet $= 0\cdot0002079$, and for any other height, the increase will be proportional to the height ; hence we may write $\log (r + h) = \log r + \dfrac{0\cdot0002079h}{10,000}$

Substituting this value in the above equation, we get

$$\log d = \log g + \log r + \frac{0\cdot0002079\,h}{10,000} - \log r$$

$$= \log g + \frac{0\cdot0002079 \times h}{10,000} = \log g + \text{N}.$$

Evidently the log of the natural number (N) to be added to $\log g$, to give $\log d$, is given by

$$\log \text{N} = \log \cdot0002079 - \log 10,000 + \log h$$

$$= \bar{8}\cdot31785 + \log h.$$

Example 4.—Let $\log g = 4\cdot6432987$,

$\qquad\qquad\qquad h = 2,435\cdot6$ feet,

find $\log d$.

$$\text{Log } 2,435\cdot6 = 3\cdot3866060$$

$$\text{Constant log} = \bar{8}\cdot3178500$$

$$\text{Sum} = 5\cdot7044560$$

$$\textbf{Corresponding number} = 0\cdot0000506$$

$$\text{Adding } \log g \quad 4\cdot6432987$$

$$\text{Gives } \log d = 4\cdot6433493$$

Problem XIX.—*To determine the height of a point above a given station.*

Case (*a*).—When the distance from the station to a point vertically under the given point can be obtained by calculation or by direct measurement.

Referring to Fig. 161, let A be the given station and C the given point. Set up the instrument over the station A, and determine the angular elevation (α) of C ; obtain also the distance from A to B. Then, clearly, B C = A B . tan α, and the height of C above A is equal to A B . tan α + height of instrument.
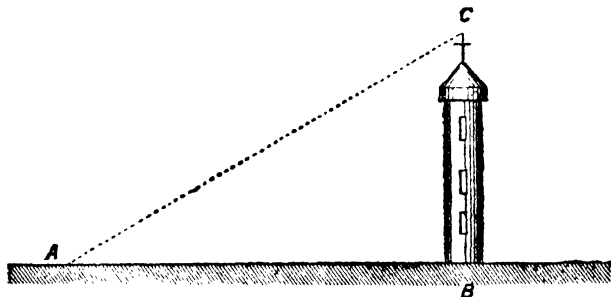
Fig. 161.

Case (b).—When the distance from the station to a point vertically under the given point is not obtained in triangulation, nor by direct measurement.

(1) Referring to Fig. 162, set the instrument over the given station A, and determine the position of a point B in the same vertical plane as A and D and at the same level as A. Obtain
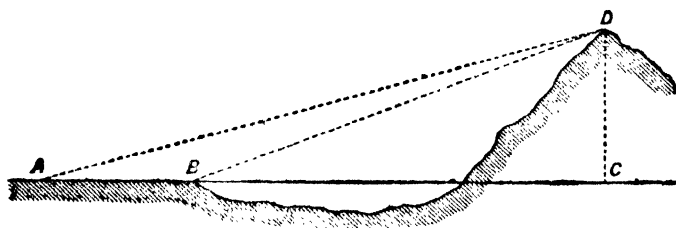
Fig. 162.

the angular elevation (α) of D at A, and the distance from A to B.

Remove the instrument to B, and obtain the angular altitude (β) of D. The height of the instrument above the ground at the stations A and B must be the same.

Obviously, the angle B D A $= \beta - \alpha$, and

$$A D = \frac{A B \cdot \sin \beta}{\sin (\beta - \alpha)},$$

Also,

$$D C = A D \cdot \sin \alpha,$$

$$= \frac{A B \cdot \sin \alpha \cdot \sin \beta}{\sin (\beta - \alpha)}.$$

The height of D above A is equal to D C $+$ the height of the instrument above the ground.

The result should be checked by first computing B D, then computing D C from the triangle D C B.

(2) **Base Inclined to the Horizontal.**—If a suitable position for the second station (E) at the same level as A cannot be

Fig. 163.

found, a convenient position for the forward extremity of the base is selected in the same vertical plane as A and D (Fig. 163), and the length of the base is measured on the slope. At A, the angles of elevation and depression α and β are observed, and at E the angle θ is determined.

In the triangle B D F, the angle B D F $= \theta - \alpha$, and

$$B D = \frac{B F \cdot \sin (\theta + \beta)}{\sin (\theta - \alpha)},$$

also,

$$D C = B D \cdot \sin \alpha,$$

$$= \frac{A E \cdot \sin \alpha \cdot \sin (\theta + \beta)}{\sin (\theta - \alpha)},$$

since A E $=$ B F.

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The height of D above A $= $ D C $+$ A B.

The height of D above E $= $ D C $+$ C G $+$ F E

$$= \text{D C} + \text{B F} . \sin \beta + \text{F E},$$

since C G $=$ H F.

We may also find the height of D above E as follows :—

From triangle B F D,

$$\text{D F} = \frac{\text{B F} . \sin (\alpha + \beta)}{\sin (\theta - \alpha)}, \text{ and}$$

$$\text{D G} = \text{D F} . \sin \theta,$$

$$= \frac{\text{A E} . \sin \theta . \sin (\alpha + \beta)}{\sin (\theta - \alpha)},$$

and the required height

$$= \text{D G} + \text{F E}.$$

(3) Base Horizontal, but not in the same Vertical Plane as the Elevated Object.—Having determined the length of the base, set
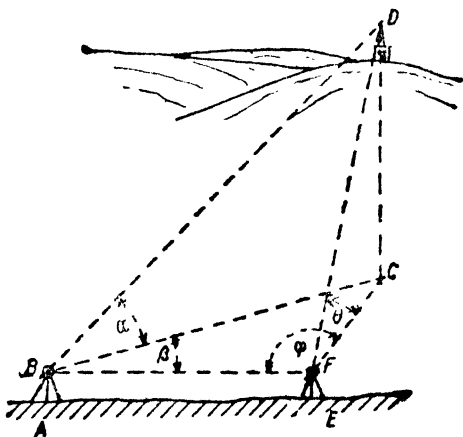
Fig. 164.

the instrument over A, observe the angular elevation (α, Fig. 164) of D, and the angle (β) between the vertical planes containing B D and B F. Remove the instrument to E, and determine the corresponding angles θ and φ.

Then, since B and F are at the same level, the plane of the triangle B C F is horizontal ; also, since the planes of the two triangles D C B and D C F are vertical, their line of intersection

D C is vertical and passes through the point C, the apex of the triangle B C F. The entire figure B F C D is a pyramid having one horizontal and two vertical faces, the oblique face being represented by the triangle B D F.

The problem of finding D C is equivalent to Case (*a*) when either B C or F C is known.

To determine B C ; from triangle B C F, we get

$$B C = \frac{B F \cdot \sin \varphi}{\sin (\beta + \varphi)}$$

$$= \frac{A E \cdot \sin \varphi}{\sin (\beta + \varphi)}, \text{ since } B F = A E.$$

Also, since the angle D C B is a right angle,

$$D C = B C \cdot \tan \alpha$$

$$= \frac{A E \cdot \sin \varphi \cdot \tan \alpha}{\sin (\beta + \varphi)}.$$

Similarly, we have

$$D C = \frac{A E \cdot \sin \beta \cdot \tan \theta}{\sin (\beta + \varphi)}.$$

(4) Base Inclined to the Horizontal, and not in the same Vertical Plane as the Elevated Object.

—The angles required for the solution of this problem cannot all be determined with a theodolite, a sextant, or other instrument for measuring oblique angles being necessary.

The solution of the problem presents no difficulty, for (referring to Fig. 163, and assuming the plane of the triangle B D F to be oblique) if β and φ be the angles measured with the sextant in the plane of B D F, we calculate B D or F D from the triangle B D F, then find D C or D G, knowing the angles α and θ.

In the solutions to the different cases of Problem XIX., no account has been taken of the effects of (*a*) the earth's curvature, (*b*) refraction, or (*c*) the height of the signal at the distant station. It has been assumed that angular altitudes have been corrected for the instrument, and horizontal angles have been obtained by face-left and face-right repetitions.

Wherever possible, in determining the difference of level of two stations from angular measurements, these measurements should be obtained as nearly as possible simultaneously by observers at the two stations. The observers reciprocally determine the angular altitudes of signals centred over their instruments, an allowance being made in the observed angles for the

difference in height of the signal, and the trunnion axis of the observing instrument.

The distance between the stations will usually be given in triangulation, or it may be determined as in Problem I.

There will be two cases to be considered, (1) when the difference in altitude is great, and the distance between the stations com- paratively short, *one angle will be an angle of elevation, and the other an angle of depression ;* (2) when the difference of elevation is small, and the distance between the stations great, *both angles will be angles of depression.*

Case (1).—Neglecting, for the moment, the heights of the signals and instruments, let A and B (Fig. 165) be the two stations,
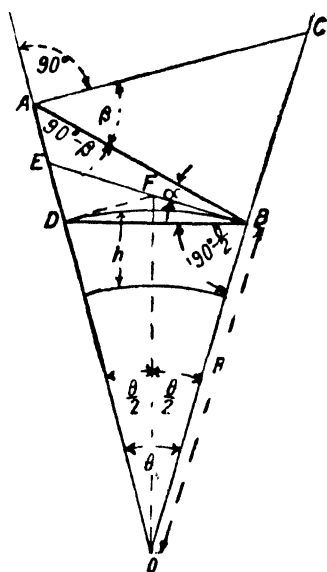
Fig. 165.

a and β = the angles of eleva- tion and depression at B and A respec- tively,

θ = the angle subtended at the earth's centre (O) by the hori- zontal distance be- tween A and B,

R_1 = the radius of a spherical surface through the lower station,

h = height of the lower station above M.S.L., and

BD = the radial projection of the horizontal distance between the two stations on a spherical surface through the lower station.

At B and D draw perpendiculars B E and D F, meeting at F; join F O. Then the angle D O F = $\angle$ B O F = $\dfrac{\theta}{2}$, and

$$DB = 2\,R_1 \sin\frac{\theta}{2} = \text{arc D B very nearly.}$$

In the triangle A B D, $\angle$ B A D = $90° - \beta$,

$$\angle A B D = \alpha + \left(90° - \overline{90° - \frac{\theta}{2}}\right)$$

$$= \alpha + \frac{\theta}{2},$$

$$\angle A D B = 90° + \frac{\theta}{2}.$$

Hence,

$$\frac{A D}{D B} = \frac{\sin\left(\alpha + \frac{\theta}{2}\right)}{\sin(90 - \beta)},$$

$\therefore$

$$A D = \frac{D B \sin\left(\alpha + \frac{\theta}{2}\right)}{\cos \beta},$$

$$= 2 R_1 \sin \frac{\theta}{2} \frac{\sin\left(\alpha + \frac{\theta}{2}\right)}{\cos \beta}. \quad (1)$$

Now, $\angle$ B D O is the exterior angle of the triangle B A D,

$\therefore \angle$ B D O, or $90° - \frac{\theta}{2} = 90° - \beta + \alpha + \frac{\theta}{2}$, or

$$\theta = \beta - \alpha, \quad . \quad . \quad . \quad . \quad (2)$$

and $\sin\left(\alpha + \frac{\theta}{2}\right) = \sin\left(\alpha + \frac{\beta - \alpha}{2}\right) = \sin\left(\frac{\alpha + \beta}{2}\right)$

substituting in equation (1), we get

$$A D = 2 R \sin \frac{\theta}{2} \frac{\sin\left(\frac{\alpha + \beta}{2}\right)}{\cos \beta}$$

$$= \frac{d \sin\left(\frac{\alpha + \beta}{2}\right)}{\cos \beta} \quad . \quad . \quad . \quad . \quad (3)$$

(very nearly), where d is the horizontal distance between the stations.

Case (2).—By changing the sign of α in the preceding investigation, we obtain the formulæ

$$\theta = \alpha + \beta, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and

$$A D = \frac{d \sin\left(\frac{\beta - \alpha}{2}\right)}{\cos \beta} \quad . \quad . \quad . \quad . \quad (5)$$

which apply when both α and β are angles of depression.

Determination of θ.—From equations (2) and (4) we note, if one of the angular altitudes is given, the other can be determined, since θ is known from the " geodetic distance " and the earth's radius. For, let D be the " geodetic distance " between A and B in miles, then

$$D : \pi R :: \theta : 180 \times 60 \times 60''.$$

Taking $R = 20,889,000$ feet,

$$\theta = \frac{180 \times 60 \times 60 \times 5,280 \times D}{\pi \cdot 20,889,000} \text{ seconds,}$$

$$= 52 \cdot 1364 \cdot D \text{ seconds.} \qquad . \qquad . \qquad . \qquad . \qquad (6)$$

Correction for Height of Signal.—If the height (h) of the signal above the instrument is small, and the angular altitude is small also, it will be sufficiently accurate to take $\tan \varphi = \dfrac{h}{d}$, where φ is the required correction, and d is the "horizontal distance" between the stations. If, however, the angular altitude is large, this value of $\tan \varphi$ must be multiplied by the square of the cosine of the angular altitude.

This rule may be demonstrated as follows:—Let A and C (Fig. 166) be the two stations, C B *the difference between the height of the signal at* C *and the instrument at* A, α and φ the angular altitude and the required correction respectively. Draw B E at right angles to A B, meeting A C produced in E ; complete the figure as shown. Clearly, from the figure (other things being equal), the angle φ diminishes as the angular altitude increases.
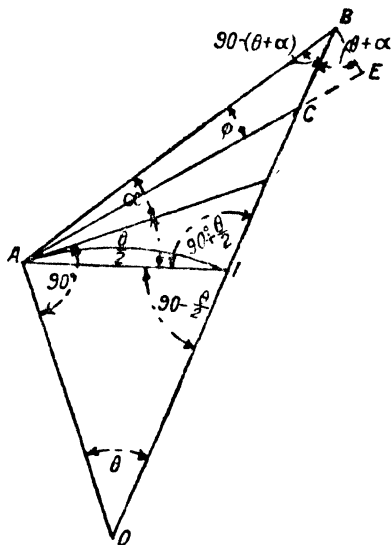
Fig. 166.

The angle A B O $= 180° - \overline{90° + \theta + \alpha}$

$$= 90° - \overline{\theta + \alpha},$$

∴ $\qquad\qquad \angle \, C \, B \, E \,=\, \theta + \alpha,$

hence, $\qquad\qquad B \, E \,=\, B \, C \cos (\theta + \alpha) \text{ (very nearly)}$

$\qquad\qquad\qquad = h \cos (\theta + \alpha) \quad \bullet \qquad \bullet \qquad \bullet \quad (a)$

In triangle A F B, $\angle \, F \, A \, B = \dfrac{\theta}{2} + \alpha,$

$$\angle \, A \, F \, B = 90° + \frac{\theta}{2},$$

∴ $\qquad\qquad \dfrac{A \, B}{A \, F} = \dfrac{\sin \left(90° + \dfrac{\theta}{2} \right)}{\sin (90° - \theta + \alpha)},$

or $\qquad\qquad A \, B = \dfrac{A \, F \cos \dfrac{\theta}{2}}{\cos (\theta + \alpha)} \cdot \quad \bullet \qquad \bullet \qquad \bullet \quad (b)$

Dividing equation (a) by equation (b), we have

$$\tan \varphi = \frac{\dfrac{h \cos (\theta + \alpha)}{d \cos \dfrac{\theta}{2}}}{\cos (\theta + \alpha)}$$

$$= \frac{h \cos^2 (\theta + \alpha)}{d \cos \dfrac{\theta}{2}}. \quad \bullet \qquad \bullet \qquad \bullet \quad (c)$$

Since in all practical cases θ will be a small angle—rarely exceeding a few minutes—we may neglect the term $\cos \dfrac{\theta}{2}$ in equation (c); further, in order that $\cos^2 (\theta + \alpha)$ may differ appreciably from $\cos^2 \alpha$, d must be large; but if d is large, the ratio $\dfrac{h}{d}$ will be small (h will rarely be more than 100 feet), and the error introduced by neglecting the value of θ in equation (c) will be less than the probable error of measurement. Hence, we may write

$$\text{Tan } \varphi = \frac{h}{d} \cos^2 \alpha. \quad \bullet \qquad \bullet \qquad \bullet \qquad \bullet \quad (7)$$

Example 5.—As an example illustrating the small amount of

error introduced by taking $\cos^2(\theta + \alpha) = \cos^2 \alpha$, let $d = 10$ miles. and $\alpha = 10° 20' 15''$. Then

$$\theta = 52\cdot1364 \times 10, \text{ seconds (p. 302)}$$
$$= 8' 41''.$$
$$\cos^2(\theta + \alpha) = \cos^2(8' 41'' + 10° 20' 15'')$$
$$= \cos^2 10° 28' 56''$$
$$= \cdot9669001.$$
$$\cos^2 \alpha = \cos^2 10° 20' 15''$$
$$= \cdot9678001.$$

Assuming $\dfrac{h}{d} = \cdot01$ (a value too great for the assumed distance and angle), and neglecting $\cos^2 \alpha$, $\tan \varphi = \cdot01$; allowing for $\cos^2 \alpha$

$\tan \varphi = \cdot009678$, and allowing for θ,

$\tan \varphi = \cdot009669$.

Taking the correct value as $\cdot009669$, the error in the first case

$$= \frac{(\cdot01 - \cdot009669) \times 100}{\cdot009669}$$
$$= 3\cdot423 \text{ per cent.,}$$

an amount, obviously, too great to be neglected.

The error in the second case $= \dfrac{(\cdot009678 - \cdot009669) \times 100}{\cdot009669}$

$$= \cdot0907 \text{ per cent.,}$$

a quantity clearly negligible in most cases. A large value for the ratio of h to d has been assumed, in order to emphasise the percentage error. If h/d be $\cdot001$ (a more probable value), the errors would be $\cdot3423$ per cent. and $\cdot00907$ per cent. respectively.

The correction (φ) for the height of the signal (sometimes called the "eye and object" correction) is small in all cases, being rarely more than 1°, and in calculating its amount it is convenient and more accurate to use the following rule than to obtain the value of the angle by interpolation from a table of tangents, or sines, in the usual way. The rule in question is as follows:—

$$\text{Log}\left(\frac{\tan \text{ arc}}{\text{arc in seconds}}\right) = \log \text{ of } 1'' \text{ in circular measure}$$

$$= 4\cdot6856 \ (4\cdot685575, \text{ more nearly})$$

or, $\log\left(\dfrac{\sin \text{ arc}}{\text{arc in seconds}}\right) = 4\cdot6856.$

From this we have,

log arc in seconds = log tan arc — 4·6856.

This rule may be verified by reference to any collection of seven-figure tables, and holds to about 2° without sensible error.

Example 6.—Taking h/d equal to ·01, find the value of φ for each of the three cases given in the preceding example.

Case (1). Tan φ = ·01.

$$\begin{aligned}
\textbf{Log } \varphi \text{ in seconds} &= \log \cdot 01 - 4 \cdot 6856 \\
&= 2 \cdot 0000 - 4 \cdot 6856 \\
&= 3 \cdot 3144 \\
&= \log 2{,}062 \cdot 5 \\
\therefore \varphi &= 34' \ 22 \cdot 5''.
\end{aligned}$$

Case (2). Tan φ = ·009678.

$$\begin{aligned}
\text{Log } \varphi &= \log \cdot 009678 - 4 \cdot 6856 \\
&= \bar{3} \cdot 9858 + 5 \cdot 3144 \ \text{(the number } 5 \cdot 3144 \\
&\qquad \text{being the arithmetical complement} \\
&\qquad \text{of } 4 \cdot 6856) \\
&= 3 \cdot 3002 \\
&= \log 1{,}996 \cdot 2
\end{aligned}$$

and $\qquad\qquad \varphi = 33' \ 16 \cdot 2''.$

Case (3). Tan φ = ·009669, and working in a similar way we find that

$$\varphi = 33' \ 14 \cdot 3''.$$

Had we taken the ratio of h to d equal to ·001, the corresponding values of φ would be 3′ 26·25″, 3′ 19·62″, 3′ 19·43″ respectively.

Refraction.—Refraction or bending of the rays of light passing from an object to an observer is caused by the varying density of the atmosphere through which the rays pass. Generally the path of the rays is a flat curve which is concave on its lower side. On rare occasions—when the rays pass close to highly heated ground—the curve is concave on its upper side; but we need not consider this case, since accurate observations under this condition of the atmosphere would be impossible.

The effect of refraction is to cause objects to appear higher than they really are. For example, in Fig. 167, where the path of the ray of light from the stations A and B is shown by the curve A C B, the observer at A sees the signal at B in the direction of the tangent line A D; similarly the observer at B sights the

signal at A in the direction B D and not in the direction B A.
In the former case the observed angle of elevation (α) would be
the angle E A D, and the angle of depression (β) in the latter,
the angle F B D.

As the rays of light in passing between two stations traverse
a very limited portion of the earth's atmosphere, it is assumed
that the curve A C B possesses the same properties throughout,
and that the refraction error at the two stations is the same—
i.e., we assume the angle B A D (Fig. 167) to be equal to the
angle D B A.
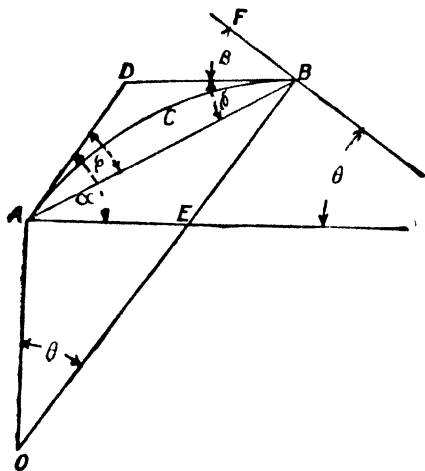
Fig. 167.

Referring to Fig. 167, let ρ = refraction error,

then
$$\rho + \beta = \theta + \alpha - \rho,$$

or
$$\rho = \frac{\theta - (\beta - \alpha)}{2}. \qquad . \qquad . \qquad . \qquad (8)$$

Similarly, when both angles are angles of depression

$$\rho = \frac{\theta - (\beta + \alpha)}{2}. \qquad . \qquad . \qquad . \qquad (9)$$

Thus we have a means of determining ρ when we know the
angles θ, α, and β.

It must be noted that the angles α and β should be determined

as nearly as possible simultaneously, otherwise the refraction may change in the interval.

If angles of elevation are considered positive, angles of depression negative, and the correction for refraction negative, the algebraic sum will be the " corrected angle."

Example 7.   Let $\rho = 8''$.

Observed angle of elevation at A, corrected for instrument

$$= + 2° 15' 42''$$
$$\rho = - 0° 0' 8''$$

Corrected angle $=$ $2° 15' 34''$

Observed angle of depression at B, corrected for instrument

$$= - 2° 40' 9''$$
$$\rho = - 0° 0' 8''$$

Corrected angle $=$ $2° 40' 17''$

Example 8.   Correct the given angular altitude for height of signal and refraction.

Data—given altitude $= 3° 40' 15''$.
Height of signal $= 20·54$ feet.
Height of instrument $= 4·58$ feet.
Refraction $= 38''$.
Log " geodetic distance " $= 4·7635489$.
Height of observer's station $= 1,486·3$ feet.

(a) Find log " horizontal distance."

Log $d = \log g + $ N (p. 295).
Log $1,486·3 =$ $3·17211$
Constant log $=$ $\bar{8}·31785$

Sum $=$ $5·48996$

Corresponding number $= 0·0000309$
Log " geodetic distance " $= 4·7635489$

Log " horizontal distance " $= 4·7635798$

(b) **Find the correction for height of signal.**

Log " horizontal distance " = 4·76358

Constant log (p. 304) = 4·68560

Sum = 9·44918

Arithmetical complement (since both the constant log, and log
d are negative) = 0·55082

2 × log cos 3° 40' = $\overline{1}$·99822

Sum = 0·54904

Height of signal − 20·54 feet.

Height of instrument +   4·58   ,,

Algebraic sum, 15·96 feet.

Log = 1·20303

56·53″, log = 1·75207

Correction for signal = − 0° 0′ 56·53″

Correction for refraction = − 0° 0′ 38·00″

Sum = − 0° 1′ 34·53″

Observed angle = + 3° 40′ 15·00″

Corrected angle =   3° 38′ 40·4″

Example 9.—From the following data determine the height
of B above A, and the refraction at the time of observation :—

Log " geodetic distance " from A to B = 4·6649732

Angular altitude of B at A = 2° 19′ 25″

Angular depression of A at B = 2° 25′ 5″

Height of signal at A = 12·45 feet.

Height of signal at B = 10·64 feet.

Height of A above M.S.L. = 364·32 feet.

Height of instrument at A = 4·62 feet.

Height of instrument at B = 5·43 feet.

Station A

(*a*) To find the logarithm of the horizontal distance.

$$\text{Log } d = \log g + \text{N (p. 295)}.$$

Height of Station A, 364·3, log = 2·561483
Add constant log = $\bar{8}$·317850

Sum = 6·879333
Corresponding natural number, N = 0·00000746
Add log " geodetic distance " = 4·66497320

Log " horizontal distance " = 4·6649807

(*b*) To find the angle (θ) subtended by the line A B.

From p. 302, θ = 52·136 D seconds.
Log θ = log 52·136 + log d — log 5,280.
Log 52·136 = 1·7171377
Log d = 4·6649807

Sum = 6·3821184
Log 5,280 = 3·7226339

456·5, log = 2·6594845

$\therefore \boldsymbol{\theta} = 7' 36''$.

(*c*) **To find the correction for height of signal. Station A.**

$$\text{Tan } \varphi = \frac{h}{d} \cos^2 \alpha \text{ (p. 303)}.$$

Height of signal at B — 10·64 feet.
Height of instrument at A + 4·62 ,,

Algebraic sum = h = — 6·02 feet.

Log 6·02 = 0·77959
2 × log cos 2° 19′ = $\bar{1}$·99929

Sum = 0·77888

$$\text{Log } d = 4 \cdot 66498$$
$$\text{Add log tan } 1'' = 4 \cdot 68560$$

$$\text{Sum} = 9 \cdot 35058$$

$$\text{Arithmetical complement} = 0 \cdot 64942$$
$$\text{To this add} \quad 0 \cdot 77888$$

$$26 \cdot 81, \text{ log} = 1 \cdot 42830$$

$\therefore$ Required correction $= -26 \cdot 81$ seconds.

Station B.

Height of signal at A $+$ 12·45 feet.
Height of instrument at B $-$ 5·43 „

Algebraic sum $+$ 7·02 „

$$\text{Log } 7 \cdot 02 = 0 \cdot 846337$$
$$2 \times \log \cos 2° 25' = \overline{1} \cdot 999227$$

$$\text{Sum,} = 0 \cdot 845564$$

$$\text{Log } d = 4 \cdot 66498$$
$$\text{Add log tan } 1'' = 4 \cdot 68560$$

$$\text{Sum} = 9 \cdot 35058$$

$$\text{Arithmetical complement} = 0 \cdot 64942$$
$$\text{To this add} \quad 0 \cdot 84556$$

$$31 \cdot 26, \text{ log} = 1 \cdot 49498$$

$\therefore$ Required correction $= 31 \cdot 26$ seconds.

(*d*) Correction for refraction.

$$\rho = \frac{\theta - (\beta - \alpha)}{2} \text{ (p. 306, Eq. 8).}$$

Angle of depression of A at B, . 2° 25′ 5″
Add correction for height of signal, 0° 0′ 31″

Corrected depression,   .   .   2° 25′ 36″   2° 25′ 36″

Angle of elevation of B at A, . 2° 19′ 25″
Subtract correction for height of
signal, ... 0° 0′ 27″

Corrected altitude, . . 2° 18′ 58″   2° 18′ 58″

Difference $(\beta - \alpha)$, . . . 0° 6′ 38″
The angle θ is . . . 0° 7′ 36″

$\theta - (\beta - \alpha) = 0° \ 0′ \ 58″$
and $\rho = 0° \ 0′ \ 29″$

Angle of elevation, . . . 2° 18′ 58″
Subtract correction for refraction, 0° 0′ 29″

Corrected altitude $(\alpha) = 2° \ 18′ \ 29″$

Angle of depression, . . . 2° 25′ 36″
Add correction for refraction, . 0° 0′ 29″

Corrected angle of depression (β), 2° 26′ 5″

(*e*) To find the difference of level of the stations.

$$\text{Required difference of level} = d \ \frac{\sin\left(\dfrac{\alpha + \beta}{2}\right)}{\cos \beta} \ \text{(p. 301, Eq. 3)}.$$

$$\alpha = 2° \ 18′ \ 29″$$
$$\beta = 2° \ 26′ \ 5″$$

$$\text{Sum} = 4° \ 44′ \ 34″$$
$$\text{Half-sum} = 2° \ 22′ \ 17″$$

Log d, from (*a*) =   4·6649807

$$\text{Log sin}\left(\frac{\alpha + \beta}{2}\right) = \text{log sin } 2° \ 22′ \ 17″ = \ 8·6167539$$

$$\text{Log sec } \beta = \text{log sec } 2° \ 26′ \ 5″ = 10·0003922$$

Sum = 23·2821268
Log 1,914·81 =   3·2821268

Required difference of level = 1,914·81 feet.

Method when Reciprocal Angles cannot be obtained.—When it is impossible to take reciprocal observations, as in the case

of an inaccessible peak, the correction for refraction cannot be
rigidly determined. In such a case, a value for refraction must
be assumed, and most writers on the subject assume the correction
to be about one-fifteenth of the angle (θ) subtended by the two
stations. It is necessary to add the assumed value to angles of
depression, and deduct it from angles of elevation.

As only one angular altitude can be observed, the other must
be obtained from equations (2) or (4), as the case may be, before
proceeding with the necessary calculations.

Simpler Formulæ for Altitude.— By allowing for curvature
and refraction, as described on p. 122, Part I., and ignoring the
value of the subtended angle, simpler formulæ giving almost
identical results (on short lines) with those derived from equations
(3) and (5) are arrived at. The investigation is as follows :—
Let A and B (Fig. 168) be the two stations, and a the observed
angular altitude of B at A. Complete the figure as shown.
From the triangle A B E, we have

$$\frac{B\,E}{A\,E} = \frac{\sin a}{\sin (90° - a + \theta)},$$

$$\therefore B\,E = \frac{A\,E \sin a}{\cos (a + \theta)}.$$

Now, $A\,E = R_1 \tan \theta$; but this will not differ appreciably
from the horizontal distance (d) between the stations, hence
without sensible error, we may write

$$B\,E = d\,\frac{\sin a}{\cos (a + \theta)}.$$

Again, on short lines, θ will rarely exceed one or two minutes,
and if σ be a fairly small angle, the error introduced by ignoring
θ will only affect the value of the cosine in the fifth place of
decimals ; hence, noting the assumption made, we may write

$$B\,E = d\,.\,\frac{\sin a}{\cos a} = d\,.\,\tan a.$$

To this we must apply the correction for curvature (D E), height
of instrument, refraction, and height of signal (all in linear
measure), to determine the required difference of level. Thus,

B D = $d \tan a$ + height of instrument — height of signal
       + (curvature — refraction)
   = $d \tan a$ + height of instrument — height of signal

$$+ \frac{4}{7}\,\frac{d^2}{5,280^2}. \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (10)$$

Let the observed angle be an angle of depression (β). Then from the triangle A D$_1$ B (Fig. 168), we have

$$\frac{\text{A D}_1}{\text{B D}_1} = \frac{\sin \beta}{\sin (90 - \beta - \theta)},$$

or
$$\text{A D}_1 = \text{B D}_1 \frac{\sin \beta}{\cos (\beta - \theta)}.$$

In this case B D$'$ = R$_1$, tan θ = d (approximately); also, assuming cos ($\beta - \theta$) equal to cos β, we get

$$\text{A D}_1 = d \tan \beta.$$
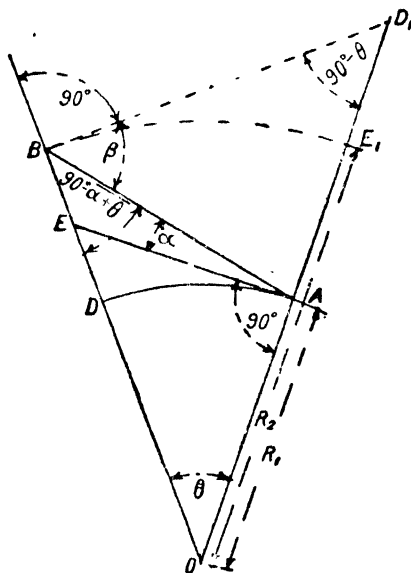
Fig. 168.

The corrections are, as before, with their signs changed ; thus,

$$\text{A E}_1 = d \tan \beta - \text{height of instrument} + \text{height}$$
$$\text{of signal} - \frac{4}{7} \frac{d^2}{5,280^2}. \qquad . \qquad . \qquad . \quad (11)$$

Formulæ (10) and (11) may be written in the general form :—
Difference of level of the stations

$$= d \tan \alpha \pm \text{height of instrument} \mp \text{height of signal} \pm \frac{4}{7} \frac{d^2}{5,280^2}.$$

21

the upper or lower signs being used according as α is an angle of elevation or depression—*i.e.*, according as there is a rise or a fall between the stations.

$$(\textit{Note.}\!-\!\text{Log} \left(\frac{4}{7 \times 5,280^2} \right) = \bar{8} \cdot 3116942.)$$

Example 10.—Find the difference of level of the stations A and B.

Data—Angle of elevation of B at A, . 3° 15′ 20″
        Height of instrument at A, . 4·83 feet.
        Height of signal at B, . . 12·40 ,,
        Height of station A above M.S.L., 228·32 ,,
        Log " geodetic distance," . . 3·8890214.

(1) *By approximate method.*

Log " geodetic distance," . 3·88902
Log tan 3° 15′ 20″, . . 8·75497
                                    ⎯⎯⎯⎯⎯⎯
                Sum, . . . 12·64399

2·64399 = log apparent height = 440·55 ft.

Curvature and refraction.
Twice log " geodetic distance," . 7·77804
To this add constant log, . $\bar{8}$·31169
                                    ⎯⎯⎯⎯⎯⎯⎯
        Sum, . . . 0·08973 = log 1·23, **1·23**

            Height of instrument, . . . **4·83**
                                                    ⎯⎯⎯⎯⎯⎯
            Sum, ... 446·61
            Deduct height of signal, . . 12·40
                                                    ⎯⎯⎯⎯⎯⎯
            Required difference of level, . 434·21

(2) *By " rigorous " method.*

(*a*) Find log " horizontal distance."

Log 228·32, ... 2·35854
Constant log, ... $\bar{8}$·31785
                                ⎯⎯⎯⎯⎯⎯
        Sum, ... 6·67639

Corresponding number, . . 0·0000047
Log " geodetic distance," . . 3·8890214

Sum = log " horizontal distance " = 3·8890261

(b) Find angle subtended by A B.

Log θ = log 52·136 + log d — log 5,280.

Log 52·136, . . . 1·7171377
Log d, ... 3·8890261

Sum, ... 5·6061638
Log 5,280, ... 3·7226339

76·48, log = 1·8835299

$$\therefore \theta = 1'\ 16·48''.$$

(c) Find correction for height of signal.

$$\text{Tan } \varphi = \frac{h}{d} \cos^2 a.$$

Height of signal, . . . — 12·40
Height of instrument, . . + 4·83

Algebraic sum, . . . — 7·57 feet.

Log 7·57, ... 0·87910
2 × log cos 3° 15′, . . . 1·99860

Sum, ... 0·87770

Log d, ... 3·88903
Log tan 1″, ... 4·68560

Sum, ... 8·57463

Arithmetical complement, . . 1·42537
To this add . . . . 0·87770

200·9, log, ... 2·30307

$$\therefore \varphi = 3'\ 21''.$$

(*d*) Find correction for refraction.

$$\text{Required correction} = \frac{\theta}{15}$$
$$= \frac{76}{15}$$
$$= 5 \text{ seconds.}$$

(*e*) Correct the given altitude.

Given altitude	$= 3° 15' 20''$
Deduct correction for signal and refraction	$0° 3' 26''$
Corrected altitude, . .	$3° 11' 54''$

(*f*) Find angle of depression.

From equation (2), $\beta = \theta + \alpha$.
$$\alpha = 3° 11' 54''$$
$$\theta = 0° 1' 16''$$
$$\text{Sum} = \beta = 3° 13' 10''$$

(*g*) Find difference of level of the stations.

$$\alpha = 3° 11' 54''$$
$$\beta = 3° 13' 10''$$
$$\text{Sum} = 6° 25' 4''$$
$$\text{Half-sum} = 3° 12' 32''$$

$$\textbf{Log sin} \left(\frac{\alpha + \beta}{2} \right) = \log \sin 3° 12' 32'' = 8 \cdot 7480035$$

$$\textbf{Log sec } \beta = \log \sec 3° 13' 10'' = 10 \cdot 0006862$$
$$\text{Log } d, \text{ from } (a) = 3 \cdot 8890261$$
$$\text{Sum} = 22 \cdot 6377158$$
$$434 \cdot 225, \log = 2 \cdot 6377158$$

∴. the required difference of level is 434·225 feet, a result in close agreement with that given by the approximate method.

If A and B are stations in a triangulation, their difference of level is also calculated from the angle of depression observed when the instrument is at B, and the mean of the two results

is taken as the difference of level of the two stations. As a further check, the stations may be arranged in circuits, closing on some one station whose level is accurately known.

The levels of all the stations are reduced to the same datum, usually the M.S.L., by simple addition or subtraction. Thus, in the preceding example, the reduced level of the station

$$B = 228{\cdot}32 + 434{\cdot}22$$
$$= 662{\cdot}54 \text{ feet.}$$

EXAMPLES.

1. Find the lengths of the sides E L and L A of the triangle E L A, given that the length of the base is 4,684·5 links, and base angles L A E and A E L 62° 2′ 53″ and 52° 13′ 34″ respectively.   (*Ans.* 4,061·9 and 4,539·2 links.)

2. In the preceding example a check line runs from A to a point J in the side E L, 2,098 links from E. The check line measured 3,777 links. What is the error of measurement, in links per thousand ?   (*Ans.* 1·4.)

3. In the triangle B′ J E, determine the remaining side and angles from the following data :—∠ J B′ E 30° 27′ 20″, sides E J and J B′, 2,098 and 3,652 links respectively.
                   (*Ans.* 4,135·55 links ; 61° 55′ 21″ ; 87° 37′ 19″.)

4. Find the length of the side C D from the given base and base angles :—Base A B, 5,845 links ; ∠ C A B, 67° 30′ ; ∠ D A B, 32° 25′ ; ∠ A B C, 44° 18′ ; ∠ A B D, 72° 41′.
                                        (*Ans.* 3,429 links.)

5. The sides of a triangle are 4,842, 5,434, and 5,240 links. Find its angles.   (*Ans.* 53° 54′ 57″ ; 65° 5′ 19″ ; 60° 59′ 44″.)

6. Determine the length of C D from the given length of base (A B) and angles.   A B, 7,520 feet ; ∠ A D C, 30° 15′ 10″ ; ∠ C D B, 33° 6′ 5″ ; ∠ B C D, 35° 18′ 24″ ; ∠ D C A, 32° 14′ 36″. C and D are on opposite sides of A B.   (*Ans.* 11,704 feet.)

7. A church tower subtends an angle of 15° 20′ at a point 300 yards from its foot. Find the height of the tower.
                                        (*Ans.* 246·8 feet.)

8. The top of a hill subtends an angle of 10° 15′ at a point A. On approaching the hill to a point B, in direct line with its top and the point A, the subtended angle is 14° 25′. If A B is 1,500 yards, find the height of the hill, and the horizontal distance from A of the point observed to.
                   (*Ans.* 914·62 and 5,057·91 yards.)

9. Find the height of the point C above the stations A and B, from the following measurements :—Angle of elevation of C at A, 20° 30' ; angle of depression of B at A, 12° 15' ; angle of elevation of C at B, 35° 40' ; length of A B (measured on the slope), 5,000 feet ; height of instrument, 4 feet. A, B, and C are in the same vertical plane. Neglect the effects of curvature and refraction.   (*Ans.* 4,971·25 ; 6,032·13 feet.)

10. The height of the signal at a distant station is 15·82 feet, the height of the instrument at the observer's station 4·36 feet, and the distance between the stations 8,452 feet. If the angular altitude of the top of the signal is 5° 14' 33", find the " eye and object " correction, and apply it to the observed altitude.

(*Ans.* 5° 9' 55·7".)

11. A base line at a height of 1,342 feet is 10,285 feet long. Find its length when reduced to sea level.

(*Ans.* 10,284·4 feet.)

12. Find the log of the " horizontal distance " of a line whose log " geodetic distance " is 4·8673254, and height 1,342 feet.

(*Ans.* 4·8673533.)

13. Determine the " subtended angle " for a line, the log of whose " geodetic distance " is 4·8673254. Assuming the correction for refraction to be one-fifteenth of the " subtended angle," find the correction for refraction, in the above case.

(*Ans.* 12' 7·5" ; 48·5".)

14. Correct the given altitude for height of signal, and refraction.

Data—Given altitude, .   .   . + 2° 45' 18"
Height of signal, .   .   .   3·45 feet.
Height of instrument, .   .   5·21   ,,
Height of observer's station,   452·81   ,,
Log " geodetic distance,"   .   4·8673254

(*Ans.* 2° 44' 34·4".)

15. From the following data determine the difference of level of A and B, and the refraction at the time of observation.

Log " geodetic distance " between A and B, 4·8976432
Angular depression of B at A,   .   . 0° 6' 47"
Angular depression of A at B,   .   . 0° 3' 54"
Height of signal at A, ... 10·86 feet.
Height of signal at B, ... 12·40   ,,
Height of instrument at A,   .   .   . 5·21   ,,
Height of instrument at B,   .   .   . 4·68   ,,
Height of A above M.S.L.,   .   .   . 385·91   ,,

(*Ans.* — 33·71 feet ; 0' 52".)

16. Find the difference of level of the stations A and B, by the approximate method, and also the reduced level of B.

Data—Angle of elevation of B at A, . 4° 18′ 39″

Height of signal at B, . . . 14·62 feet.

Height of instrument at A, . . 4·92 ,,

Height of station A, . . . 140·43 ,,

Log " geodetic distance," .   3·7469852

(*Ans.* 411·90 feet ; 552·33 feet.)

17. Solve example 2, p. 285, by the precise formula,

$$\cos (\theta + x) = \frac{\cos \theta - \sin h \,.\, \sin h'}{\cos h \,.\, \cos h'},$$

(*Ans.* 60° 01′ 01·507″.)

18. The distance between a satellite station E and the corresponding principal station A is 10 feet ; B C, the side of the triangle A B C, is 2,000 feet long ; the angles A B C and A C B are 60° and 50° respectively, and the angles observed at E are A E B = 90°, B E C = 70° 14′ 45″ ; find the angle B A C.

(*Ans.* 69° 59′ 58″.)

19. Determine the spherical excess in the triangle A B C, and correct the observed angles by Gauss's rule. Length of A B, 158,456·34 feet ; observed angles—

A.	B.	C.
62° 28′ 45″,	55° 40′ 28″,	61° 50′ 48″
44	30	46
47	29	49
46	27	

Take the mean radius of the earth. 20,889,000 feet.

(*Ans.* Spherical excess   4·93″

Corrected angles, A, 62° 28′ 46·39″

B, 55° 40′ 29·39″

C, 61° 50′ 49·15″.)

20. Determine the length of the sides C B and A C of the triangle given in Question 19.

(*Ans.* a = 159,382·88 feet ; b = 148,420·62 feet.)

21. Correct the following observed angles of the polygon A, B, C, D, E, centre O, so that the polygon shall satisfy the three equations of condition :—

Triangle.		Central Angles.	Left-hand Angles	Right-hand Angles.
(1)	A O B	67° 44′ 29″	59° 15′ 32″	53° 00′ 05″
(2)	B O C	74° 40′ 28″	38° 00′ 25″	67° 19′ 13″
(3)	C O D	75° 36′ 18″	66° 05′ 30″	38° 18′ 20″
(4)	D O E	69° 00′ 30″	52° 04′ 42″	58° 54′ 41″
(5)	E O A	72° 58′ 24″	54° 28′ 30″	52° 33′ 19″

22. Solve the polygon in Question 21, given that A B = 10,062·54 feet.

23. In a certain small trigonometrical survey, wooden pegs were driven into the ground exactly 2 feet towards the magnetic north of the stations, and the signals were erected at the true stations. At one station (A) it was desired not to disturb the signal, so the theodolite was set up over the centre of the wooden peg, the instrument was set to magnetic north as zero, and the magnetic bearings of stations B and C observed as 317° 21′ and 21° 17′. By plotting, A B and A C were found to be 1,360 feet and 1,870 feet. Find by an approximate method, to the nearest minute, the value of the angle B A C; also indicate the exact method. (*B.Sc., London*, 1907.)

(*Ans.* (1) 63° 51′ ; (2) see Prob. XIV.)

CHAPTER XII.

SURVEYING WITH THE THEODOLITE AND OTHER INSTRUMENTS.

General Considerations.—In the case of chain surveys (*vide* Chap. III.), we have seen that the outlines of the several figures forming the system of main lines are fixed by the direct measurement of their sides, the figures being proved by the measurement of other lines, which are redundant as regards the outlines of the figures, but are necessary for checking purposes. In surveys of limited extent, conducted by aid of the theodolite, the outlines of the several figures and the relative directions of the main lines are fixed by the measurement of their contained angles. The correct summation of the interior angles of each of the figures gives a sufficient check, thus largely obviating the measurement of tie and check lines. In estate surveys, this is probably the chief advantage obtained by using this instrument; but in more extended surveys, whether intended for purely topographical or for engineering purposes, greater accuracy will result from its use than is possible if the angles are fixed by chained distances. In town work accurate surveying over a considerable extent of ground is impossible without the aid of the theodolite, or some other similar instrument of precision.

Position of Main Lines.—No hard and fast rules can be laid down for conducting a survey with the theodolite, the positions of the main stations, and consequently the system of lines adopted will depend on (*a*) the nature of the ground to be surveyed, (*b*) the positions of the more important surface features, and (*c*) the shape and extent of the ground. Clearly, the system of lines adopted in the case of a railway survey will be very different from that used in the survey of (say) the County of Yorkshire.

Systems of Lines.—The systems of lines will consequently vary from (1) a mere chain of lines forming a closed or unclosed traverse, as in surveying a road or river ; (2) a system of triangles having a common base, useful in surveying the head of a valley

for an impounding reservoir, or the ground for the site of a pro-
posed dock ; (3) a chain of triangles or polygons, as in the case
of the survey of the ground intended for a railway, or as used
in the Survey of India, where chains of polygons, roughly fol-
lowing the directions of the meridians were first worked out,
the chains being afterwards connected by cross figures ; to (4)
a complete network of interlaced triangles, as in the Trigono-
metrical Survey of these islands.

The Number of Angles to be Measured.—As to the number of
angles which should be measured in a particular case, no fixed
rule can be given ; but in all cases sufficient angles must be read
to check the work in a satisfactory manner. For example, in
the case of a simple triangle, although all the quantities apper-
taining to the triangle may be calculated from the measurement
of its base and base angles, yet the work would not be satis-
factory, as no check is given by the data ; but if we measure
the three angles of the triangle we have a check on the angular
measurements, since their sum—if the triangle is small—must
be 180°. If the triangle is large—$i.e.$, greater than 10 miles
side—this equation should hold, $E = A + B + C - 180°$, where
A, B, and C are the measured angles and E is the spherical
excess. E may be calculated as explained on p. 273.

Spherical excess may be ignored on all ordinary surveying
operations, as the correction will, in general, be far smaller than
the probable error of measurement. The spherical excess in the
case of an equilateral triangle of 10 miles side is only 0·568 second.
Clearly, it would be absurd to allow for spherical excess when
using an instrument reading to 10 or even 5 seconds.

Again, if we know two sides and three angles of a four-sided
figure, we can calculate its remaining sides and angle ; here
also no check is given on the result. A check on the measure-
ment of the angles is, however, given if the fourth angle be
measured, since their sum must be 360°.

Similarly, in the case of a polygon, if all its angles be measured,
a check on the accuracy of the measurements is at once given,
since the sum of the interior angles of any polygon is = 180°
(N — 2), where N is the number of sides in the figure.

In surveying with the theodolite, these facts must be con-
stantly kept in mind.

Traversing.—In traversing with the theodolite, the following
three methods (or their combinations) of fixing the relative
directions of the traverse lines are open to us ; the relative
directions may be fixed by determining (1) the **magnetic bearings**

of the lines; (2) their "true bearings"; and (3) the angular deviation of each line relative to the preceding one.

Traversing with the Magnetic Needle.—In the first method the direction of each line is referred to the magnetic meridian, the fore- and back-bearings of each line are determined, the operations following the same general lines as in compass traversing (*vide* p. 72), except that the bearings are read from the North and not the South end of the needle. One face-right and one face-left repetitions of each fore- and back-bearing should be made, and the needle examined before and after the bearings have been read.

Limits of Accuracy in taking Bearings.—Bearings obtained with a prismatic compass cannot be read with accuracy nearer than about ± 15′ of their true value. On a line 10,000 units long, this produces an error in direction of ± 43·63 units, the error in chaining the same length, under good conditions, would be about ± 10 units; hence, with this instrument the errors in direction are greater than the errors of chainage displacement. The displacement of the forward station, due to these errors, is shown in Fig. 169, where B represents the position of the forward station. If the length of the line A B is chained with perfect accuracy, the position of B as given by the compass may be at either of the points C or D; but, as the distance is imperfectly measured, the position of B as given by the measurements may be at E, F, G, or H. All that we can say with regard to the true position of the station is that it lies somewhere within the area E F G H. This area is sometimes called the "area of uncertainty," in the position of the station.
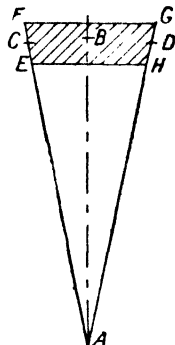
Fig. 169.

With a 5″ theodolite reading to single minutes, the error of bisection should not be more than 30″, but this accuracy, when taking magnetic bearings, is vitiated by the unavoidable error in setting the needle. After many trials, made under varying conditions, it is the author's opinion that the edge needle, as fitted in the trough compass of most 5″ theodolites, cannot be set to within ± 3 to ± 4 minutes of its true position. Consequently, the magnetic bearings obtained with such an instrument will be in error by a like amount. This error will not affect angles obtained by difference of bearings at any one

station, since all bearings read (on one setting) at the station will be equally affected; but it will affect the traverse as a whole. If we accept the smaller value of the above errors, as the probable magnitude of the error in setting, the probable error in direction on a line 10,000 units long will be $\pm 8\cdot727$ units, as against a probable error of ± 10 units in chainage. Obviously, the area of uncertainty in this case is much smaller than in that of the prismatic compass.

Traversing by True Bearings.—The directions of the traverse lines may be fixed relative to the geographical meridian through some one station by a suitable manipulation of the instrument, at each station. This method is independent of the compass needle, and consequently of its errors of setting; the errors arising are those due to (a) inaccurate centering, (b) incorrect bisection, and (c) faulty setting or reading of the verniers or micrometers. All these errors may, with care, be reduced to negligible quantities.

We assume the direction of the meridian at one of the stations to be known either from astronomical observation, or from the known back-bearing of the station, from some distant object. Referring to Fig. 170, let A be the known station on the traverse, and N S the direction of the meridian through A. The instrument is centred over A, the limbs are clamped at zero, and the vertical axis is then clamped with the telescope pointing accurately in the direction of the meridian, the object glass being to the north. Leaving the vertical axis clamped, unclamp the limbs, direct the telescope to the forward signal B, accurately bisecting it with the cross wires. The bearing of A B—*i.e.*, the angle O A B—will be given by the vernier originally at zero. Leaving all the clamps tight, remove the instrument and set it up at B. Unclamp the vertical axis and bisect the rear signal A, using the axis clamp and tangent screw only. Since the verniers have remained clamped during these operations, it is obvious that the angle O A B is equal to the angle S_1 B A, hence, if the limbs are unclamped and the same vernier brought again to zero, the telescope will be placed in a direction parallel to the meridian through A, but pointing south. If the telescope be now turned (clockwise) to bisect the signal at C, the next forward station, the bearing of B C, diminished by 180°, will be given by the vernier, consequently the vernier reading must be increased by 180° to give the bearing of B C. This will be true so long as the line has a westerly bearing; but, if the line has an easterly bearing, the vernier reading must be diminished

by 180°. The displacement of the zero by 180° occurs at the second, fourth, sixth, etc., stations, and care must be exercised at these stations to add 180° to the observed bearing if it is less than 180°, and to subtract a like amount if it is greater than 180°.

Having observed the bearing of B C, the instrument is removed and set up at C, with the vernier limb still clamped. The vertical axis is then unclamped, and the rear station B is bisected. On unclamping the limbs and bringing the *same* vernier to zero, the telescope will again point north, and when the next forward signal D is bisected the vernier will give the true bearing of C D.
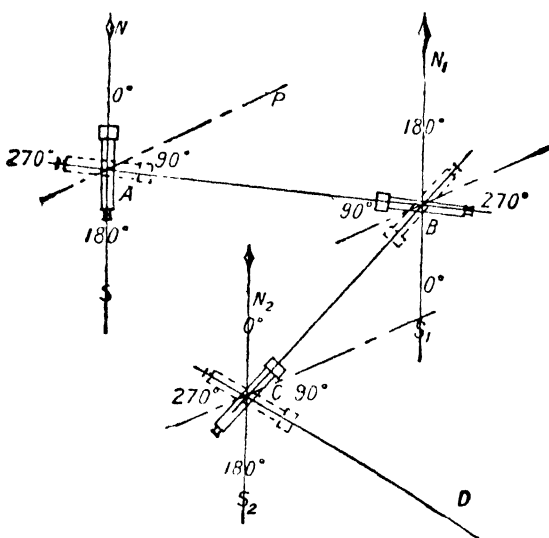
Fig. 170.

Obviously, this method of manipulation carries forward automatically the meridian of A to the successive stations of the traverse, and on returning to A and sighting the referring object in the meridian, the vernier should read zero or 180°, depending on whether the last observation is odd or even in the series.

With an instrument having two verniers (usually marked **A** and B), the correct bearings will be observed if the surveyor uses the A and B verniers alternately ; but, if he is using a three-vernier instrument, the bearings must be read on the *same* vernier and 180° added (or subtracted), as required to the readings

at the even stations. If at a particular station he uses one of the other verniers, the bearing will be in error by 60° or 120°, and this great error can only be discovered in computation or in plotting. This error will not affect the bearings at subsequent stations, provided the proper vernier be used in obtaining them, since the mechanical setting of the instrument is correct, although the recorded bearing is in error, hence no clue is given in computation to the erroneous side. Since the error is large, probably the quickest way to discover the faulty line would be to retake the bearings with a compass.

Geographic Meridian not Necessary.—This system of traversing may also be used in cases where the direction of the geographic meridian is not known. The standard line of direction, or "meridian," may be any line we please, provided its direction is accurately fixed, and the manipulation of the instrument will remain the same. For example, if A and P (Fig. 170) are fixed stations, and P be taken as the referring object at A, the traverse will be fixed relative to the line A P and the "latitude" through A, and would be computed and plotted with reference to these two directions in the usual way.

Traversing by Included Angles.—In this system the angle between each pair of lines meeting at a station is directly measured. It is a matter of indifference whether the interior or exterior angles of the traverse polygon be determined, but in order to prevent confusion, the same mode of procedure in reading the angles must be adhered to throughout the whole traverse. It is obvious that if the surveyor, using a theodolite, works around the traverse in a clock-handed direction, and at each station sights the back signal first, he will determine the *exterior* angles of the figure, while the same mode of procedure will determine the *interior* angles if he works around the polygon anti-clockwise. If the forward signal is used as the referring object at each station, the *interior* and *exterior* angles will be determined in the two cases.

Great Care Necessary.—Great care must be exercised in determining the angles; at least one face-left and one face-right repetitions of each angle should be made, as all the errors are carried forward, and appear in the final closing error on summation.

Checking the Angles.—The accuracy of the angular measurements may be checked by the well-known rule :—" *The sum of the interior angles of the polygon is equal to twice as many right angles as the figure has sides less four right angles.*" If the exterior angles have been measured, the corresponding rule is :—" *The*

sum of the exterior angles of any polygon is equal to twice as many right angles as the figure has sides plus four right angles."

Reduction to Bearings.—For computation purposes, it is necessary to know either the true bearing of some one line of the traverse or the bearing of a line relative to a known object, and from this and the included angles, to compute the bearings of the remaining sides. The method of computing the bearings is as follows:—Let the bearing of A B (Fig. 171) be a, and θ be the exterior angle A B C. The bearing (β_1) of B C is equal to the angle N_1 B C, and this, as shown by the figure, is equal
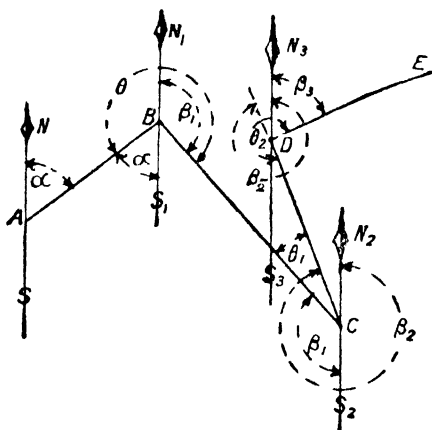
Fig. 171.

to $a + \theta - 180°$. The bearing (β_2) of C D $= \beta_1 + \theta_1 + 180°$
The bearing (β_3) of D E $= \beta_2 - 180°$ — interior angle C D E

$$= \beta_2 - 180° - (360° - \theta_2)$$
$$= \beta_2 + \theta_2 - 180° - 360°.$$

In the first case $a + \theta$ is greater than 180°; in the second case the sum of β_1 and θ_1 is less than 180°; and, in the third case, the difference of $\beta_2 + \theta_2$ and 180° is greater than 360°, hence the rule for deducing the required bearings is:—*Add the known forward bearing to the angle between the line to which it refers and the next in order. If the sum is greater than 180° deduct 180° from it, if less than 180°, add 180° to it. If after deducting 180° from the sum of the angles the remainder is greater than 360°,*

deduct 360 *from it*. The result in each case will be the bearing of the line.

The method of computing the bearings is precisely the same in the case of interior angles. The computed bearings are, of course, whole circle bearings.

The system of traversing by included angles should always be used where a high degree of accuracy is required in angular measurements, as it offers greater facilities for checking the angles by repetition and reiteration than the previous system. It is the system most generally used in surveying the ground intended for a railway, and should be adopted in all cases in which the traverse does not close.

Traversing with the Box Sextant.—As the box sextant is not intended for the taking of bearings, the only system of traversing in which it can be used is that by included angles. Great care is necessary, not only in using the instrument, but also in recording the angles ; with each entry a statement should be made as to whether the record indicates an interior or exterior angle. Since the instrument will not measure angles greater than 120°, in the case of obtuse angles the lines should be prolonged, and the supplementary angles determined ; their mean value, deducted from 180°, gives the required interior angle. If the lines cannot be prolonged as indicated, some object in the interior of the figure must be located and its angular deviations from the lines determined, the required angle is then given by the sum of the measured angles. Beyond the difference in the method of measuring the angles, traversing with the box sextant in nowise differs from that carried out with the theodolite.

Checking the Work as it Proceeds.—The operations involved in traversing with the magnetic needle do not lend themselves to a system of checks as the work proceeds. In fact, really accurate work cannot be done with the needle over an extended course. In addition to the unavoidable errors in setting the needle at each station, it is liable to accidental perturbations owing to the presence of magnetic substances near the stations, and although these accidental disturbances will in many cases be detected by the error in the back-bearings, yet they may pass undiscovered until the traverse is plotted, or computed. Little can be done to check the work as it proceeds beyond care in checking the agreement of the fore- and back-bearings.

In traversing by " true bearings," the meridian at the initial station is automatically transferred parallel to itself, to all the different stations in the series. The slipping of the vertical axis,

or the accidental turning of the wrong tangent screw would vitiate the whole of the work. Hence, frequent checks should be put in as opportunities present themselves. It will frequently happen, owing to bends in the forward course of the traverse or to the nature of the ground, that from a given station one or more stations, several stations forward, are visible; when this occurs, a check bearing (α, Fig. 172) should be read, and on arriving at the corresponding forward station (U) the back-bearing of the check line (U P) should be determined. The agreement of the fore- and back- bearings (± 180°) ensures that no gross error has been made in working between the two stations.
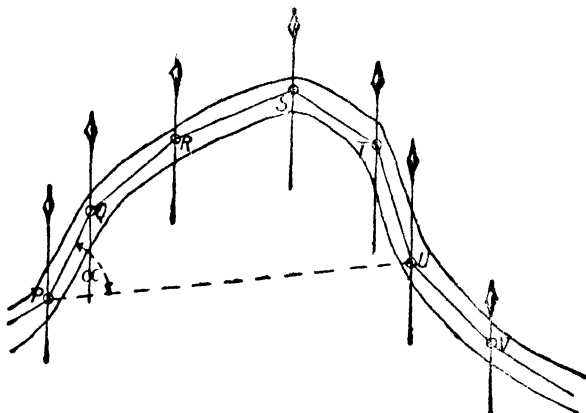
Fig. 172.

Subsidiary Angles and Bearings.—The best method of checking a traverse while the operations are in progress is that afforded by taking the angles or bearings at each salient, of some prominent object outside the course. The object selected may be anything we please, a church tower, the gable end of a house, a prominent tree, or a rock ; but, to be of service, the object must be visible from at least three consecutive stations. For example, in the angle A B C (Fig. 173), suppose there is a prominent tree visible from A, B, and C, and in the angle B C D a church tower visible from B, C, and D. While the instrument is at A, the angle B A P, or, the bearing of A P, is determined. On arriving at the point B, the round of angles Q B C, C B P, P B A, A B Q is taken, the same mode of procedure being followed at each of the subsequent stations. From the known

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length of the first side A B, and the measured angles, the lengths
of the remaining sides are computed, by repeated applications
of the sine rule.   Thus, referring to Fig. 173, we have

$$\frac{B P}{A B} = \frac{\sin \alpha}{\sin (180° - \alpha + \beta)},$$

or

$$B P = \frac{A B \cdot \sin \alpha}{\sin (\alpha + \beta)},$$

and

$$\frac{B C}{B P} = \frac{\sin (180 - \theta + \varphi)}{\sin \varphi},$$

$$\therefore \qquad B C = \frac{B P \cdot \sin (\theta + \varphi)}{\sin \varphi},$$

i.e.,

$$B C = \frac{A B \cdot \sin \alpha \cdot \sin (\theta + \varphi)}{\sin (\alpha + \beta) \cdot \sin \varphi}.$$
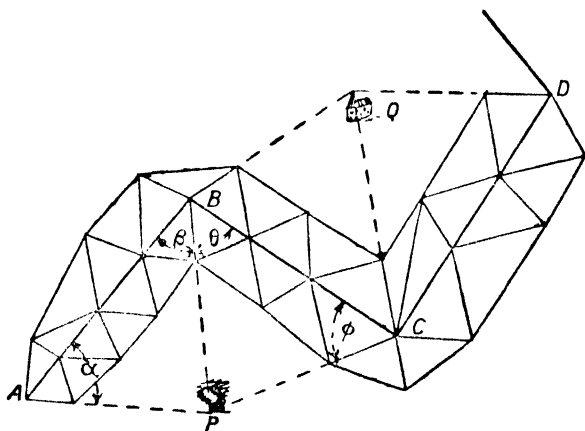
Fig. 173.

B C now forms a new base for the computation of the next side,
which in its turn also becomes a new base, and s) on.   Obviously,
if the computed length of any side agrees with its measured
length, no gross error can have been made in the work up to and
including that side.

By adopting this method, and checking his work day by day,
the surveyor will have the assurance that no gross error has passed
undetected.

Railway Surveys.—The ground to be surveyed for a proposed railway is in all cases a comparatively narrow strip of land, about a quarter to half-a-mile wide, which will seldom be straight for any considerable part of its course. The surveying of such a strip of ground must be done by traversing, and usually the work is carried out by the method of included angles.

The main traverse lines may or may not form the final centre line of the road, but if all the main stations are carefully marked, they will be found very useful in assisting the engineer to its final location, and also for checking purposes.

As the width of the ground to be surveyed is too great for fixing the relative positions of the surface features by direct offsets, triangles and other figures having their bases on the traverse lines (Fig. 173) are built up as required, and from the sides of these figures the detail is fixed in the usual way.

Surveying Rivers.—In surveying the course of a river, the same mode of procedure is followed as in the case of a road, except that the traverse lines will, from necessity, follow one side of the survey, and not run along its centre. If the river is too wide to chain across, traverses must follow both banks, the two traverses being tied to each other either by cross angles or bearings (Fig.
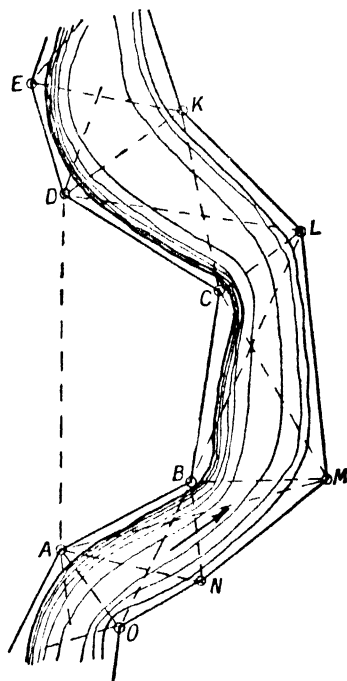
Fig. 174.

174). These cross angles need only be read from one side of the river, as ample checks are provided in each figure. For example, if while the instrument is at C (Fig. 174) the base angles D C L and D C K are measured, and when the instrument is at D the angles L D C and K D C are obtained, the distance C D being known, K L may be obtained either by computation, or by plotting. The agreement of the measured and computed lengths

of K L completely checks the figure D K C L. The same reasoning applies to the other figures. Working in this way, all the angular measurements may be obtained from one bank of the river, and the working time be much shortened in consequence.

Entering Field Notes.—The field notes of distances and offsets are entered in the field book as in chain surveying, the angles or bearings being entered on the pages of the lines to which they refer.

For convenience of the computer, it is usual to collect the angles and distances as they are obtained, and enter them in tabular form on a spare page of the field book.
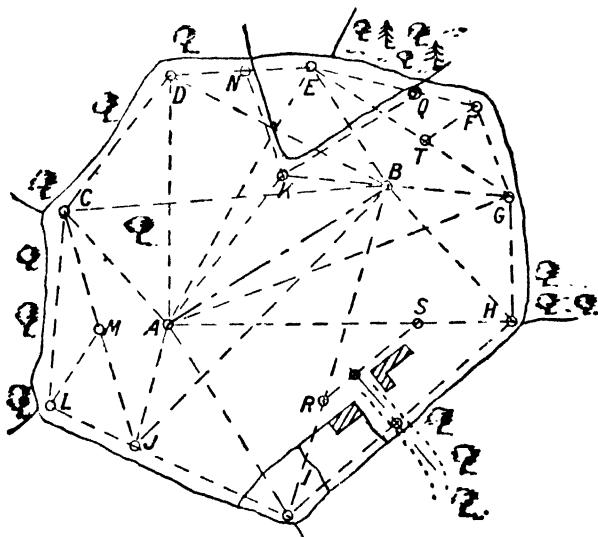
Fig. 175.

Computation and Errors of Closure.—The computation of "latitudes" and "departures," and the various methods of dealing with the errors of closure, have been fully dealt with in Chap. IV., Part I., to which the student is referred.

Surveys from a Single Base Line.—When the ground to be surveyed is of a compact form, and not encumbered by many surface features, the survey may be conveniently carried out from a single base line, to which the distant stations are fixed by measured base angles. In this system the base line A B

(Fig. 175) is laid down on the most suitable stretch of ground, the principal stations C, D, E, etc., are arranged as required for the purpose of filling-in the detail, and the base angles of the triangles A C D, A D B, etc., are obtained with the instrument. The base line should be chained at least twice and the mean length used in computation.

In plotting the survey, the triangles having their bases on the line A B are first laid down, either with the protractor, or from the computed lengths of their sides. The agreement of the scaled (or computed) lengths of the lines (C D, D E, E F, etc.) joining the vertices of the triangles, with their measured lengths, checks the survey.

Where necessary, filling-in lines are placed in position with their extremities on the sides of the triangles, and are fixed by chaining the distances of their extremities from the nearest angular point. Thus, in Fig. 175, the line R S is fixed in position by measuring the distance of R from I on the side I B, and S from II on the side II A.

This system can only be used under very favourable conditions.

Estate Surveys.—On estate surveys, the theodolite is mainly used to determine the interior angles of the triangles or other principal figures forming the groundwork of the survey, and also for any traversing that may be necessary. If all the angles and sides of each figure are determined, tie and check lines may be dispensed with, since the data furnished by each figure is sufficient for the purpose. Additional checks will be furnished by the filling-in lines passing between known stations on the main lines.

The disposition of the main lines will be governed by the relative positions of the principal surface features and the shape of the ground, as in chain surveying, and with the exception of the measurement of tie and check angles, the operations will proceed exactly as in that system.

Town Surveys.—For accurate work in the survey of towns, the theodolite is a necessity. The instrument is used for the purpose of measuring the angles between the survey lines at the corners of all the principal streets, and at the stations where the direction of a main street or road changes. If this be done, and the angles of each principal figure checked as already described, the measurement of the angles at the junction of the main and filling-in lines, will not be necessary. For example, in Fig. 176, if all the angles at the corners A, B, C, and D of the

principal figure A B C D are obtained, the chain measurements on the lines E H, F E, I J, etc., will be sufficient both to fix and check their relative positions.

The intermediate stations (E, F, G, H, Fig. 176) on the main lines should be lined-in with the instrument while it is set over a station at the end of the main line on which they occur.
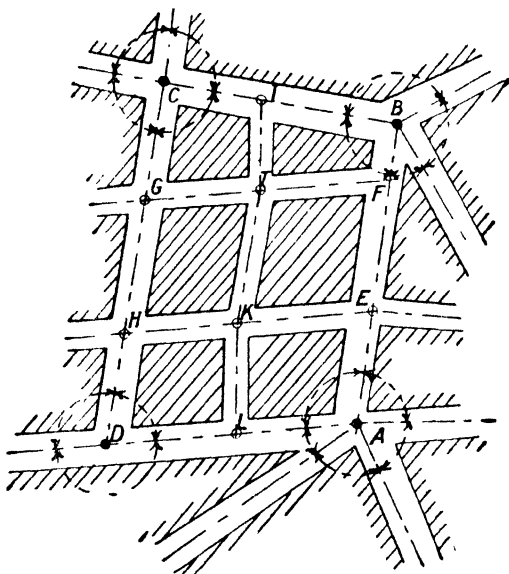
Fig. 176.

Marking Stations.—The principal stations should be carefully marked and have their positions clearly defined, so that they may be readily found. If the station is on the road, an iron spike should be driven into the road, the top of the spike being driven slightly below the road surface, and the exact position of the station marked with a centre punch on the top of the spike. Stations on the pavement are marked by cutting a fine cross in its surface. In all cases the exact position of the station must be fixed by careful measurements to definite points on the nearest permanent objects, and the measurements must be entered on a sketch in the field book

The work of filling-in the detail proceeds as described in Chap. III., Part I., to which the student is referred.

Mine Surveys.—Underground or mine surveying forms an important and special branch of surveying. To deal fully with the subject an entire volume would be required ; consequently, a brief general description only of the principles involved can be given here.

The objects of mine surveying are, (1) to show by plans and sections the relative positions of the underground galleries, main roads, and air passages ; (2) to exhibit those portions of the mine from which the ore or other mineral has been extracted , and (3) to show the positions of the undermined and solid portions of the mine relative to objects on the surface.

In addition to preparing plans and sections satisfying the above conditions, and the solution of many difficult underground problems arising in the pursuit of his craft, the mine surveyor must be able to carry out surface surveys, and set out, when required, the necessary roads to connect the mine-head with the nearest rail or other main line of communication.

Mining Dial.—The principles involved in both surface and underground surveying are the same ; also, the instruments employed are the same, except that a special instrument is used for measuring angles. This instrument—known as the mining dial—differs considerably from the theodolite in its construction, as may be seen in Fig. 177. The construction of the instrument depends somewhat on the maker, but broadly speaking, the general arrangement of the parts is the same in all.

The tripod is constructed either with framed telescopic legs, or with round solid legs provided with a screw joint at the centre of each leg, so that the height of the tripod may be reduced when working in places which are low overhead.

The tripod head is provided with a ball and socket joint (Fig. 178), the friction in which is governed by drawing the two parallel plates closer together by a screw. The stem of the ball is prolonged, the prolongation is turned conical, and fits into a corresponding socket in the base of the instrument. The turning of a clamp screw closes the socket slightly, thereby fixing the instrument to the tripod. For reading horizontal angles, the instrument is provided with two limbs, as in the theodolite, the lower limb is fixed to the socket ; the upper limb carrying the verniers is in the form of a cylindrical box, the sides of which cover the divisions on the lower limb, with the exception of a short length in contact with the vernier. Within the box is centred a delicate needle, whose ends move over a graduated circle, the movements of the needle being observed through the

protecting glass lid.   Centred on diametrically opposite trunnions
fixed to the circular case is a swinging frame carrying a pair
of folding sights of the slit and window pattern.   These sights
partake of the movement of the vernier limb, hence horizontal
angles are read as with the theodolite.

There are various devices for reading vertical angles, the one
shown in Fig. 177 consists of a divided semicircle hinged to the
circular case, a vernier or (in many cases) an index mark, fixed
to the sight frame, is arranged to move over the scale, thus
giving the inclination of the line of sight.

The instrument is levelled by swivelling it about the ball
and socket joint until the bubbles of two small spirit levels

Fig. 178.   Fig. 177.

(placed below the needle and viewed through the glass lid) are
in the middle of their respective runs.

Centering is done in the usual way by means of a plumb-bob
suspended from the centre of the tripod head.

Sighting objects consist of lamps or candles, either supported
on a tripod, centred over the distant station, or suspended from
cross byats of timber, in the desired position.

Mining dials are usually constructed to read by verniers to
three minutes of arc, both on the horizontal and vertical circles,
if no vernier is fitted to the vertical circle the reading is either
degrees or half-degrees.

Fixing Underground to Surface Operations.—One of the first problems which confronts the mine surveyor in commencing the survey of a mine is the accurate co-relation of his underground and surface operations. To solve this problem it is necessary to place underground a line whose position and direction relative to some known line on the surface is accurately fixed. When this has been done, the underground line becomes a base to which the directions of all the other lines in the survey are referred. It is convenient to have the direction of the base line that of the meridian, but this is not necessary, any direction will suffice provided it is clearly marked out on the surface, as its true bearing, and consequently, the true bearings of all the lines tied to it, may be obtained at any convenient time.

The method adopted in transferring the surface line underground will depend on whether the entrance to the mine is by an adit (*i.e.*, an incline leading into the mine) or by a vertical shaft. In either case no special difficulty arises if the needle may be used, since in the first case the surface line may be fixed to the underground line by a compass traverse along the adit, and in the second case, all that would be necessary would be to take the bearing of the underground line from a point at the bottom of the shaft, vertically under a fixed point at the surface ; but in such an important operation it would not be wise to rely solely on the magnetic needle.

Discarding the use of the needle, a traverse by the system of included angles may be run along the adit, and where a second adit exists in underground communication with the first, it should be taken advantage of to close the traverse. If the traverse cannot be closed, it must be repeated until no doubt exists as to its accuracy.

When the entrance to the mine is by one vertical shaft the problem is more difficult. One method which has been much used is as follows :—Planks are fixed across the top of the shaft, near its sides, and the direction of the surface line is accurately marked on the planks by aid of the theodolite or the mining dial. Fine wires passing down the shaft are then suspended from the planks in accurate contact with the marks on them. The lower end of each wire carries a heavy weight to pull the wire straight and ensure its verticality, and to still the vibrations of the weight it is placed, in free suspension, in a bucket of water. After suspending the wires, they are examined throughout their whole length, to see that no projecting object interferes with their free suspension.

The surface line is now defined at the bottom of the shaft by the line joining the centres of the two wires, and its position may be fixed by placing the instrument accurately in line with the lower ends of the wires and setting the line on two permanent marks, over or under which the instrument may be placed as required.

The line joining the suspended wires may be connected to the underground survey by a triangle in the manner illustrated by Fig. 179, which shows the underground arrangement. A and B, are the suspended wires, C E a line in the cross cut. By measuring the three sides of the triangle A B C, its angles may be calculated and the point C fixed, the accuracy of the work may be checked by measuring the angle A C B. By measuring the angles B C E and E C A, the angle between C E and A B may be obtained, thus fixing the point C and the line C E to the surface line.
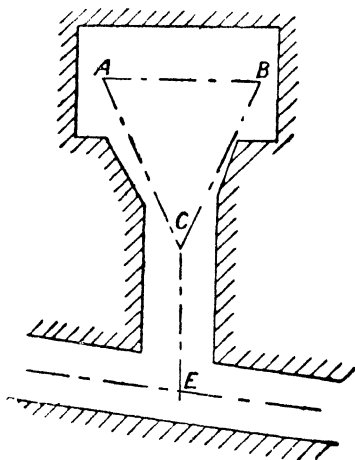
Fig. 179.

Where two shafts exist in underground communication, a plumb-line may be suspended at a known point in each shaft and the two points connected by underground and surface traverses, which check each other. The direction of the line joining the points, relative to the sides of the underground traverse being known, any side and station on it may be used as a line of reference and point of departure respectively.

The accurate solution of this problem is by no means easy in practice, as, in addition to the physical discomfort caused by working in cramped positions, in semi-darkness, the surveyor is confronted with the further difficulty of observing to a slowly oscillating line, as a plumb line suspended in a deep shaft, is never completely at rest.

While the wires are in position the line may be set out at all the different levels of the mine, in the above manner.

After the underground line has been fixed, a convenient point on it is selected as the point of departure, for the subsequent

operations. The position of this point may be clearly defined by measurements to one or both of the suspended wires.

Having obtained his standard direction and point of departure, the surveyor may carry out his operations by any method he pleases ; but, as these operations are, of course, confined to the narrow passages of the mine, which generally give only infrequent and irregular cross communications, they must be in the nature of a traverse. Mine traverses are conducted on both the compass and included angle systems, the latter system, however, has now largely superseded the former, owing to the increasing use of electrical machinery, in mining operations.

The method of carrying out the survey is much the same as that of a surface survey ; angles are read at each station to all the forward stations directly connected with it, distances are chained and offsets taken in the usual way to all the objects in the neighbourhood of the chain line. The inclinations of inclined lines are obtained with the dial, the lines are chained on the slope, and are afterwards reduced to the horizontal.

The survey may be plotted with the protractor or by the system of " latitudes " and " departures," the latter being that most generally used.

Transferring the Levels Underground.—When the entrance to the mine is by an adit, the levels may be obtained with the spirit level in the usual way, by working along the adit ; but if the adit is very steep it would be quicker to measure its inclination and length, and obtain the required difference of level by calculation.

For the method of transferring the levels down a vertical shaft, see Chap. XV.

Use of Theodolite in Marine Surveying.—In surveying the outline of a rocky coast, where the application of ordinary methods would be both dangerous and tedious, or in fixing the positions of points on sub-surface contours, obtained by soundings, the theodolite is very useful.

On many parts of a rocky coast, the ground shelves steeply from a considerable altitude to the sea, and the position of the water edge cannot be fixed, in the usual way by distances and offsets, to the tide mark. A convenient way of overcoming this difficulty is obtained by the use of two instruments placed over stations (A, B, Fig. 180) commanding the outline of the coast between them. The instruments having been placed in position, an assistant (in a boat or landed at the foot of the cliff) erects a signal at the selected point (D, Fig. 180), and the observers

at the instruments determine its angular deviation from the
base line, A B. These operations are repeated at a sufficient
number of points to fix the coast outline. If several points are
invisible from a station, they may be left marked until one
instrument is moved forward to its next station, when their
positions may be fixed relative to the new base line. For example,
if the points 1, 2, and 3 (Fig. 180) are invisible at A, but visible
at B and C, their positions may be fixed by moving the instru-

Fig. 180.

ment at A to C and determining their angular deviations from
the extremities of the new base, C B.

The positions of the points P, Q, R, S, etc., on the sub-surface
contour are fixed in a similar manner. The movements of the
boat, carrying the assistants engaged in taking soundings, are
followed by the observers at the instruments, and when a new
point on the contour is determined, a signal is erected in the
boat and observed to from the instruments.

This method is also useful in fixing the positions of soundings taken on a sub-surface line of section across a river or harbour, and in fixing the positions of buoys, lightships, and other floating marks.

In taking soundings, the rise or fall of the tide must be allowed for. This may be done by noting the time at which each sounding is made; the height of the tide at that instant is then obtained from records of observations on a tide gauge made by an assistant on shore.

The observer's stations A, B, C, etc., may be stations fixed either by a traverse of the coast, or by a triangulation.

Trigonometrical or Geodetic Surveying.— The methods employed and the instruments used, in this the highest form of surveying, are warranted only when the extent of the ground to be surveyed is very large. Trigonometrical surveying is undertaken by the Government of the Country concerned, and not by private citizens, as the work is, of necessity, very costly, and requires a great lapse of time for its completion.

A brief description only of the principles involved, and the methods employed, is here given.

General Principles.—A survey of this description, whether intended for the measurement of a parallel of latitude, an arc of a meridian, or for the formation of a territorial map, must commence from an accurately measured base line.

Principal Triangles.—From the extremities of the base line, angular observations are made to other points previously fixed on as survey stations, the lines joining the extremities of the base to these stations become new base lines, from the extremities of which observations are made to other selected points; the lines joining these points become in turn new base lines, the process being repeated until the country is covered with a net-work of principal triangles.

Bases of Verification.—As the work extends, new base lines are laid down and carefully measured, and the triangles in the neighbourhood of these bases are caused to close on them. From the data furnished by the measurement of the original base and the chain of angular measurements, the lengths of the new bases are computed and compared with the corresponding measured lengths, thus giving a rigorous check on the principal operations.

Secondary Triangles.—Within the principal triangles other points are chosen to form a secondary series of triangles. The positions of these secondary stations are fixed in computation from angular measurements obtained with smaller instruments,

or are fixed by angular measurements and chained distances. If desired, a secondary series of secondary triangles may be laid down within the first, and from the sides of these figures the interior detail is filled-in by chain measurements and offsets, or by partial measurement and sketching with the aid of some portable instrument.

Length of Base Line.—The length of the base line bears no fixed relation to the total length of the survey. Six principal bases were used in the Ordnance Survey of these islands ; their lengths vary from 5 to 8 miles.

In India ten bases were used, nine of these vary from 6·4 to 7·8 miles, the remaining base, at Cape Comorin, being 1·7 miles.

There are several bases of about a mile and a half long, on the Spanish Survey, the principal base, near Madrid, having a length of 9·1 miles. The principal base on the French Survey, near Ensisheim, has a length of about 11·8 miles.

Selection of Ground and Demarcation of Base Line.—The ground selected for a base line should be fairly level and free from obstacles throughout the whole of its course, the extremities should be mutually visible, and should command views of selected objects intended to form stations for the purpose of gradually enlarging the base, by triangulation. The line is first approximately set out, and an accurate section of the ground is obtained by spirit levelling.

An accurate alignment then follows, instruments are set up at one or both ends of the base for the purpose of lining-in stout pickets, which are driven into the ground at intervals to delineate the base line clearly. Two or more points are selected in positions suitable for dividing the line into segments, and are accurately ranged in position. The segments of the base need not be in the same straight line, if the angle between them is accurately determined, but the measurements on each segment must be made in the vertical plane containing its extremities.

The terminal points—and sometimes also the points selected for dividing the base into segments—are preserved in various ways, in some cases by a fine cross cut in a piece of brass, cemented in a block of stone, which is surrounded by brickwork.

Measuring the Base Line.

Wooden Rods.—The instruments used in the measurement of base lines have varied considerably. The earliest bases were measured with wooden butt rods, the contact surfaces being

formed by metal studs, driven into their ends. This apparatus was used by Maupertuis to measure a base line of 8·9 miles on the frozen surface of the River Tornea, in Lapland, in 1736 ; also in the measurement of the base line on Hounslow Heath, the base having a length of 5·2 miles. The measurement of the latter was considered unsatisfactory owing to the alteration in length of the rods due to moisture, and the base was remeasured with glass tubes ; this measurement was also considered un-satisfactory, and the base was again measured with a steel chain, in 1791.

Steel Chains.—Steel chains were used in the measurement of five of the six principal bases on the Ordnance Survey of this country. These chains were 100 feet long, constructed with forty links half an inch square in cross-section. When in use the chain was supported in troughs, and stretched with a load of 28 lbs. ; each end of the chain carried a flat metal plate, on which a fine line was engraved, the distance between the lines giving the length of the chain at the standard temperature. The length was transferred to the ground by stout pickets carrying on their upper ends a plate also engraved with a fine line, which was adjusted to position by a screw into accurate alignment with the corresponding line on the end of the chain. Alterations in length produced by change of temperature were allowed for, from readings given by thermometers in contact with the chain, and the actual length of the chain was tested from time to time, by reference to a similar chain, kept as a standard.

One of the chief difficulties encountered in base line measure-ments is due to the alteration in length of the measuring instru-ment, caused by changes of temperature, and various attempts have been made to embody in the apparatus either a ready means of measuring the temperature, or an arrangement for automatically eliminating its effects.

Borda's Rod.—The earliest measuring rod designed for the former purpose was constructed by a French artificer of the name of Borda, and the rod was used for base line measurements by Delambre, in 1792. The rod consists of a strip of platinum overlaid by, and in free contact with, a strip of copper, the former being about 6 inches longer than the latter. The strips, $\frac{1}{2}''$ broad by $\frac{1}{12}''$ thick, are fixed together and to a stout beam of wood at one end only, being free to expand and contract for the remainder of their length. The changes of temperature are inferred from readings given by a scale engraved on the edge of the copper strip, and a corresponding vernier on the platinum

strip. One free end of the platinum strip projects over the end of the supporting beam and is formed into a tongue, the other end a corresponding groove is cut for the reception of the projecting tongue on the next bar. The tongue and groove are provided with a scale and vernier for the purpose of setting one rod in accurate corelation with the preceding rod, in the order of measurement. Microscopes were used in reading the two verniers.

During the measurement of a base line each rod was supported on an iron tripod fitted with levelling screws, the inclination of the rod was determined by a vertical arc of 2 feet radius, fitted with a spirit level, and applied in reversed positions.

The length of the rod on measuring is two toises, one toise being equal to 2·13151116 standard yards.*

Struve's Bars.—Seven of the ten bases on the Russian Survey were measured with the apparatus designed by the Russian astronomer, F. W. Struve. He used four bars of wrought iron, each two toises in length, one end of each bar terminates in a short steel cylinder with a convex end, the cylinder being coaxial with the bar. At the other end is a contact lever centred on a horizontal pin at the end of the bar, the contact arm being bent upwards; the upper end of the lever, carrying an index line, moves over a graduated arc; the normal length of the bar is indicated when the index line is in contact with a certain division on the scale, but its length is known for each scale division.

To protect the bars from injury, they are encased in wooden boxes, the ends only being exposed, and as a further protection from changes of temperature, the bars are covered with raw cotton, kept in place by wrappings of cotton cloth. Alterations in temperature were obtained by two thermometers, the bulbs of which were let into the body of each bar.

When in use, the ends of the bars were brought into contact, and the contact maintained by the action of a spring acting on the lever, the boxes containing the bars were supported on tripods and trestles, and were raised or lowered as required by means of screws.

The probable errors in the measurement of the seven bases on which the apparatus was used vary from $\pm 0\cdot73\,\mu$ to $\pm 0\cdot91\,\mu,$* μ being one-millionth of the measured length.

Bessel's Rod.—Bessel's arrangement consists of two superimposed rods, one of iron and the other of zinc. The zinc rod

being fixed to the iron rod at one end, terminates in horizontal knife edges, while the iron rod carries at one end two vertical knife edges, the outer knife edge fixing the length of the bar, and the inner forming with the horizontal knife edge of the zinc a small distance which served as a metallic thermomenter. This interval is measured by a small glass wedge, 4″ long, 0·07″ thick at the thin, and 0·17″ at the thick end, and on its face 120 divisions 0·03″ apart are engraved. The component bars or rods are supported on a deep iron bar carried by seven pairs of rollers, the whole being protected by a wooden case, from which the ends of the bars project.   For setting the bar in contact with the preceding bar in the measurement, a small horizontal movement of the bar relative to its protecting case is obtained by a slow motion screw.

In addition to its use on the Prussian bases, this apparatus was used in the measurement of the Belgian bases near Ostend and Beverloo, by General Nurenburger, in 1852-53. The computed mean error on the former, 2,488 metres, is $\pm$ 0·45 μ, and on the latter, 2,300 metres, $\pm$ 0·59 μ.[*]

Colby's Apparatus.—In Colonel Colby's apparatus we have an attempt to arrive at a measuring rod whose length is invariable.

Starting from the known fact that the coefficient of expansion of iron is to the coefficient of expansion of brass as 3 is to 5, Colonel Colby arranged two bars ($1\frac{1}{2}$″ by $\frac{5}{8}$″), one of brass and the other of iron, parallel to each other, the distance between the bars being fixed by two ferrules, $1\frac{1}{4}$″ long, through which pass the rivets for uniting the bars. Pinned to each extremity of the bars, but not rigidly fixed thereto, is a short lever (A B C, Fig. 181), and near the end of each lever is a small platinum **dot,** so placed that

$$\frac{A\ B}{A\ C} = \frac{\text{coefficient of expansion of iron}}{\text{coefficient of expansion of brass}}$$

$$= \frac{3}{5}.$$

If the bars expand owing to a rise in temperature and C moves to D, then the point B will move to E such that B E : C D :: 3 : 5. Hence, owing to the dimensions of the lever, the line D E produced will pass through the point A, or the position of A is invariable.

The length of the bar is defined by the distance between the platinum dots, and is 10 feet.

The bars resting on two rollers are encased in a strong wooden box, longitudinal motion being prevented by a pin passing up between the ferrules F (Fig. 181), the ends of each bar project from the protecting box.

In measuring, the boxes carrying the bars were supported on trestles and tripods provided with adjusting screws, the bars were adjusted so that the interval between the platinum dots on two consecutive bars was 6 inches, this distance being measured by compensated microscopes, so arranged that the lines of collimation of the two vertical microscopes coincided with the compensated points of two short bars, the length of which (6'') formed part of the measurement, and gave the distance between two consecutive measuring bars.

The weak point in the construction of these bars lies in the practical difficulty met with in *making the bars absorb and reject heat at the same rate*, for it is obvious that if either bar gains or loses heat at a quicker rate than the other, the length of the
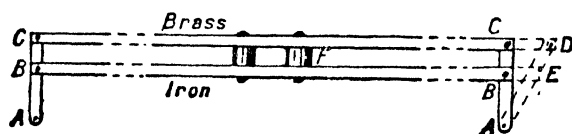
Fig. 181.

measuring bar will no longer be invariable. The difficulty is to a great extent obviated by clouding and varnishing the bars, until by trial their rates of heating and cooling are the same.

Two bases have been measured in this country with this apparatus; one at Lough Foyle, in Londonderry, the other on Salisbury Plain. The apparatus has also been used in the measurement of ten bases in India; but owing to the detection of inaccuracies, due to the apparatus, instead of measuring a base of the usual length at Cape Comorin, a base of about one-fourth the usual length was selected, and measured four times. From these measurements, it appears that the probable error of measurement is about $\pm 1.5 \mu$.*   It should, however, be noted that the compensation principle was not fully relied on during the measurements, the temperatures of the measuring bars were regularly observed, by aid of two thermometers.

* Clarke's " Geodesy."

Bache's Apparatus.—Professor Bache, in 1845, devised an apparatus which combined the principle of Borda's rod, the contact lever of Struve, and Colby's compensation lever. The apparatus was used on the United States Coast Survey. The arrangement of the component parts of the apparatus is shown in Fig. 182. The component bars are placed edgewise, with the iron above and the brass below, and are firmly united together at one end. The iron bar is supported on small rollers, which are fastened to it, and run on the brass bar. At the free end of the bars is a lever of compensation (L), pivoted on the lower bar ; the lever is provided with two knife edges A and B, which are pressed into contact with prepared surfaces on the upper bar and the central slot in a sliding rod respectively ; the pressure of contact being maintained by the action of the spring
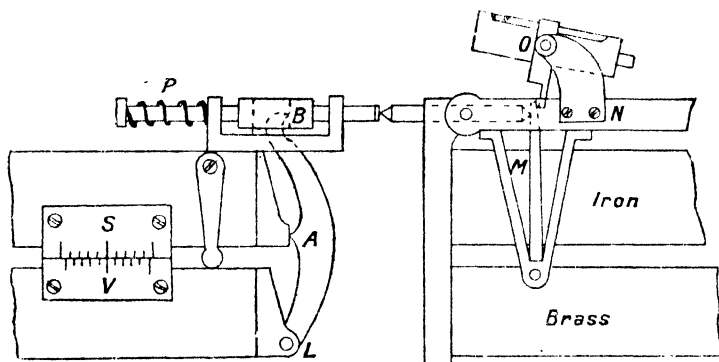
Fig. 182.

P. The end of the sliding rod carries an agate plane for contact with the next bar.

A metallic thermometer is formed by the scale S, and the vernier V fixed to the bars as shown.

At the end, where the bars are united, there is a corresponding sliding rod terminating in blunt horizontal knife edges, the outer knife edge abuts on the agate plane of the preceding bar, and the other abuts on the lever M, which has its fulcrum on and below the sector arm N. The upper end of this lever comes in contact with the projection on the spirit level O, which turns on centres, and is thrown out of balance by a small weight. When the desired pressure of contact is obtained, the bubble of the spirit level comes to the middle of its run ; this arrangement ensures

that the pressure of contact between any two bars is always the same.

The arm N is connected to a sector for indicating the inclination of the bar in measuring.

Each compound bar rests on rollers, in stirrups, suspended in the interior of a spar-shaped double tin tube, which is suitably strengthened by longitudinal and cross diaphragms, the ends of the bars projecting from the closed ends of the tube.

There are two measuring bars in the apparatus, and the length of each bar is 6 metres.

Eight bases have been measured with this apparatus, the last of which, at Atalanta, in Georgia, was measured three times, twice in winter and once in summer, the range of temperature varying from 18° F. to 107° F. The probable error in the measurement is $\pm 1 \cdot 76 \mu$; on the other seven bases the probable errors vary from $\pm 1 \cdot 8 \mu$ to $\pm 2 \cdot 4 \mu$.*

Measurement of Base Lines with Steel Bands.—For trigonometrical surveys of moderate extent, the elaborate and expensive base measuring apparatus described on the preceding pages is not necessary; careful measurement with a steel band will suffice. With ordinary care, keeping the band as nearly horizontal as possible by hand, the ends of the band being marked with a plumb-bob on uneven ground, and the chainman keeping the pull on the band as nearly constant as can be estimated, an accuracy of about 1 in 5,000 can be obtained. An accuracy of 1 in 50,000 is obtainable by regulating the pull on the band by spring balances, stretching the band over accurately aligned pegs, and allowing for the slope and temperature of the band. Still greater accuracy may be obtained by suitably supporting the band on posts, and in addition to the above allowances, introducing corrections for elongation due to pull, and shortening due to the sag of the band between the points of support.

The most accurate results will be obtained on cloudy windless days, when the variation in temperature is small.

" Konstat " Steel Bands.—Steel band tapes intended for the measurement of base lines are now made of " Konstat " steel, the highest grade of nickel steel originally introduced as " Invar," and which contains 36 per cent. of nickel. This steel has the smallest known rate of expansion of any metal or alloy, its coefficient of expansion being only 0·0000005 per degree Fahrenheit. The use of this steel for measuring tapes reduces the errors in steel band measurements caused by the uncertainty of tape

* Clarke's " Geodesy."

temperatures. The tapes are usually either 100 feet or 30 metres long, but 300 feet and 100 metre lengths are often used, and when intended for base line measurements they are made a few
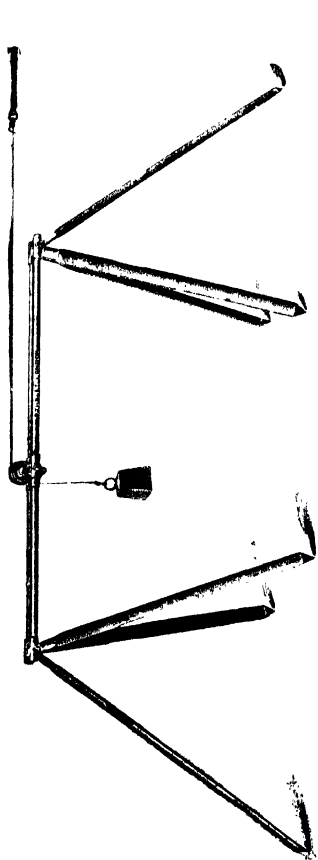
Fig. 183.
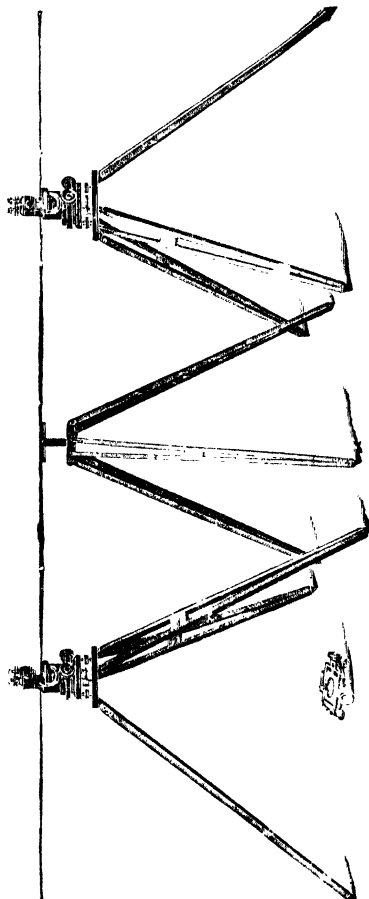
Fig. 184.

feet longer than these measurements, so that the end rings are clear of the reading lines.

The base line apparatus illustrated in Figs. 183 and 184 consists of a straining support (Fig. 183), one of which is placed at each end of the tape, two reading microscopes (Fig. 184),

and a number of supporting tripods (Fig. 184), which are placed under the tape at intervals of about 10 feet to reduce the sag. The straining weights are connected by silk cords (passing over pulleys turning on ball bearings) to the end rings of the measuring band. The pulley carriages are adjustable along the two supporting steel bars for distance, and vertically for height, in the final adjustment of the band. The reading microscopes are supported on rigid tripod stands, and are provided with levels and levelling screws, traverse screw motions and movement in azimuth, and also with aligning telescopes. The microscope is rigidly fixed over a small table, on which the tape rests ; the coincidence of the line of collimation and a certain division on the tape is made by the traversing screw. In exact collimation with the microscope is a plumbing telescope, and this sets the distance on a transferring apparatus (Fig. 184), which rests in contact with the metal plate either on the top of a marking post or on the ground.

The Transferring Apparatus.—This consists of a plate fitted with levels, levelling screws, and traversing screws. At the centre of the plate, with its axis guided rigidly at right angles thereto, is a spring centre punch. The top of the centre punch has a small disc of platinum let into a recess, and on this disc a fine cross is cut, the centre of the cross being in the axis of the centre punch. When the cross exactly coincides with the cross wires of the plumbing telescope, a light blow from a mallet on the punch transfers the mark accurately to the object intended to receive it.

The line to be measured is first marked out with short oak posts, which are placed at distances apart corresponding to the length of the Konstat tape used. These posts are ranged in line with a theodolite, and are set out with an ordinary steel band. A piece of zinc is fixed by screws to the top of each post for the reception of the distance mark. The measuring band is then placed in position, with its ends over the straining supports placed clear of the terminal stations and the first intermediate post. No. 1 reading microscope is then adjusted in position over the permanent plate, No. 2 microscope is adjusted over the first intermediate post, and the two microscopes are aligned upon each other by their aligning telescopes. The intermediate stands are then placed in position and adjusted both for height and line, and the tape is moved lengthwise over the pulleys until the starting division is nearly bisected by No. 1 microscope ; an exact bisection of the mark is then obtained by the traverse

screw of the microscope. The transferring apparatus is then placed on the metal plate of the permanent station, and is adjusted in position by its transverse screws until the cross on the top of the centre punch is exactly bisected by the webs of the plumbing telescope; the centre punch is then struck, and the mark thus made on the metal plate has a line (or lines at right angles) engraved through it. No. 2 microscope is, meanwhile, adjusted over the end division on the tape, and the corresponding distance mark is made on the zinc plate on the top of the first marking post, as given by the plumbing telescope of this microscope. No. 2 microscope is then left undisturbed. No. 1 microscope is set up over the second marking post and aligned with No. 2 microscope, the band (with its straining supports and intermediate stands) is moved forward and set in both level and line, the rear end division is brought into exact coincidence with the webs of No. 2 microscope by shifting the band lengthwise, and No. 1 microscope is adjusted until the forward end division on the tape is exactly bisected by the sighting webs. The distance mark is next transferred to the top of the second marking post by sighting in with the plumbing telescope of No. 1 microscope. No. 2 microscope is now moved forward, No. 1 being left undisturbed, and the operations are repeated until the end of the line is reached; the last centre punch mark is made on the metal plate at the end station, and lines are engraved through it.

For the purpose of allowing for change in length of the tape due to change of temperature, a few divisions either 0·01″ or 0·1 mm. apart are engraved on the tape at each side of one end division. The temperature of the band is obtained from thermometers generally one suspended on the tape at each end.

Slight differences of level are allowed for in setting up the tripod stands so that the tape remains level; if the change of level is too great for this, the inclination of the tape must be obtained and the measured length reduced to the horizontal.

In the method employed by Mr. O. B. Wheeler, U.S. Assistant Engineer on the Missouri River Survey,* posts with their sides in the line were driven into the ground at distances varying from 20 to 100 feet. From these posts the band was supported on hooks suspended from nails driven into the sides of the posts; the nails were set at a uniform gradient, or their levels determined. At the ends of each band length, marking posts were driven in the line with their tops 2 feet above the ground. On

* Report of Missouri River Commission.

the tops of these posts zinc strips about $1\frac{1}{2}$ inches wide were nailed, and on these strips the extremities of the band were marked with a fine steel point. Behind the rear and ahead of the forward marking posts straining posts were fixed. The latter consisted of three posts suitably stayed and prepared for receiving the straining apparatus.

Straining Apparatus.—This consists of a block A B C (Fig. 185) turning on a knife edge at B. The end of the band is attached by a chain to the block at the point C, and from the point A a weight W is suspended. The support for the knife edge B is carried on a slide actuated by the screw F. When correctly set the bubble of the spirit level D is in the middle of its run, and the tension (P) on the band is equal to W. In order that P may be equal to W, the arm B C must be such that

$$W . B C = W . A B + w . B H,$$

or, $$B C = A B + \frac{w}{W} B H,$$

where w is the weight of the block A B C and B H is the horizontal

Fig. 185.

distance of the centre of gravity of the block from the knife edge B.
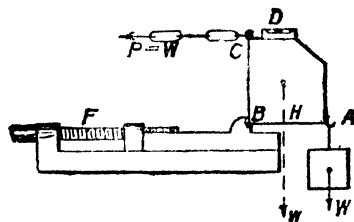
At the rear straining post, the band is connected to a slide similar in construction to that shown in Fig. 185.

In the measurements the band was set in position by turning the screw at the rear post until the mark on the end of the band was in line with the mark on the marking post. The screw on the straining apparatus was then turned until the bubble came to the middle of its run and a mark was made on the zinc plate on the forward marking post. After noting the temperature of the band as given by the thermometers, the band was moved forward and the operations repeated.

The posts were left in position, and as a check the base was remeasured in the same direction.

Accuracy of Steel Band Measurements.—Using a steel band 300 feet long, and proceeding in the above manner, Mr. O. B. Wheeler obtained a high degree of accuracy in his base line measurements. On three bases, on the Trigonometrical Survey of the Missouri River, having mean lengths of 7,923·320, 9,870·485,

and 9,711·903 feet, the probable mean errors were $\pm\,7\cdot15\,\mu$, $\pm\,1\cdot87\,\mu$, and $\pm\,1\cdot25\,\mu$ respectively, μ being one-millionth of the measured length.

Prof. J. B. Johnson,* working on cloudy days with a steel band on bases of about $\frac{1}{2}$ mile, obtained a probable error of $\pm\,1\cdot0\,\mu$, the error being calculated on the mean of three or four measurements.

Correction for Change of Temperature.—The length of the band at some standard temperature must be known. This will be determined at the Government Standards Office, for the payment of a small fee, or it may be obtained by comparison with a band whose length is known, at a given temperature. In addition to this, the amount the band expands or contracts for a change of 1° must be known. This may be determined by laying the band on a level surface on which is inscribed a mark at each end of the band, the alteration (l) in the relative positions of the marks is measured, and the corresponding temperature change (T) is noted ; from this data we have, the expansion or contraction per unit length per degree is equal to $\dfrac{l}{L\,T}$, where L is the length of the band.

This method presupposes that the surface on which the band is laid does not expand or contract as the temperature changes, and can only give approximate results. The increase of a body per unit length per degree is called the coefficient of expansion, and for steel its value varies from ·0000055 to ·000007 per degree Fahrenheit.

If the coefficient of expansion of the band is not known, an average value of ·0000063 may be taken.

When base line measurements are to be corrected for temperature, the temperature of the rod or band used is taken at two, three, or more places on its length. The sum of all the temperature readings is obtained, and this sum divided by the product of the number of temperature readings per rod or band and the number of rod or band lengths in the base gives the average temperature to be used in the correction. This mean temperature will also require correction for the known errors of the thermometers used, such errors being obtained by comparison of the thermometers with a standard instrument.

When a steel band is used it is usual to compute the correction

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* Johnson's " Theory and Practice of Surveying."

for that part of the base measured in whole band lengths, **and** that for the remaining fractional part separately.

Let L = length of the part measured in whole band lengths,
  l = length of remaining fractional part,
  T = temperature at which the length of the band is correct,
  T_1 = mean corrected temperature of the length L,
  T_2 = mean corrected temperature of the length l,
  a = the coefficient of expansion,

then the correction to be applied to the length

$$L = a\,(T_1 - T)\,L, \text{ and on the length}$$
$$l = a\,(T_2 - T)\,l.$$

Example (1).—The sum of the corrected thermometer readings on the part of a base line measured in whole band lengths is 6,825° F., and on the fractional part 134° F. The length of the base as measured is 2 miles, and the length of the band 300 feet. Three thermometer readings were taken on each whole band length, and on the fractional part two readings only. Find the correction for temperature, assuming the standard temperature is 55° F. and coefficient of expansion ·0000063.

Here, the number of band lengths $= \dfrac{2 \times 5,280}{300}$,

$$= 35\cdot2.$$

The average temperature $T_1 = \dfrac{6,825}{35 \times 3}$,

$$= 65° \text{ F.}$$

The average temperature $T_2 = \dfrac{134}{2}$,

$$= 67° \text{ F.}$$

Therefore, the correction for the
   length L = ·0000063 (65°— 55°) 35 × 300
       = ·6615 foot.
  Correction for length l = ·0000063 (67°— 55°) ·2 × 300
       = ·004536 foot.
   Total correction = ·6615 + ·0045
       = + ·666 foot, nearly.

Correction for Pull—Modulus of Elasticity.—To find the extension of the band under the constant stretching pull, we must know the stress required to elongate the band unit length, per unit length, on the assumption that the cross-sectional area

is unit squared. This quantity is known as the modulus of elasticity, and is a constant for the same material. For steel, it may be taken at 30,000,000 lbs. per square inch.

Let P = pull on the band in lbs.,

  l = length of the band in inches,

  A = cross-sectional area of the band in square inches

  X = extension of the band in inches,

  E = modulus of elasticity.

Then,

$$E = \frac{\frac{P}{A}}{\frac{X}{l}},$$

∴

$$X = \frac{P \times l}{A \times E},$$

and

$$n \times X = \frac{P \times n \times l}{A \times E},$$

or,

$$e = \frac{P \times L}{A \times E},$$

where n is the number of times the length of the band is contained in the base whose length is L, and e is the total extension.

Example (2). If the constant pull on the band in the preceding example is 20 lbs., the width of the band 0·15 inch, and thickness 0·02 inch, find the correction for the pull on the band. Assume E = 30,000,000 lbs. per square inch.

Here,

$$e = \frac{20 \times 5,280 \times 2 \times 12}{·15 \times ·02 \times 30,000,000} \text{ inches}$$

$$= + 2·346 \text{ feet.}$$

Correction for Sag.—The effect of the sag on the band between its supports is opposite to that of the pull upon it, and tends to make the measurement appear longer than it really is, hence the correction for sag must be subtracted, not added.

This correction is the difference between the length measured on the curve the band assumes and the distance between the supports.

Very approximately, the correction for one sag is given by the formula,

$$s = \frac{l}{24} \left(\frac{w \times l}{P} \right)^2,$$

where s = correction required,
l = distance between the supports,
w = weight of band per unit length in lbs., and
P = pull on the band in lbs.

If there be N equal sags in a given length L, the total correction is

$$S = \frac{N \times l}{24} \left(\frac{w \times l}{P} \right)^2$$

$$= \frac{L}{24} \left(\frac{w \times l}{P} \right)^2.$$

This formula assumes that the curve assumed by the suspended band or chain is a parabola; the real shape of the curve is a catenary.

Example (3).—In example (1) find the correction for sag, assuming the band to be supported at every 50 feet and the weight of 1 cubic inch of steel ·28 lb.

Weight of band per foot = ·15 × ·02 × 12 × ·28
$$= ·01008 \text{ lbs.,}$$

$$\therefore S = \frac{2 \times 5,280}{24} \left(\frac{·01008 \times 50}{20} \right)^2,$$

$$= - ·2794 \text{ foot.}$$

Tension Necessary to Eliminate Corrections for Pull and Sag.—
The corrections for pull and sag being of opposite sign, it is interesting to note that they may be eliminated by making the tension on the band dependent on the length between the supports of its segments. To find the tension necessary to do this, we must make $e = S$,

then
$$\frac{P \times L}{A \times E} = \frac{L}{24} \left(\frac{w \times l}{P} \right)^2,$$

or,
$$P = \sqrt[3]{\frac{A E}{24} (w \times l)^2},$$

and for a given band we have
$$P = K l^{\frac{2}{3}},$$

where K is a constant, and is equal to $\sqrt[3]{\frac{A \cdot E \cdot w^2}{24}}.$

Example (4).—Taking the data given in examples (2) and (3),
find what value of P is necessary to eliminate the corrections
for pull and sag.

$$\text{Here, P} = \sqrt[3]{\frac{\cdot 15 \times \cdot 02 \times 30,000,000}{24}(\cdot 01008 \times 50)^2}$$
$$= 9 \cdot 84 \text{ lbs.}$$

Checking Base Line Measurements.—For the purpose of
checking the measurements of a base line, it is divided into
two or more segments, and from
the extremities of these segments
angular observations are made to
favourably placed objects outside
the line. Assuming the measure-
ment of some one segment to be
correct, the length of the other
segments are calculated, by repeated
applications of the sine rule. Thus,
in Fig. 186, let A B be the segment,
the length of which is assumed
correct, knowing the three angles

Fig. 186.

of the triangles A B E and E B C, we have (referring to figure),
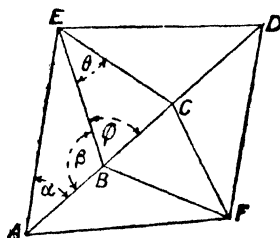
$$\frac{E\,B}{A\,B} = \frac{\sin \alpha}{\sin (\alpha + \beta)},$$

$$\therefore \quad E\,B = \frac{A\,B \cdot \sin \alpha}{\sin (\alpha + \beta)},$$

and

$$\frac{B\,C}{E\,B} = \frac{\sin \theta}{\sin (\theta + \varphi)},$$

or

$$B\,C = \frac{E\,B \cdot \sin \theta}{\sin (\theta + \varphi)},$$

that is,

$$B\,C = \frac{A\,B \cdot \sin \alpha \cdot \sin \theta}{\sin (\alpha + \beta) \cdot \sin (\theta + \varphi)}.$$

Prior to the calculation, the errors of the angles are worked out
and allowed for.

The calculation of B C is checked from the triangles A B F
and B F C. The length of the segment C D may be obtained
in a similar way from the triangles B C E and C E D, and checked
from the triangles A F C and C F D.

Prolonging a Base Line.—The methods employed in checking the measurement of the segments of a base line may also be employed when it is desired to prolong a base line, by triangulation.

For example, let A C (Fig. 186) be the measured portion of the base, and D the extremity of the base when prolonged. The station D is accurately ranged in position with an instrument centred over A or C ; on A C and C D as base lines triangles A E C, C E D, A F C, and C F D are built up, and all their angles are measured. From the data thus obtained the length of C D is calculated, exactly as the segments of the base are calculated in the preceding case.

Reduction of the Base to Sea Level.—When the length of the base line has been determined, its equivalent length at the sea level is computed prior to the enlargement of the base by triangulation. The method of reducing the base to sea level has been dealt with on p. 293.

Principal Bases on the Ordnance Survey.—The following table gives a list of the principal measured bases of the United Kingdom, with the year in which each base was measured ; also the situation and length, both measured and computed, of each base :—

Date.	Place.	Measured. Feet.	In Triangulation. Feet.	Difference.	County.
1791	Hounslow Heath.	27,406·19	27,406·36	+0·17	Middlesex.
1794	Salisbury Plain.	36,576·83	36,577·66	+0·83	Wilts.
1801	Misterton Carr.	26,344·06	26,343·87	−0·19	Lincoln.
1806	Rhuddlan Marsh.	24,516·00	24,517·60	+1·60	Flint.
1817	Belhelvie.	26,517·53	26,517·77	+0·24	Aberdeen.
1827	Lough Foyle.	41,640·89	41,641·10	+0·21	Londonderry.
1849	Salisbury Plain.	36,577·86	36,577·66	−0·20	Wilts.

The last two bases were measured with the Colby apparatus. the others with steel chains.

Enlarging a Base Line.—Even the longest base line is short compared with the sides of the principal triangles ; hence, in order to have good conditioned triangles in its neighbourhood,

it is necessary to enlarge the base gradually by triangulation. In enlarging a base line (A B, Fig. 187), suitable points C and D are selected on opposite sides of the base as the vertices of well-conditioned triangles having A B as base. From the measured base and angles of the two triangles the length of the diagonal C D is calculated. C D now becomes a new base, which is further enlarged to E F by means of the triangles D E C and D F C; this process is repeated as often as required. The final enlarged line is used as the base of a triangle, whose sides are also used as base lines for the construction of other triangles, the sides of which, in turn, become new bases for the further extension of
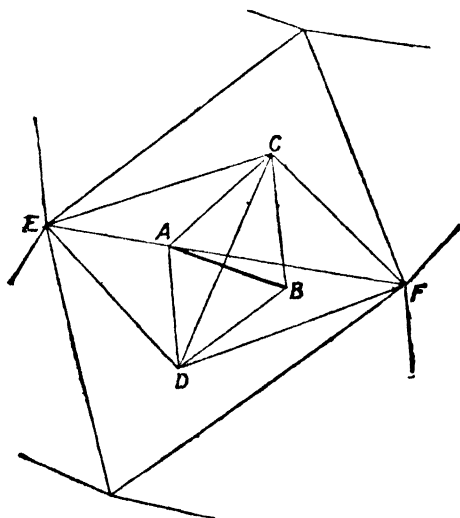
Fig. 187.

the system, the process being repeated until the whole country to be surveyed is covered by the principal triangles.

The secondary triangles are next set out with smaller instruments, the principal detail within each triangle is then surveyed either by traversing or by distances and offsets, in the usual way.

Plotting the Survey.—In plotting the survey, the principal and secondary triangles are laid down from the computed lengths of their sides (see Chap. XVII.), the detail is then plotted within each triangle, the whole of the plotting being done to a scale

larger than that intended for final representation. Copies of small portions of the plan are then made and are checked on the ground they represent, corrections are pencilled-in where required, and at the same time the minor detail, such as ditches, footpaths, trees, etc., is inserted either entirely by sketching, or by partial sketching and measurement.

After correction and the insertion of the minor detail, the plans are reduced by photography to the scale previously fixed on for final representation.

Instruments used on Geodetic Surveys.—The instruments used for measuring the angles of the principal triangles in the Ordnance Survey of the United Kingdom comprised two large theodolites having horizontal circles 3 feet diameter, one theodolite 18 inches diameter, and one 2 feet diameter. The three former were made by Ramsden, and the latter by Troughton and Simms. The angles of the secondary triangles were measured with a 12-inch instrument, while 5-inch and 7-inch instruments, chiefly the latter, were used for measuring the angles of the tertiary triangles.

In the principal triangulation for the survey of India, Colonel Everest used two theodolites with horizontal circles 3 feet diameter and vertical circles 18 inches diameter, besides two " Astronomical Circles " having double vertical circles 36 inches diameter, used chiefly for determining latitudes from star observations.

In the United States Coast Survey the larger instruments were 24 and 30 inches diameter.

On the European Continent, smaller instruments found favour. The instrument used by Struve had a diameter of 13 inches only, with a vertical circle of 11 inches. The eye-piece of this instrument was placed in one of the trunnions of the telescope, the rays of light being bent at right angles by a prism placed in the centre of the telescope.

In the principal triangulation of Spain the theodolites used for terrestrial observations have diameters of 12·5 and 14·5 inches, and for astronomical work a transit theodolite having horizontal and vertical circles of 12·5 and 10·25 inches respectively. These instruments have their lines of collimation bent at right angles, as in Struve's theodolite.

In most of the above instruments the subdivision of the scale is obtained by the use of micrometer microscopes.

Marking Trigonometrical Stations.—The centres of trigonometrical stations are usually indicated by a clear mark cut in the upper surface of a block of stone, placed at a sufficient depth

below the surface of the ground. The marks in the vicinity of the base line are often made on a piece of brass set in the block of stone, and are microscopic in size.

Heights of Instruments and Signals.—When the distance between two stations is great, and their elevation small, it is necessary to elevate both the instrument and the signals to clear intervening obstacles, and to overcome the earth's curvature. In hilly country, where the principal stations are placed on the tops of hills or mountains, there will, as a rule, be no necessity to raise the instrument above the ground, but in flat country this necessity will often arise. When required, the instrument is elevated on a triangular scaffold, built to the required height This scaffold is surrounded by a rectangular scaffold to carry the observer and observatory at its summit, the two scaffolds are built quite independent, so that any movement of the outer scaffold, due to wind pressure, or to the movements of the observer, is not communicated to the instrument. A vertical trough running from the top to the bottom of the inner scaffold is arranged to protect the plumb line from the wind. The trough is arranged to rotate on a vertical axis, so that its open side may be placed on the lee side.

For stations on the ground, three strong stakes are driven as far as possible into the ground, and are then cut off level with the surface, thus forming an immediate support for the tripod. On scaffolds the tripod is often dispensed with, a strong table with firmly braced uprights is formed, and on the table the instrument is supported on a tribrach plate having three pointed projections on its lower side, which are firmly pressed into the table. A hole is left in the centre of the table, through which the plumb line passes.

In all cases the instrument is sheltered from the sun and wind by an observatory either in the form of a tent or a more solid structure built of wood. The floor of the observatory must not be allowed to come in contact with the instrument stand.

Many cases requiring high scaffolds occurred on the Ordnance Survey of the United Kingdom. At Thaxted Church, the station was over the church spire, the outer scaffold rising over 170 feet above the ground, the inner scaffold was carried on timbers passing through the spire at 140 feet above the ground; at Misterton Carr, the height of the scaffolding was about 120 feet.

The average height of the scaffolds at 243 stations on the United States Survey of the Great Lakes was 58 feet; 35 of these scaffolds varied from 100 to 124 feet in height.

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On p. 122, Part I., we have seen that the allowance for curvature and refraction is approximately equal to $4/7 \cdot d^2$ feet; hence, if h is the height of the top of the scaffold at station A (Fig. 188), and E C F be drawn tangent to the earth's surface at C, then

$$d = \sqrt{\frac{7}{4}h} = \frac{\sqrt{h}}{\cdot 756}.$$

Similarly,

$$d_1 = \frac{\sqrt{h_1}}{\cdot 756},$$

consequently,

$$d + d_1 = \frac{\sqrt{h} + \sqrt{h_1}}{\cdot 756},$$

that is,

$$D = \frac{\sqrt{h} + \sqrt{h_1}}{\cdot 756},$$

where D is the distance between the stations in statute miles. This equation gives the distance of mutual visibility (neglecting
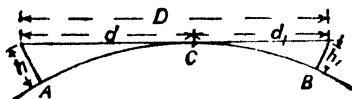
Fig. 188.

intervening obstacles) of two stations whose heights in feet are h and h_1 above mean sea level. The actual height of the signals and instruments will, of course, be made a few feet greater than that given by the equation.

Example (5).—The height of the signal above station B is 54 feet, the height of B above M.S.L. 120 feet. Find the approximate height of the scaffolding at A, if the station is 250 feet above M.S.L. and the distance between the stations 40 miles.

Here

$$\frac{\sqrt{54 + 120} + \sqrt{h + 250}}{\cdot 756} = 40,$$

∴

$$\sqrt{h + 250} = 40 \times \cdot 756 - \sqrt{174}$$
$$= 17 \cdot 05,$$
$$\text{and } h = 40 \cdot 7 \text{ feet.}$$

In the foregoing investigation we have assumed that the intervening ground between the two stations presents no obstruction on the line of sight. Over land surfaces, a profile

along the proposed line of sight will determine whether that line is practicable, and if practicable, the points above which the line of sight must pass for mutual visibility. The necessary heights of the scaffolds cannot be computed exactly owing to the uncertainty due to refraction. An approximate method of solution must, therefore, be made use of, and this method is best shown by an example :—

Example (6). Two stations, A and B, are 60 miles apart ; the top of the scaffold at A is 75 feet above M.S.L. and the height of the ground at B is 2,080 feet above the same datum. The highest intervening point is at C, 25 miles from B, at a height of 750 feet above M.S.L. Find the height of the scaffolding at B in order that the line of sight may clear the point C by 10 feet.
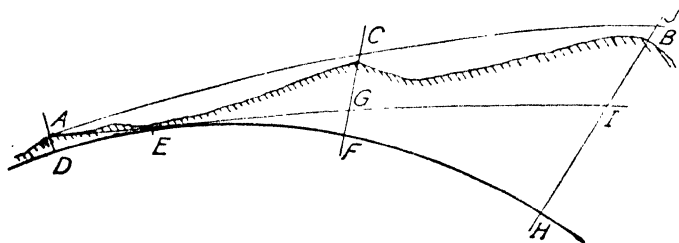
Fig. 188A.

Referring to Fig. 188A, we have—

$$A D = \frac{4}{7} D E^2$$

$$\therefore \quad D E = \sqrt{\frac{75 \times 7}{4}} = 11 \cdot 46 \text{ miles}$$

$$E H = 60 - 11 \cdot 46 = 48 \cdot 54 \text{ miles}$$

$$E F = 48 \cdot 54 - 25 = 23 \cdot 54 \text{ miles}$$

$$D F = 23 \cdot 54 + 11 \cdot 46 = 35 \cdot 00 \text{ miles}$$

$$F G = \frac{4}{7} \times 23 \cdot 54^2 = 316 \cdot 6 \text{ feet}$$

$$H I = \frac{4}{7} \times 48 \cdot 54^2 = 1,347 \text{ feet}$$

$$G C = 750 + 10 - 316 \cdot 6 = 443 \cdot 4 \text{ feet}$$
$$I B = 2,080 - 1,347 = 733 \text{ feet}$$
$$I J = 733 + h.$$

Assuming the triangles A G C and A I J are similar, we have :—

$$\frac{733 + h}{443 \cdot 4} = \frac{60}{35}$$

from which $\qquad h = 27 \cdot 1$ feet.

Signals.—The form of the sighting object, in geodetic operations, varies with its distance from the observer, its local surroundings, and the atmospheric conditions under which it is viewed ; but, whatever be the form of signal adopted, it is essential that it be symmetrical about a vertical line passing through the station, so that an observation of the signal is equivalent to an observation of the station mark.

As the result of experience, it is found that a signal to be easily seen must subtend an angle of at least 30″ at the observer's station, hence the height of the signal must not be less than $\frac{1}{7000}$ of the distance. The diameter of the signal at its base should be about $\frac{1}{8}$ of its height. Signals must be easily seen, and should be constructed so that the instrument may be placed exactly over the station they indicate.
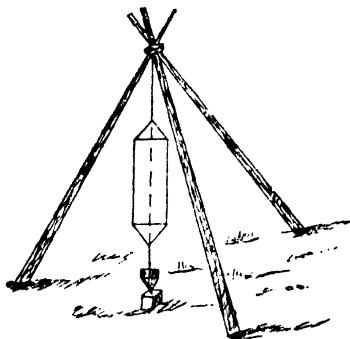
Fig. 189.

When seen against the sky, a tall pole, stayed with guy ropes, forms an efficient signal, and is visible for fairly long distances in a clear atmosphere. The foot of the pole should be jointed so that it may be easily removed when the instrument is to be placed in position.

A signal of white canvas stretched over hoops forming a cylinder (Fig. 189), terminated by cones and suspended from the apex of a tall tripod by a rope, has been used for stations seen against a background of hills or trees. The lower end of the suspending rope is heavily weighted, and accurately centred over the station. When illuminated by the sun the signal is almost phaseless, thereby tending to reduce the errors of bisection.

In other cases a vertical pole carrying a flag or other distinguishing object is lashed to the head of the tripod, the centre of the pole being vertically over the station ; the lower end of the pole may be held in position by cross pieces fixed to the legs of the tripod, or by stones piled around its foot.

The tripod type of signal is very convenient in use, as the instrument—on removing the pole—may be placed over the station without removing the tripod.

Heliostats and Heliographs.—For observing over long distances in daylight, special instruments are used, which flash the sun's rays in the required direction. For this purpose two types of instrument are in common use ; one type sends a continuous beam of light, the other sends short and long flashes for transmitting messages. Instruments of the former type are known as heliostats and of the latter as heliographs.

One of the commonest forms of heliostats consists of a circular mirror mounted so that it is free to turn on vertical and horizontal axes. At the centre of the mirror a small circular portion $\frac{1}{2}''$ to $\frac{3}{4}''$ diameter is left unsilvered. The mirror is carried on a tripod stand, which may be centred over the station. Attached to the vertical axis of the tripod head, and capable of rotating around it, is a horizontal arm, which carries at its free end a vertical open frame fitted with stout cross wires, having a white cardboard disc, about an inch diameter, at their centre. This frame is adjustable for height, and, like the horizontal arm, may be clamped in any desired position.

In using the apparatus, the arm is turned so that the line joining the centres of the mirror and cross wires passes through the distant station. The mirror is then turned—and adjusted in position from time to time by the " heliotroper ' —so that the beam of reflected sunlight is bisected by the cross wires on the horizontal arm. That the reflected ray is bisected by the cross wires is tested by sighting through the clear glass at the centre of the mirror, when the ray is accurately directed, a small dark circle is seen at the centre of the cardboard disc.

Night Signals.—In the earlier portion of the Ordnance Survey, General Roy used Bengal lights, to render the distant stations visible at night. These were replaced by Argand oil burners fitted with parabolic reflectors ; later still, this apparatus was improved by the addition of a large plano-convex lens, from which a powerful parallel beam of light emerged, visible under good conditions for about 50 miles. The emergent beam passed through a vertical slit, the width of which was varied according to the distance of the observer's station.

In India lamps formed of shallow earthenware dishes, containing raw cotton saturated with cotton-seed oil, were often used. Each lamp was provided with an earthenware cover,

from which the light emerged by a vertical slit turned in the direction of the observer's station.

The powerful light (invented by Lieutenant Drummond) obtained by playing an oxy-hydrogen jet on a ball of lime was used on several occasions on the Ordnance Survey.

Any kind of light will serve for observations at night, provided it be sufficiently steady, is easily controlled, and sufficiently powerful to throw a bright beam the required distance.

Minor Triangulation.—For the survey of places of relatively small extent, such as the smaller colonies, islands, etc., where very great accuracy is not required, and where the expense attendant on geodetic operations would not be justified, a triangulation system formed of small triangles of one to two miles side is best suited. A triangulation of this kind is called a " minor triangulation."

The method of carrying out the survey is similar to that of a great trigonometrical survey, but much smaller instruments are used, and less accomplished observers are needed.

Size of Instruments.—The instruments employed are of the transit type, having horizontal circles 5 or 6 inches diameter, and reading to 5 or 10 seconds of arc by micrometer microscopes.

As in great trigonometrical surveying, the system of triangles commences from a carefully measured base line, which may be measured with suitably laid steel bands, allowance being made for alteration in length of the band due to sag, temperature changes, and tension. Bases of verification are marked out in suitable positions, and may be measured in the same way.

The triangles do not usually interlace, but each station is arranged so that the lines joining the stations around it form a polygon. The angles of each polygon are checked and adjusted as explained in the preceding chapter, and the sides of the triangles are laid down on the plan from their computed lengths.

The interior detail of each triangle is surveyed by traversing, or by distances and offsets, in the usual way. If the climatic conditions are suitable, the interior detail may be surveyed by means of the plane table.

The Plane Table.—In surveying with the plane table, the surveyor works between stations fixed by the larger operations, and plots the plan of the ground surveyed as he proceeds.

The plane table (Fig. 190) consists of a drawing board mounted on a tripod fitted with levelling screws, so that the plane of the board may be set quite horizontal. On the drawing board a sheet of drawing paper or thin cardboard is fixed either by drawing pins or by suitable clamps. The working surface of

the drawing paper or board should be tinted a faint green or grey, in order to reduce the strain on the surveyor's eyes.

In the simpler forms of the apparatus, the sighting instrument consists of a brass straight edge (the Alidade) about 18 inches long, fitted with a pair of sights of the slit and window pattern, the line of sight being in the same vertical plane as the fiducial edge of the alidade. In the best instruments the alidade is fitted with a telescope provided with a vertical circle and spirit level; the line of collimation of the telescope lies in the vertical plane containing the fiducial edge of the alidade. The telescope
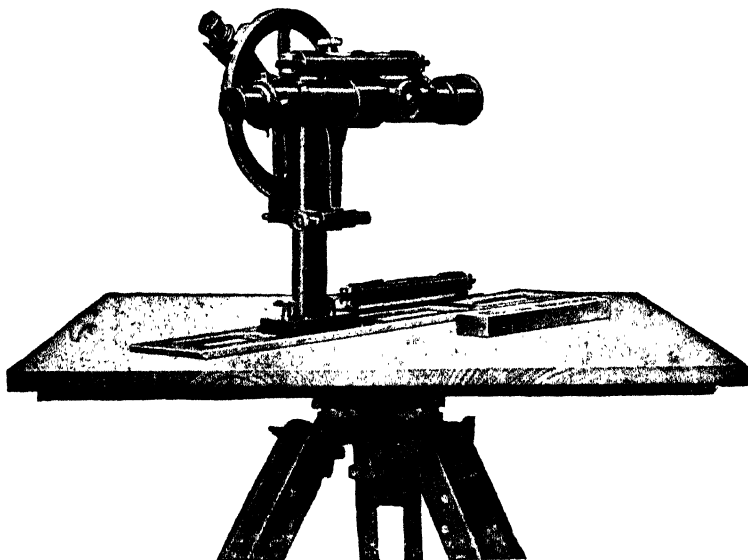
Fig. 190.

is usually fitted with stadia wires for distance reading. A trough compass having its index line parallel to the long sides of the case is used for taking magnetic bearings.

The principle underlying the method of using the plane table is as follows:—Let A, B, C, and D (Fig. 191) be four points to be surveyed. Selecting the line joining A B as a base line, its length is chained and a corresponding line (*a b*) is laid down on the drawing paper, to the proper scale. The plane table is then placed in position with the point *a* on the paper vertically over the station A on the ground, and the line *a b* is sighted in the

direction of A B with the alidade. The centering of the point *a*
is effected by means of a plumb-bob attached to the lower prong
of a fork-shaped piece of metal ; the upper prong, carrying an
index point or line, rests on the paper with the index in coin-
cidence with the point *a* on the paper. The board is then
adjusted in position until the plumb-bob is vertically over the
station, and *a b* is in correct alignment with the base line A B.

A fine needle is now stuck upright in the paper at *a*, the
alidade is placed with its fiducial edge in contact with the needle,
and turned until it is in true alignment with A D), and the ray
a d is drawn on the paper. The ray passing through the station
C is drawn in the same way. The plane table is next removed
and placed over station B, the point *b* is centred over the station
B, and the line *b a* brought into accurate alignment with B A.
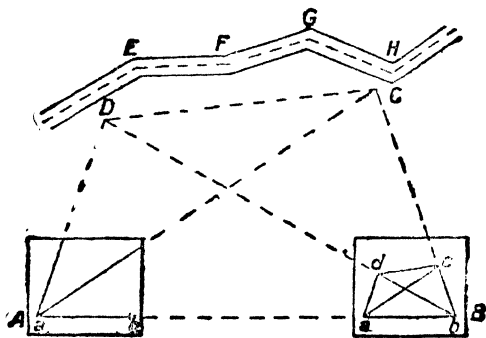
Fig. 191.

On drawing the rays *b d* and *b c*, their points of intersection
with the rays *a d* and *a c*, fix the positions of the points *d* and *c*.

Clearly, the same method will apply no matter how many
points we wish to fix on the plan, the intersection of two rays,
one from each extremity of the base, being sufficient to fix any
one point.

The methods of fixing the positions of points surveyed, by
radiating lines, is combined with that of distances and offsets,
for it is obvious that a great number of rays would be necessary
to fix the outlines of irreg lar objects, and this tends to confusion
and inaccuracy. For example, the irregular road shown in
Fig. 191 may be surveyed either by fixing the points of change
E, F, G, H, etc., by rays, and the sides of the road by distances
and offsets from the lines E F, F G, G H, etc., or a straight line

(D C), having its extremities fixed by rays, may be laid down, and the road surveyed by distances and offsets from this line. The surveyor must decide which method to employ in each case as it arises, and he must also decide whether particular detail is sufficiently important to warrant him in representing it on the plan by mere visual observation and sketching.

Detail fixed by distances and offsets should be plotted while in the neighbourhood of the object, so that its plan may be readily checked by inspection.

If the telescope is fitted with stadia wires, the distance and height of the observed object may be obtained from readings on a levelling staff held at the object. The stadia wires are usually placed on the diaphragm so that the required distance is 100 times the staff reading, when the collimation line is horizontal. The method of obtaining the distance of the staff when the collimation line is inclined, is dealt with in Chap. XIII. Levels are obtained by readings on the central wire in the usual way.

To fix the position of a point by this method, the point is sighted from one end of the base, and the ray through it is drawn. The distance and level are computed from the staff readings, the computed distance laid off along the ray fixes the plan of the observed point. The level of the point may be indicated by a figured height, written against it.

The plotting of points whose distances are obtained by stadia readings is much simplified if the appropriate scale is engraved on the fiducial edge of the alidade.

When all the detail in the neighbourhood of the base has been plotted, the instrument is moved to the forward end of the new base, and the detail in its neighbourhood is surveyed as before. The process is repeated until the work closes on the pre-arranged trigonometrical station.

The new base line may be fixed by rays from the extremities of the preceding base to its forward extremity. Thus, in Fig. 191, the rays $a\,c$ and $b\,c$ fix the position of the point c, and $a\,c$ and $b\,c$ may be used as new base lines. This method is, however, not always convenient in practice, and the method adopted to fix the position of the observer's station will depend on the available data. Three cases are here dealt with.

(1) *When only one ray has been drawn through the station.*— The instrument is set up in the line indicated by the ray, and the ray is brought into accurate alignment with the preceding station. A station is then selected on the ground whose plan has already been fixed, and the straight edge is placed in alignment with the object (B) and its plan (b, Fig. 192). A ray is now drawn back-

wards from *b* to cut the ray *c x* from the preceding station. The point sought is the point of intersection of the two rays. As a check, another ray *a* A should be drawn through a point in line with the object it represents. This line should also pass through the point (*x*) already found.

(2) *To fix the position of a station when no ray has been drawn through it.*—In default of compass bearings, this can only be done when the station is visible from at least three other stations already fixed on the plan. To find the position of the station at which the instrument has been set up, lay a sheet of tracing paper on the plan, and on the tracing paper draw rays from any point on it to three known stations A, B, and C (Fig. 193). Now move the tracing paper about until the three lines pass through the plans (*a, b,* and *c,* Fig. 194) of the three selected stations, then
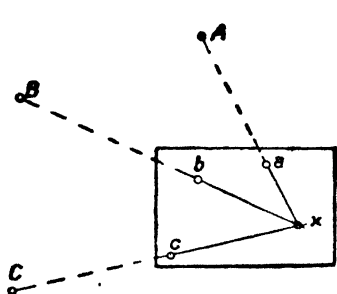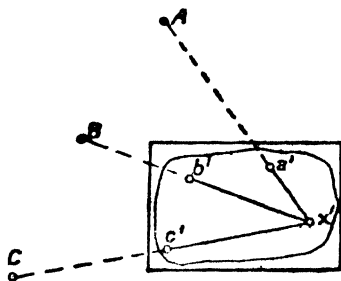
Fig. 192.   Fig. 193.

prick through the paper at *x'*, thus fixing the position of the station on the plan. Remove the tracing paper, lay the straight edge along the line joining *x'* to *a,* and turn the board round until *x', a,* and A are in the same straight line. The plan should now be in its proper position relative to points already plotted; as a check, on laying the straight edge along the line joining the points *x'* and *b,* the line of sight should bisect the signal at B.

This method will not give an accurate result unless the three stations A, B, and C are about the same distance from the station whose position is to be determined.

(3) *To fix the position of a station when the direction of the magnetic meridian at the preceding station has been drawn on the plan.*—To solve this case, the new station must be so placed that it is visible from at least two stations whose plans have been plotted.

The instrument having been set up in the desired position, the trough compass is placed on the plan with the long side of the compass case in contact with the meridian line drawn on the plan. The plane table is then turned on its vertical axis until the needle settles in the magnetic meridian with its ends opposite the index marks in the compass case. The plan will now be in its proper position relative to the objects represented.

To fix the position of the station, set the straight edge in line with any object (A) and its plan (a, Fig. 195), and draw a line on the plan, backwards ; repeat this, using a second object B and its plan b. The point of intersection (x) of the two lines is the point sought. As a check, place the straight edge in line with the point x and the plan of a third point c, the line of sight should now bisect the object C.
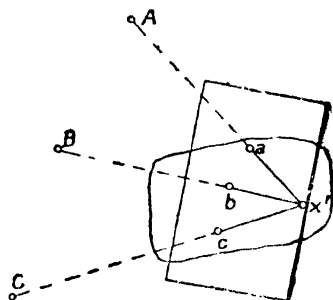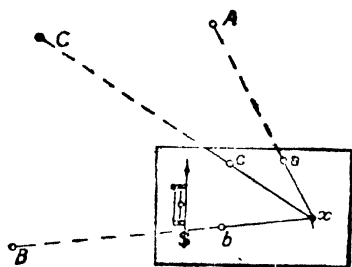
Fig. 194.   Fig. 195.

Reduction to Final Scale.—Plane table surveys are always drawn to a much larger scale than that intended for final representation. The separate sheets obtained in the survey are joined together and are reduced by photography, to the predetermined scale ; they are then ready for insertion in the final plan.

Photographic Surveying.

Since about the middle of last century many attempts have been made to make use of the art of photography for surveying purposes, but it was not until comparatively recent years that a successful system of photographic surveying was evolved. Under the direction of Mr. E. Deville, the Surveyor-General of Canada—who published a practical treatise on the subject in 1895—many thousands of square miles have been surveyed in the north-west of Canada with great success.

Photographic surveying is not suitable for flat or for heavily wooded country, neither can it be used with great success in the older countries in which the ground is divided by a multiplicity of enclosures ; its chief area of usefulness lies in undeveloped hilly or mountainous countries possessing a clear atmosphere, or in unhealthy malarial districts.

The Camera for taking the photographs is mounted on a tripod of the construction usually adopted for surveying instruments, and should possess the following qualities :—(1) The lower part of the instrument should be constructed like a theodolite, with clamps and tangent screws to the vertical axis and horizontal graduated circle, so that the optical axis of the lens may be (a) set truly horizontal, and (b) correctly oriented either with a known station or the meridian, for which purpose a telescope or a pair of sights should be fitted to the instrument.

(2) The camera body should be of metal in preference to wood, so that its dimensions may not be altered by shrinking or warping. The fixed focus and not the collapsible type of camera body should be used.

(3) A good lens having a focal length of not less than 6 inches and an aperture of 45°.

(4) A pair of hair lines, one horizontal and the other vertical, should be placed in front of the photographic plate to mark on the negative the traces of the principal horizontal and vertical planes of the perspective view.

The photo-theodolite, invented by Mr. J. Bridges Lee, possesses the qualities specified above, and in addition it is constructed so that the magnetic bearing of the principal vertical plane is automatically photographed on each picture obtained with the instrument.

Principle of the Method of Photographic Surveying.—The principle underlying the method of photographic surveying is precisely the same as that underlying plane table work—viz., if the direction of a ray from each end of a base line to an object be known, the position of the object relative to the base line is given by the point of intersection of the two rays. If a photograph showing the traces of the principal perspective planes be taken from a station whose position is known, the direction of the optical axis and the focal length of the lens be also known, then if the photograph be laid down on the paper on which it is desired to draw the plan with the trace of the principal vertical plane properly oriented and passing through the station, and with the horizon line at a distance therefrom equal to the focal

length of the lens, rays drawn through the station, and the projections on the horizon line of points shown in the picture, will give the directions in which the points are situated. If these operations be repeated at a second station, using a photograph taken at the station and showing the same objects, the intersections of the rays passing through the same object gives the position of the object in plan. It is obvious that by taking a sufficient number of photographs from suitable stations, the country surveyed may be mapped out.

Fig. 195A has been drawn to illustrate the principle of this method of photographic surveying. In the figure the angles
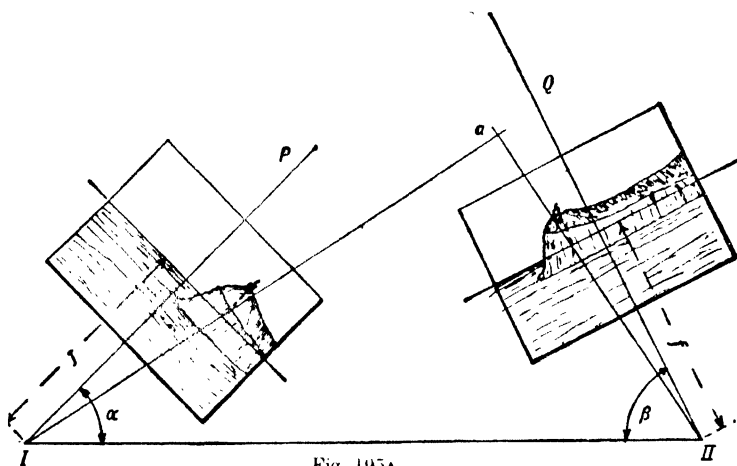
Fig. 195A.

α and β are the angles the principal vertical plane (*i.e.*, the vertical plane containing the optical axis) makes with the base line at the stations I and II respectively at the time of exposure. The base line I-II having been drawn to the given scale, the angles α and β are set out with a protractor, and we thus get the principal (or distance) lines IP and IIQ. From IP and IIQ the focal length (f) of the lens is cut off (full size) and through the points thus obtained lines are drawn at right angles to the principal lines. The corresponding photographs are then arranged as shown in the figure. To determine the plan of any point (say the top of the tower shown), its image is first projected on to the principal horizontal line in each photograph and through the points thus obtained radials are drawn through

the corresponding stations (I and II), the intersection of these radials giving the plan (α) of the point. In a similar way the plans of other points shown on both photographs may be found.

It must, however, be clearly understood that the photographs are not usually laid down on the drawing board in this manner, but are used and the plotting carried out as described on page 380. If the photographs shown in the figure be rotated through a right angle about their principal horizontal lines, their plans would be represented on the paper by the horizontal lines (or picture traces) themselves, and the student must consider the picture trace drawn for any station as the plan of a photograph taken at that station, and consequently the plans of all points on the photograph would be represented by points on its picture trace.
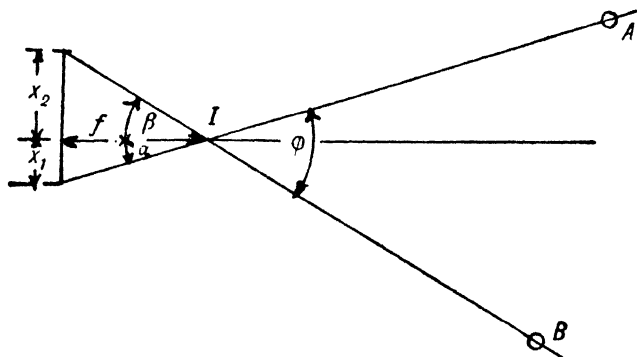
Fig. 195B.

There are certain problems which arise from time to time in connection with this method of photographic surveying. The principal problems are as follows :—

1. *To determine (or check) the focal length of the lens.*

It is obvious from the description referred to, that an accurate knowledge of the focal length (f) of the lens used is a prime necessity. This is usually supplied by the maker of the instrument, but the surveyor should be prepared to determine this quantity either for checking the value supplied to him, or for establishing it in the case of a lens of unknown focal length. To do this (by Deville's method) a photograph is taken of two well defined distant points A and B (Fig. 195B) and the angle (φ) they subtend at the station I is measured with a theodolite. Let the abscissæ of the points as measured on the photograph be x_1 and x_2, and the

angles they subtend at I be α and β respectively, then from the figure we have

$$\tan \alpha = \frac{x_1}{f}, \tan \beta = \frac{x_2}{f},$$

$$\tan \alpha \tan \beta = \frac{x_1 x_2}{f^2}$$

Now,

$$\tan \varphi = \tan (\alpha + \beta)$$

$$= \frac{\dfrac{x_1}{f} + \dfrac{x_2}{f}}{1 - \dfrac{x_1 x_2}{f^2}}$$

From which, solving for f we get,

$$f = \frac{x_1 + x_2}{2 \tan \varphi} + \sqrt{\frac{(x_1 + x_2)^2}{4 \tan^2 \varphi} + x_1 x_2}$$

The result should be checked by taking corresponding quantities from other points in pairs.

2. *To determine (or check) the position of the principal line.*

The principal line is the line of intersection of the plane of the negative and the vertical plane containing the optical axis of the lens.

Obtain the photograph of three well defined points (A, B and C) whose positions and altitudes (H_1, H_2, and H_3, required for problem 3) relative to the camera station are known, also let the photograph show the image of a plumb line (VV', Fig. 195c), placed a short distance in front of the camera. Scribe fine lines on the negative through a, b and c (Fig. 195c), parallel to the image of the plumb line, and any line mn at right angles thereto. Lay a paper strip with its lower edge on mn and mark on this edge the points a_1, b_1, c_1. Now place the strip on the plan of the three radials through the plans of the points A, B and C (Fig. 195D), and adjust it in position until a_1, b_1 and c_1 fit exactly on the three radials as shown. The strip should now be tangent to an arc of radius f having I as centre. From I drop a perpendicular IO on the strip and mark the point O. On placing the strip in its original position on the negative the point O' may be marked thereon and a line drawn through this point, parallel to VV', is the principal line required.
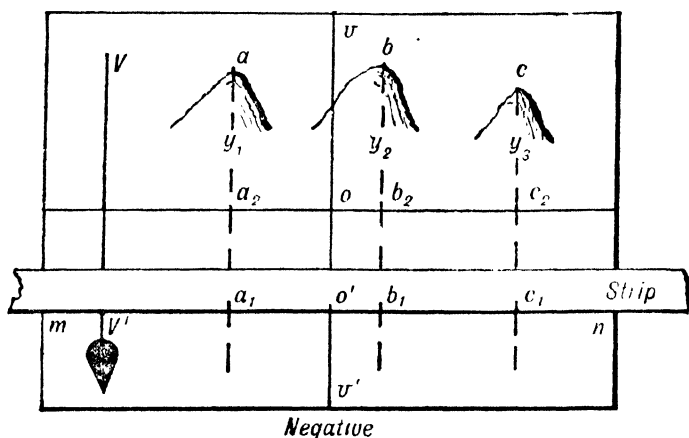
Negative

Fig. 195c.

3. *To determine (or check) the horizontal line.*

On the plans of the three radials (Fig. 195D) measure Ia_1, IA ; Ib_1, IB ; Ic_1, IC. Let y_1, y_2 and y_3 be the ordinates of a, b and c on the negative. Then

$$y_1 = aa_2 = H_1 \frac{Ia_1}{IA} \; ; \; y_2 = bb_2 = H_2 \frac{Ib_1}{IB} \; ; \text{ and } y_3 = cc_2 = H_3 \frac{Ic_1}{IC}$$

Mark these ordinates on the negative along the verticals through a, b and c ; the required horizontal line is now obtained by drawing a line through the points a_2, b_2 and c_2 thus obtained.

It is obvious that the third station (say B) is only required as a check.

Alternatively, a level may be set up near the camera, the optical axes of the camera and level being adjusted to the same horizontal plane. Well defined points on the horizon line are found with the level and their positions and nature are noted. These are subsequently identified on the negative and a line drawn through their images establishes the position of the horizontal line.

The positions of the principal and horizontal lines having been determined and checked for a given negative, on placing the negative in the dark slide exactly in the position it occupied at the time of exposure, the ends of the lines may be scribed on the

slide, thus defining the positions of V notches or other marks the images of which will mark the positions of the lines on all subsequent exposures.
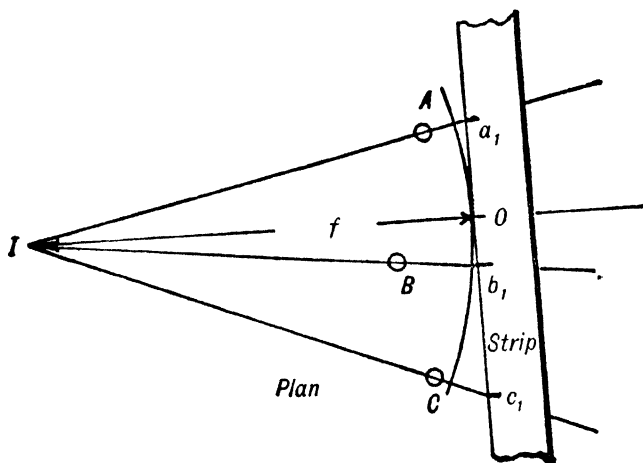
Fig. 195D.

4. *To orient the picture trace when a known point is pictured, but no angles or bearings have been obtained.*

The pictured point may be a camera station or any point whose position is known relative to the station at which the exposure was made.

In Fig. 195E let I be the camera station, P the plan of the terrene point pictured at p on the negative exposed at I ; also let the abscissa of p be x, as measured on the negative. To orient the picture trace join IP and cut off the length $Ip = \sqrt{x^2 + f^2}$, and on Ip describe a semi-circle ; with centre I and radius f describe an arc cutting the semi-circle in the point O_1 ; join O_1I and O_1p ; the former is the direction of the principal line and the latter that of the picture trace. If the plotting is done from enlargements, m times the size of the original, then Ip' is made equal to m times Ip and IO' equal to m times f, otherwise the construction is the same as is shown by the dotted lines.

Fig. 195E is also drawn to illustrate the system of plotting described on p. 380. If x_1 and x_2 are the abscissæ of any terrene point A obtained from the negatives exposed at the stations I and

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II, the intersection of the radials Ia_1 and IIa_2 gives the plan **a** of this point.

5. *Given the angles (α and β) between the principal lines and the base line at two consecutive stations, and the distance between the stations, to find the lengths of the radials and the height of the pictured point above the stations.*

From the two negatives measure the abscissæ x_1 and x_2 of the points.
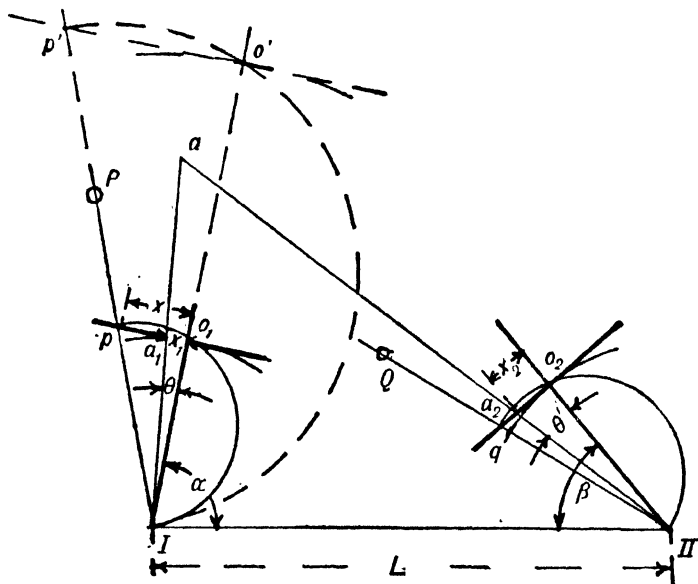
Fig. 195ᴇ.

Referring to Fig. 195ᴇ, let D = Ia and D_1 = IIa, and the length of the base I.II = L, then it is obvious from the figure that

$$\tan\,\theta = \frac{a_1\,o_1}{f} = \frac{x_1}{f}\,;\quad \tan\theta' = \frac{a_2\,o_2}{f} = \frac{x_2}{f},$$

$$\text{and D} = \frac{\text{L}\sin\,(\beta - \theta')}{\sin\,(\alpha + \beta - \theta' + \theta)}\,;\quad \text{also D}_1 = \frac{\text{L}\sin\,(\alpha + \theta)}{\sin\,(\alpha + \beta - \theta' + \theta)}$$

which are the radials required.

To find the heights (H and H_1) of the pictured point, let y and y_1 be the ordinates of the point as measured on the photographs obtained at the stations I and II respectively, then if φ and φ_1 are the inclinations of the rays passing through the point we have

$$\tan \varphi = \frac{y}{\sqrt{x_1^2 + f^2}}, \text{ and } \tan \varphi_1 = \frac{y_1}{\sqrt{x_2^2 + f^2}};$$

therefore $\quad H = \dfrac{Dy}{\sqrt{x_1^2 + f^2}}, \text{ and } H_1 = \dfrac{D_1 y_1}{\sqrt{x_2^2 + f^2}}.$

The heights H and H_1 are measured from the level of the optical axis of the lens; to these heights it is necessary to add (algebraically) the height of this line above the ground at the respective stations to obtain the differences in ground level.

Good Intersections Necessary.—On each picture there will be a certain area (usually about mid-distance), in which the intersections will be well-conditioned, while in parts of the picture the intersections will be too acute or too obtuse to give reliable results. In arranging his camera stations, the surveyor should indicate the area which gives the best intersections on his field sketches, and should arrange, wherever possible, that the more important points fall within this area; failing this, such points should be shown on at least three views taken at different stations. Good intersections rather than a multiplicity of views should be aimed at.

Field Work.—If the country to be mapped has been triangulated the surveyor first obtains a plan showing the trigonometrical points, from any one of which he may commence his operations; but if no triangulation has been made, the surveyor must proceed to make one with a theodolite in the usual way. In the latter case, the stations may be selected with a view to their use as camera stations during the subsequent operations; if not conveniently situated for this purpose, if their positions are clearly defined, they are useful for orienting the camera when taking the photographs and in fixing the position of the camera station.

When the triangulation has been completed, the photographic survey commences from a selected camera station. The instrument is set up over the station, the photographs of the views around the station are obtained, the direction and height of the optical axis of the lens being noted for each view. The number of views required at each station will depend on the angular aperture of the lens employed (*i.e.*, on the ratio of the focal

length of the lens to the size of the plate used). When all the necessary exposures have been made at a station, and a sketch plan of the chief detail in its neighbourhood completed, the surveyor moves on to the next station and repeats his operations.

In selecting the stations and in deciding on the number of views to be taken, in addition to obtaining good intersections for all important points, the topographer must keep constantly in mind that the plotter in the office can only fix the position of points which are visible from *at least two* stations.

Office Work.—This consists of the development of the plates exposed in the field, the preparation of enlargements from the negatives, and in plotting the topography from the enlargements.

By using great care, the plotting may be done from the short focus pictures obtained by contact printing from the negatives ; but, as a rule, it is better to do the plotting from enlargements, as this not only increases the scale of the picture, thus enabling the plotter to take off his distances more accurately, but it also places the horizon line outside the area to be plotted, which is a great convenience, and tends to greater accuracy. There is, however, a limit to the scale of the enlargement for convenient handling, and this is usually made 3 to 4 times that of the original picture. The scale of enlargement should be the same for all the pictures ; this is a great convenience, but not a necessity, so long as the plotter knows the scale of enlargement in each case.

The exact positions of the camera stations are first plotted, and through the stations " distance lines " are set out with a good protractor in the directions successively occupied by the optical axis of the lens when the exposures were made. Along these lines the (enlarged) focal length of the picture is laid off, and through the points thus obtained lines are drawn at right angles to the distance lines, these forming the horizon lines of the pictures to be dealt with. The views taken at a pair of stations are then carefully examined, and the corresponding points on the same object are marked with red ink dots ; a letter or other distinguishing mark is written against each dot for identification. The plotter next takes two long narrow strips of paper, the length of the strips being as long as the enlargement is broad, and draws a fine line across the middle of each strip. On the edge of one strip the distances of the dots right and left of the principal vertical plane of the picture are marked. This strip is then laid down on the plan with the edge carrying the marks exactly in coincidence with the horizon line drawn

thereon, and the transverse line exactly in coincidence with the distance line. The strip is then fixed in place, and the other strip is marked from the corresponding picture at the second station, and is similarly fixed in position relative to the proper horizon line at that station. A pin is next stuck upright at each station, a long fine silk thread having a loop at one end and a piece of elastic attached to a paper weight at the other, being passed over each pin. The threads are drawn tight and adjusted in position so that they accurately pass through corresponding points on the paper strips; the intersection of the threads indicates the position of each point successively indicated by each dot. At this stage the plotter can note if the intersections are good, and decide whether other intersections are necessary for checking the positions of the plotted points.

The above process can be repeated indefinitely with other pictures from the same or other stations, and the outlines of the plotted work may be filled in as the work proceeds.

Levels.—The approximate height (H) of any point relative to the ground at the camera station may be obtained as shown in problem (5), page 378.

Clearly, the levels obtained in this way can only be approximate, as their computation is dependent on distances measured from the plan and the photograph. On a scale of 200 feet to 1 inch, distances less than about 2 feet cannot be accurately measured, and to this scale the levels would be within about 1 or 2 feet of the truth.

To reduce the levels to a common datum, it is necessary to obtain first the heights of all the camera stations relative to the datum point.

Contours.—Assuming that the optical axis of the lens has been set truly horizontal at each station, it is evident that the trace of the principal horizontal plane drawn on each picture will pass through all points at the same level as the optical axis, and if a line be drawn joining the plans of all the points which fall on the horizon line of any photograph, the line so drawn will be a contour line. If this be done at each station a series of contour lines will be obtained at irregular vertical intervals, since the heights of the stations relative either to each other or to a common datum will naturally be irregular. If special contours at given levels be required, the surveyor must select his stations so that the optical axis of the lens is successively at the given levels at which the contours are required. In this way approximate contours may be obtained for any given vertical interval.

The methods described above are those which have been used in Canada and elsewhere, and in careful hands they have been found to give satisfactory results.

For a full discussion of the subject, the student is referred to the treatise by Mr. Deville on Photographic Surveying, already referred to, and published by the Government Printing Bureau, Ottawa, Canada.

Stereo-photographic Surveying.—In this system—which is of comparatively recent development—stereoscopic views of the same terrain features are taken in pairs at opposite ends of a base line of known length and position. The two exposures forming a stereoscopic pair must be made in a vertical plane coincident with, or parallel to, the base line which forms the stereoscopic base. As in the older system, the negatives carry the images of the principal and horizontal lines or marks denoting their positions. The length of the base varies between 100 and 400 feet, being made proportional to the accuracy required, the square of the distance of the points being located, and inversely proportional to the focal length of the camera lens. The length of the base is determined either tacheometrically or by measurement with a steel tape, and the error of measurement should not be greater than 1 in 1,000.

Instruments.—The instruments used in this system must be of the camera-theodolite type. The Zeiss photo-theodolite has been specially designed for stereo-photographic surveying and consists of a fixed-focus camera having a metal body mounted on the upper or vernier plate of a theodolite. The frame supporting the trunnion axis of the telescope is attached to the top of the camera, and is fixed in position so that the line of collimation of the telescope is at right angles to the optical axis of the photographic lens. Vertical and horizontal angles are read as on a theodolite; bearings are obtained by the aid of a trough compass and distances may be obtained by reading on a horizontal stadia rod which forms part of the equipment.

Field Work.—The camera is set up at a station and adjusted to give the required view and an exposure is made. The base line is next set off by sighting through the telescope and the base is measured. The instrument is then set up at the forward end of the base and the optical axis is set parallel to its previous position by a backsight to the rear station through the telescope, and the second exposure is made. The two exposures form a stereoscopic pair. Obviously two such pairs may be obtained from each base line; it is also obvious that the base lines may be

arranged in the form of triangles or other figures to suit the terrain.

Plotting.—In laying down the plan it is usual to refer both distances and heights to the left-hand station of each pair. The principal line and picture trace are drawn for the left-hand station as in the older system, these lines being now at right angles and parallel to the base line respectively, in all cases. The radials through the points to be plotted are set off from the corresponding abscissæ (x, x_1, etc.), measured from the left-hand
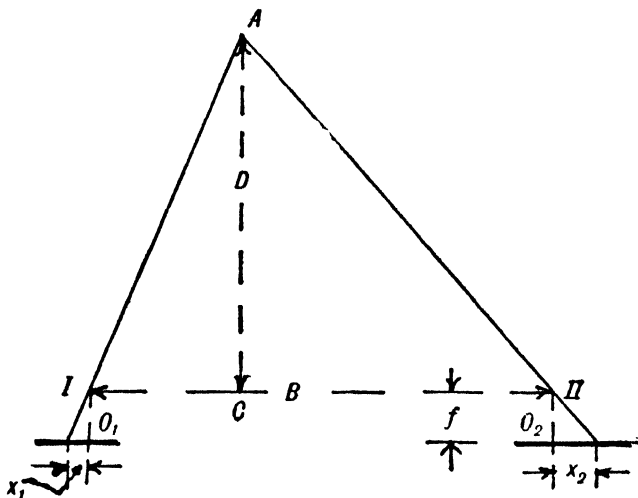
Fig. 195ғ.

photograph; the perpendicular distances (D, D_1, etc.) of the points from the stereoscopic base are laid down to scale along the principal line and the projections of these points, parallel to the base line, on the corresponding radials give the plans of the points to be plotted.

The distance (D) of any terrene point is computed as follows: Let I and II (Fig. 195ғ) represent the optical centre of the camera lens and O_1 and O_2 the positions of the corresponding principal points at the ends of the base of length B; also let the abscissæ of the terrene point A, as measured on the photographs, be x_1 and x_2, as shown on the figure. Then

$$\frac{D}{I\,C} = \frac{O_1\,I}{x_1} = \frac{f}{x_1} \text{ and } \frac{D}{B - I\,C} = \frac{O_2\,II}{x_2} = \frac{f}{x_2},$$

hence

$$D = \frac{B\,f}{x_1 + x_2},$$

which gives D in terms of known quantities. If in a particular case D is known, then B may be evaluated. Clearly the sign of x_1 will be negative when A is to the left of O_1 and similarly x_2 will be negative when A is to the right of O_2. The algebraic sum of the abscissæ of any pictured terrene point is called the linear parallax (p) of that point, hence we may write the above equation in the form

$$D = \frac{B\,f}{p} \qquad . \qquad . \qquad . \qquad . \qquad (\alpha)$$

If y is the ordinate of any pictured point from the horizon line of the photograph at any station, then the elevation (H) of the corresponding terrene point with respect to that station is given by the expression

$$H = \frac{D\,y}{f} \qquad . \qquad . \qquad . \qquad . \qquad (\beta)$$

$$= \frac{B\,y}{p} \qquad . \qquad . \qquad . \qquad . \qquad (\theta)$$

By differentiating equation (α) we get :

$$dD = f\frac{B}{p^2}\,dp \; ; \text{ but } p = \frac{f\,B}{D},$$

∴

$$dD = \frac{D^2}{B\,f}\,dp.$$

This shows that the error in locating points varies directly as the square of the distance and inversely as the base and the focal length. Hence if limits of error be assumed, the necessary length of base may be determined. For example, let the limit of error (Δ D) be $1 : 750$, the error in parallax $0 \cdot 0004$ inch, the focal length 9 inches and the limit of control 3,000 yards, then since

$$B = \frac{D^2}{f} \cdot \frac{\Delta p}{\Delta D}$$

we have

$$B = \frac{3{,}000^2}{\dfrac{9}{36}} \times \frac{0 \cdot 0004}{36 \times \dfrac{3{,}000}{750}}$$

$$= 100 \text{ yards.}$$

It is usual to make the base not more than 80 per cent. of its computed length.

The Stereocomparator.—It is obvious that the measurement of the quantities x, y and p must be made with a considerable degree of accuracy if the system is to be a success. These measurements are made on an apparatus called the stereo-comparator, designed by Dr. Pulfrich and made by the well known firm of Carl Zeiss, Jena. Essentially this apparatus consists of a table to which the two negatives are fixed and illuminated from below. The table is arranged to move in a slide parallel to (*i.e.*, left to right, or right to left of) the observer, the movement (X) being controlled by a screw. The right-hand negative is carried on a separate slide mounted on the table and is capable of two movements (p and h) parallel to and at right angles to (*i.e.*, to or from) the observer. The two movements are controlled by screws at right angles to each other. The two negatives are viewed through a special form of stereoscope, both eyepieces of which are fitted with exactly similar indices. This stereoscope is carried on a slide controlled by a rack and pinion and capable of a movement (Y) at right angles to the observer. The four movements are measured either by scales and verniers, or by micrometer heads. To obtain the quantities x, y and p : (1) The negatives are clamped in an erect position with their horizon lines parallel to the direction of movement of the table. The table and right-hand negative are then adjusted in position by manipulating the three screws and the pinion until the principal points O_1 and O_2 are coincident with the indices of the stereoscope. These indices appear as a single point (the " wandering mark "). (2) Read the verniers and micrometers which, if the instrument be in perfect adjustment, should all read zero. (3) Next select any point in the stereoscopic view and bring the wandering mark to this point by manipulating the movements X, Y and p. (4) Read corresponding scales which now indicate the co-ordinates x and y and the parallax p of the point selected. The movement h of the right-hand negative is necessary to allow for the difference in height of the stations at the ends of the stereoscopic base.

The Stereoautograph.—By the aid of gears, slides, screws and levers, the four movements of the stereocomparator have been combined and the resultant movement given to a pencil which moves over a sheet of paper and draws the plan and contours of the pictured terrain. A second pencil may be arranged to draw contour lines on a photographic print fixed to the sliding table to

the left of the two negatives. The print so marked may be used for locating the contours on the ground and for checking purposes.

The name stereoautograph was given to this apparatus by its inventor, Major von Orel, of the Austrian Geographical Institute.

The stereoautograph has been further improved by Dr.Pulfrich of the Zeiss firm and the result of his labours is embodied in the apparatus known as the stereoplanigraph, in which by ingenious but exceedingly complicated optical-mechanical arrangements the exact orientations of the negatives at the time of exposure are reproduced, and hence pictures may be treated stereographically which otherwise present no stereographic impression. Like the stereoautograph this apparatus draws the plans and contours automatically (if we neglect the human factor in setting the wandering mark).

Sections and Contours.—It is evident from equation (α) that, for a given base, if p be kept constant, D will be constant also, and if the wandering mark be set to points in the stereographic view having the same parallax, all such points will be at the same distance from the base line ; hence the movement of the pencil in the stereoautograph will be a straight line parallel to the photographic base. From equation (β) we observe that if D be constant H will vary as the ordinate y ; hence if the value of the parallax (p) be set on the apparatus for a given distance (D) and the wandering mark be brought to successive points on the stereographic view by the X and Y movements only, the pencil will trace out the longitudinal section or profile of the ground made by a vertical plane parallel to the base and at a distance D therefrom. Again, from equations (α) and (θ) we note (1) that D varies inversely as the parallax p, and (2) for given values of p, H varies directly as y ; hence if the wandering mark be set to any terrene feature at a height H above datum as given by equation (θ), any other points in the stereographic view which can be brought to the wandering mark (using the X movement only) will be, (1) at the same height above datum, and (2) on a vertical plane parallel to and at a distance D from the base, the distance being that corresponding to the value of p used in determining H. It is obvious that other points may be found at the same altitude but at varying distances from the base by using suitable values of y and p. In this way the pencil of the stereoautograph—whose movements are controlled by the movements X, Y, and p—may be caused to take up successive positions which indicate the trend of the contour line at the given altitude. In

the same way contour lines may be traced out for any given vertical interval and heights above datum.

Accuracy.—The accuracy of stereographic surveying is greater than that of the older system and approaches that of good tacheometric work, measurements to 1 in 1,000 in a horizontal distance of 600 feet, and of 0·23 feet in level at the same distance, being not uncommon.

Aerial Photographic Surveying.—In this system overlapping photographs are taken automatically from an aeroplane which flies as near as possible at a constant height in a given direction. On arriving at the end of the course marked out, the aeroplane returns on a parallel course and photographs overlapping each other and those of the first series are taken, the process being repeated until the whole series of photographs covers the ground to be surveyed. The usual amount of overlap is about 50 per cent. in the direction of flight and about 25 per cent. in a direction at right angles thereto. If the object of the survey is limited to the planimetry of the ground surveyed, the camera axis is kept as nearly as possible vertical, but where the resulting map is to show both plan and contours, the former being the more important of the two, the axis is given an inclination (or a " tilt " in a forward direction) of about 30 degrees to the vertical.

In general all the photographs will require correction for height, tilt, swing, *i.e.*, inclination at right angles to the direction of flight, and height of terrain features, which produces distortion near the edges of the photograph. The first three are known as the plate constants and are determined by the aid of apparatus specially constructed for the purpose ; the last is allowed for by using the central area of each photograph only.

The Ordnance Survey Department of Great Britain has recently (1935) placed a contract for the photographic Air Survey of about 400 square miles in the Midland Counties, the area including the towns of Coventry, Birmingham and Wolverhampton. The aeroplane will be fitted with an automatic pilot to ensure greater accuracy in the flying conditions, and the contact prints will be made on a new type of paper which incorporates a thin sheet of aluminium foil in order to prevent distortion of the print under varying conditions of humidity and temperature. The results are to be used in the revision of the 25in. Ordnance Sheets, and it is expected that the methods adopted will result in the saving of several years in the revision of the Ordnance Survey Maps.

Plotting.—If accuracy is a secondary consideration the plan

may be formed by cutting the consecutive photographs to fit each other in their proper order, thus producing a " mosaic " plan.

When the object of the survey is the production of a topographical map a system of ground control is necessary. Each photograph must contain three or more easily recognised points whose positions have been fixed by trigonometrical means. If only three known points are pictured on a photograph the procedure is operose ; with four control points the procedure is as follows :—

(1) The figure formed by the points is first drawn to scale on the map ; its inclined sides are then produced in pairs to meet, thus in general obtaining two points outside the figure ; a third point may be obtained by producing a diagonal of the figure to intersect the line joining the points first found ; by drawing the diagonals of the figure and radials from the outside points through their point of intersection, other four-sided figures are formed by the intersections of these radials and the sides. If desired the process may be carried further by drawing radials through the points of intersection of the diagonals of the smaller figures. In this way a " grid " is built up for the map and an exactly similar grid is drawn on the photograph. The detail given on the photograph is transferred to the map by reference to the lines of the grid either by estimation or by the aid of proportional dividers.

(2) The photograph is mounted on a vertical board which can be given the movements necessary to allow for the plate constants. The mapping board rests on a table in front of the vertical board, and photograph and mapping board are viewed simultaneously by the aid of a camera lucida ; by adjusting the relative positions of the photograph, mapping board and camera lucida, the points of control on the photograph are brought into coincidence with the corresponding points on the map and the detail is then drawn by going over the lines of the projected image.

(3) The photographic plate is placed in an enlarging lantern and the image is projected on a vertical board which carries a sheet of paper on which the points of control have been laid down to the given scale. By adjusting the position of the board the control points on the image are made to coincide with those on the paper. The latter is then replaced by a sheet of photographic paper, the resultant print being sensibly true to scale. The topographical detail on the print is inked-in and copied on tracing paper. Finally the detail is transferred to the map, its position being determined by other control points already laid down.

This system has proved the cheapest and most accurate of the three.

All the above methods are dependent on a system of ground control which, in unmapped countries, is slow and expensive. Most recent investigators are all endeavouring to perfect a system of automatic plotting which is independent of ground work and based on the stereoscopic principle. One outcome of this is seen in the stereoautograph of Dr. Pulfrich already referred to. The problem is much more difficult than that of stereoscopic surveying from ground bases, and its final solution will render this system capable of superseding the other two.

EXAMPLES.

1. Discuss the relative merits of the different methods of traversing with the theodolite, and point out the method which you consider the best for use in an assumed case.

2. If in taking bearings with the needle the fore- and back-bearings on a particular line could not be made to agree within the limits of permissible error, what steps would you take to obtain the correct values of the bearings ?

3. Soundings are being taken along a line of section crossing a broad river. Explain how you would fix the points of sounding, (a) by a box sextant used in the boat, (b) by a box sextant used on the bank of departure, and (c) by two theodolites.

4. Give a brief description of the mining dial, and point out its special advantages for use in mining surveys.

5. In mine surveying, how are the surface and underground surveys connected to each other when the entrance to the mine is by (a) an adit, and (b) a vertical shaft ?

6. What are the principal objects to be kept in view in selecting the ground for a base line in a large survey ? Enumerate in sequence the operations necessary before the measurement of the base line commences.

7. Explain why Colby's base apparatus did not give complete satisfaction in practice. If a base line were measured with this apparatus in sunny weather, would the direction in which the line proceeded have any influence on the result ? Why is dull, cloudy weather the best in which to undertake base line measurements ?

8. Describe how the measurement of the segments of a base line are checked by triangulation.

9. Briefly describe the plane table and the method of using it.

10. A plane table has been moved from a station A to a station B, and a ray has not been drawn from a in the direction of the

new station. Give a method of fixing the position of b, assuming that three points already surveyed are visible at B and the direction of the magnetic meridian is (a) known and (b) unknown.

11. The height of the ground at a station A is 150 feet above M.S.L., and the height of the scaffolding over the station is 30 feet. Find the approximate height of the scaffolding at a second station B, whose height above M.S.L. is 50 feet, the distance between the stations being 30 miles.   (*Ans.* 35·93 feet.)

12. From the following data correct the given base line measurement for (a) temperature, (b) pull on the band, and (c) sag : length of base, 11,200 feet ; length of band, 300 feet ; sum of thermometer readings, 6,882° F. ; on fractional part of base, 122° F. ; standard temperature, 55° F. ; number of readings on each band length, 3 ; and on fractional part, 2 ; pull on band, 20 lbs. ; section of band, ·20 × ·02 inches. Assume the coefficient of expansion = ·0000063 ; E = 30,000,000 lbs. per square inch ; weight of band per cubic inch = ·28 lb. ; and the band to be supported at every 100 feet of its length. Write down the corrected length of the base.

<div align="center">(Ans. (a) + ·4933 foot ; (b) + 1·866 feet ;
(c) − 2·108 feet, and 11,200·2513 feet.)</div>

13. In Example 12, find the pull on the band in order that the corrections for pull and sag may balance each other.

<div align="right">(Ans. 20·83 lbs.)</div>

14. Give a brief description of a method of making a photographic survey. Explain clearly how you would plot the plan of a point whose position is shown on two photographs taken at the extremities of a known base.

15. The distance of a point B is 2,540 feet from a station A, as measured on a plan. B is shown on a photograph, taken at A, 0·75″ above the trace of the principal horizontal picture plane. If the focal length of the picture is 3 feet, and height of camera 4·5 feet, what is the height of B above A ?

<div align="right">(Ans. 57·5 feet, nearly.)</div>

16. Two proposed stations A and B, 70 miles apart, are at altitudes of 1,850 and 2,075 feet above M.S.L. Two intervening points C and D are at altitudes of 965 and 1,760 feet above the same datum, their distances from A being 25 and 60 miles respectively. If the height of the instrument at A is 25 feet above the ground at the station, find the height of the instrument at B, so that the mutual ray may clear the ground at D by 10 feet. By how many feet will the mutual ray clear the point C ?   (*Ans.* 77·5 feet ; 366·4 feet.)

CHAPTER XIII.

TACHEOMETRY.

Preliminary Remarks.—The term "tacheometry" is applied to the measurement of distances by telescopic observations on a staff held at a distant point.

The instruments used for this purpose may be divided into two classes—(a) tacheometers or omnimeters, and (b) telemeters. The former are essentially theodolites in construction, the latter are chiefly reflecting instruments.

The indirect measurement of distances by telescope is based on the solution of a triangle. In Fig. 196, if the height B C of
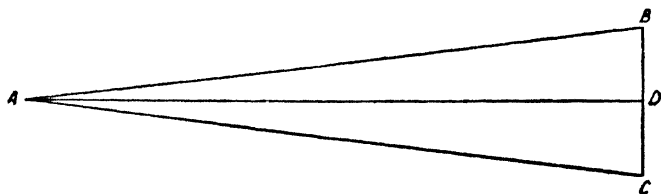
Fig. 196.

the isosceles triangle B A C, and the angle B A C are known, then the distance A D may be calculated. Three types of instrument are used. In one the distance B C (marked on a staff) is fixed and the angle B A C is variable; in the second B C is variable, being read on a graduated staff, and the angle B A C is fixed; and in the third type, both B C and the angle B A C vary. The second type is that most commonly used.

In the first type of instrument a staff carrying two vanes, or targets, at a fixed distance apart is held, by an assistant, vertically over the distant station, and the angular altitudes of the two vanes are determined with the instrument. From this data, and knowing the height of the lower vane above the foot of the staff, both the horizontal distance and the height of

the distant station relative to the instrument may be determined.
In Fig. 197, let A and B be the two stations ; C and D′ the vanes
fixed on the staff at a distance S feet apart ; also, let the angular
altitudes of D′ and C be α and β.

Then,   E D′ = G E tan α,

                          E C = G E tan β,

∴·   E D′— E C = G E (tan α — tan β).

But,   E D′— E C = S, and G E = D,

∴   S = D (tan α — tan β),

or, $$D = \frac{S}{\tan \alpha - \tan \beta}.$$
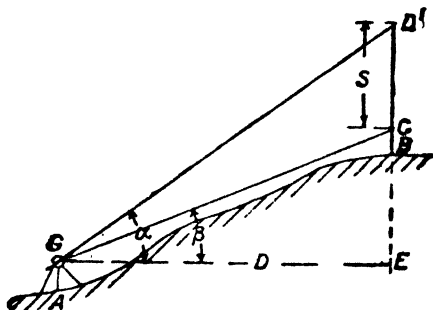
Fig. 197.

If β is an angle of depression this equation becomes

$$D = \frac{S}{\tan \alpha + \tan \beta}.$$

Also,   B E = C E — C B

                      = D tan β — C B,

tan β changing sign when β is an angle of depression.

To simplify the calculation of the required distance, two
methods are open to us, (a) we may vary the distance S and employ
certain standard angles, the sum or difference of whose tangents
is a simple number, such as ·01, ·02, etc. ; or (b) keep S fixed, and
read off the sum or difference of the tangents from a scale fitted
to the instrument, and specially graduated for the purpose.

Barcena's Method.—The former is the method first proposed
by Barcena, a Spanish writer on tachcometry. He had the

selected angles engraved on a special scale on the vertical circle of a theodolite, the scale being fitted with verniers. Angles such as the following are suitable for this purpose :—

Angle.						Tangent.
0° 34′ 23″,	.	.	.	.	.	. 0·01
1° 08′ 54″,	.	.	.	.	.	. 0·02
1° 43′ 06″,	.	.	.	.	.	. 0·03
2° 17′ 36″,	.	.	.	.	.	. 0·04
etc.						etc.

In this case, if both angles are angles of elevation or depression, and the angles selected have tangents, 0·03 and 0·02 respectively,

then
$$D = \frac{S}{\cdot 03 - \cdot 02} = 100\ S.$$

The height, if a " rise " $= 100\ S \times \cdot 02 -$ lower reading on staff

The height, if a " fall" $= 100\ S \times \cdot 03 +$ lower reading on staff

If β is an angle of depression, then $D = \dfrac{S}{\cdot 03 + \cdot 02} = 20\ S.$
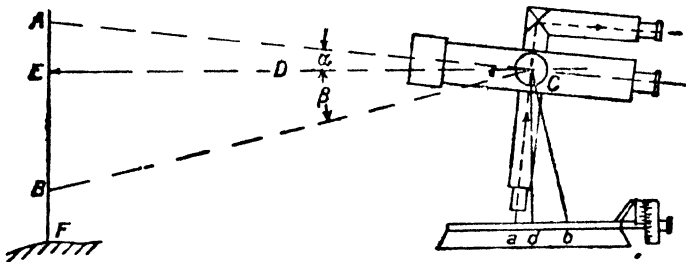
Fig. 198.

This method has not come into general use, because angles cannot be read with sufficient accuracy on the vertical arc of a theodolite.

Eckhold's Omnimeter.—The second method is exemplified in the omnimeter of Eckhold. This instrument is essentially a theodolite with the addition of a reading microscope fixed to the horizontal axis at right angles to the telescope (Fig. 198), and turning with it, and a scale fitted in a slide, placed parallel to the telescope on the horizontal surface of the upper vernier limb. For convenience in reading, the line of collimation of the microscope is bent at right angles, as shown diagrammatically

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in Fig. 198. The movement of the scale in its slide is governed by a screw of fine pitch, actuated by a micrometer head. When using the instrument, the telescope is directed to the upper vane on the distant staff and the scale is read (at a) with the microscope; a second reading on the scale (at b) is obtained when the telescope is directed to the lower vane. Since the remaining quantities in the calculation are constants, the difference of the scale readings is the measure of the required distance.

From the construction of the instrument, it is evident that the lines of collimation of the microscope and telescope turn through the same angles, hence in Fig. 198, the angle A C E $= \angle a\,\mathrm{C}\,d = \alpha$, and the angle B C E $= \angle b\,\mathrm{C}\,d = \beta$. If $h =$ height of the axis above the scale, then

$$\tan\ \alpha = \frac{a\,d}{h}, \tan\beta = \frac{d\,b}{h},$$

and $\tan\ \alpha + \tan\ \beta = \dfrac{a\,d + d\,b}{h}.$

Consequently, $\quad \mathrm{D} = \dfrac{\mathrm{S}}{\dfrac{a\,d + d\,b}{h}} = \dfrac{h\,\mathrm{S}}{a\,d + d\,b} = \dfrac{h\,\mathrm{S}}{a\,b}.$

Since the scale reads continuously from one end $a\,b$ is equal to the difference (l) of the scale readings, therefore $\mathrm{D} = \dfrac{h\,\mathrm{S}}{l}.$

The height of any point may be obtained with the instrument, for when the telescope is horizontal the microscope reads at a point of departure on the scale. The height of F (Fig. 198) relative to the axis of the instrument is equal to E F, and

$$\begin{aligned}
\mathrm{E\,F} &= \mathrm{F\,B} + \mathrm{B\,E} \\
&= \mathrm{F\,B} + \mathrm{D}\tan\beta \\
&= \mathrm{F\,B} + \mathrm{D}\frac{d\,b}{h} \\
&= \mathrm{F\,B} + \mathrm{S}\cdot\frac{l'}{l},
\end{aligned}$$

where l' is equal to the difference of the scale readings at b and the point of departure.

From the above, we note that we are merely concerned in obtaining the ratio of h to l from the instrument; consequently, the scale may be divided to any system of units we please. As

usually constructed, the scale is 4 inches long, and the height of the base of the instrument is 6 inches. The scale, with the aid of the micrometer head, is divided into 100,000 equal parts, and h is then represented by 150,000 of the same parts.

To determine the distance between two stations, the instrument is set-up over one station and the staff is held vertically at the other. The micrometer is set to zero, the upper vane is sighted, and the reading on the scale obtained. Suppose the cross wires of the microscope appear to cut the scale beyond the 72nd division, the micrometer head is then turned, either way, until the next division on the scale is in the centre of the cross wires of the microscope; let the micrometer head now read 341. If the middle point of the 72nd division has not been reached, the reading is 72,341 ; but if beyond the mid-point, the reading is 72,841. The telescope is now directed to the lower vane on the staff, and the reading (say) 70,452 is obtained in the same way; lastly, the telescope is set truly horizontal, and the reading (say) 50,006 obtained at the point of departure.
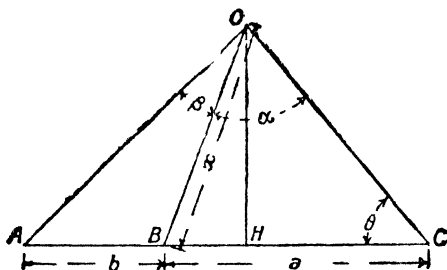
Fig. 199.

Let the distance between the vanes on the staff be 10 feet (the usual distance),

then

$$D = \frac{10 \times 150,000}{72,841 - 70,452}$$

$$= 627 \cdot 7 \text{ feet.}$$

$$\text{The height} = \frac{10 \times (70,452 - 50,006)}{(72,841 - 70,452)}$$

$$= 85 \cdot 55 \text{ feet — the height of the lower vane above the ground.}$$

A negative value indicates a " fall," a positive value a " rise.'

The accurate determination of distance with this instrument depends on knowing with exactness the length of the base on the instrument and the point of departure on the scale, in terms of the scale unit. These quantities are liable to change in value, owing to wear and other causes, and it is necessary to redetermine them from time to time. To compute these quantities proceed as follows:—Determine by trial the distances subtended on the scale by the known angles α and β (Fig. 199), let these distances be a and b, and O H be the required length. Also, let the angle O C H $= \theta$ and O B $= x$.

Then,
$$\frac{a}{x} = \frac{\sin \alpha}{\sin \theta},$$

also,
$$\frac{x}{b} = \frac{\sin (\alpha + \beta + \theta)}{\sin \beta},$$

∴
$$\frac{a}{b} = \frac{\sin (\alpha + \beta + \theta) \sin \alpha}{\sin \theta \sin \beta}$$

$$= \frac{\sin \alpha \{ \sin (\alpha + \beta) \cos \theta + \cos (\alpha + \beta) \sin \theta \}}{\sin \theta \sin \beta}$$

and
$$\frac{a \cdot \sin \beta}{b \cdot \sin \alpha} = \sin (\alpha + \beta) \cot \theta + \cos (\alpha + \beta),$$

∴
$$\cot \theta = \frac{a \sin \beta}{b \sin \alpha \sin (\alpha + \beta)} - \cot (\alpha + \beta), \text{ which gives } \theta.$$

Also,
$$\frac{C O}{(a + b)} = \frac{\sin (\alpha + \beta + \theta)}{\sin (\alpha + \beta)},$$

or,
$$C O = \frac{(a + b) \sin (\alpha + \beta + \theta)}{\sin (\alpha + \beta)},$$

and,
$$O H = C O \sin \theta.$$
$$C H = C O \cos \theta,$$

thus determining the base of the instrument and the point of departure.

The result of the calculation should be checked by taking a new set of values for α, β, a, and b.

Accuracy of Results.—The instrument has been largely used on Revenue surveys in India, and on railway surveys in America, with satisfactory results. It does not, however, compare very favourably with the ordinary tacheometer. In his report on the

omnimeter,* Colonel Laughton says:—" It has been found to give very accurate heights of buildings, etc. ; also to be wonderfully accurate when used as a levelling instrument ; but it is not so accurate for measuring distances over 600 feet, and even at this distance the error sometimes amounts to as much as 1 foot. It is recommended as admirably adapted for city surveys and traversing ; also in hilly and jungly countries, and for railway and similar purposes."

This defect is no doubt due to the difficulty of the manipulation of the instrument. To determine a distance, it is necessary to set the vertical arc twice, once for each vane on the staff, and also to read and focus the microscope twice, and it is difficult to perform these operations without disturbing the centre of the instrument by pressure. In distant readings there is more risk of errors caused by change in atmospheric refraction when the readings have to be taken separately than when read almost simultaneously, as with the tacheometer, and, further, the effects of wear, or of defects in the construction of the instrument, militate seriously against its readings.

Field Book.—The table on next page is a convenient form of field book.

Remarks on the Field Book.—(1) The stations of the instrument are indicated by capital letters, and the staff stations by corresponding small letters with suffixes. (2) The height of the trunnion axis at each station is measured with the tape, and entered in the third column. (3) The bearings are " whole circle " bearings, and may be either magnetic or true bearings. Before leaving any station, the fore-bearing from that station to the next in order must be read, and also the back-bearing to the preceding station. The fore- and back-bearings should agree within the limits of permissible error. (4) The staff stations should be selected so that the staffman moves continually clockwise around the instrument, thereby saving time both in the field and drawing office. (5) The computation of distances and altitudes is performed as explained on p. 395. In computing altitudes, care must be exercised to subtract 2 feet from the computed altitude if it be a " rise," and to add a like amount if it be a " fall." (6) In computing the reduced levels, we note that the numbers in the altitudes' columns are altitudes of the ground at the staff stations relative to the axis of the instrument. Hence, to find the reduced level of the axis at A, subtract the

* Report on Omnimeter, by Major G A. Laughton, Superintendent, Bombay Revenue Survey.

Staff = 10 feet. Lower vane 2 feet above ground. Base = 150,000. Point of departure = 50,006.

| Station | | Height of axis above ground. | Bearing. | Reading on Scale. Upper vane. Lower vane. | Difference. | Distance (feet). | Altitudes. Back-sight. | | Intermediate. | | Fore-sight. | | Reduced Levels. Of Axis. | Of Staff Station. | Remarks. |
Of Inst.	Of Staff.						+	−	+	−	+	−			
A	a	5·00	50° 30′	72,841 / 70,452	2,389	627·8	83·55						166·45	250·00	B.M. on top of left-hand gate post, at P on plan.
	a₁		94° 15′	68,456 / 66,321	2,135	702·6			74·42					240·87	
	B		187° 20′	53,826 / 52,658	1,168	1,284·24					20·75			187·20	
B	A	4·82	7° 20′	47,839 / 46,672	1,167	1,285·34		36·57					192·02	161·45	

rise (83·55) from A to *a*, from the reduced level (250) of the
B.M., thus obtaining 166·45. The reduced levels of all staff
stations read from A are then obtained by adding all the rises
and subtracting all the falls from 166·45. The reduced level of
the ground at station A is obviously 166·45 — 5 = 161·45.
The reduced level of the axis at B is equal to 187·20 + 4·82
= 192·02, and on subtracting the fall from B to A from 192·02,
we again obtain the number 161·45, thus checking the levels
at the main stations A and B.

Each folio of the field book should be accompanied by a sketch
of the ground surveyed. On the sketch the positions of the staff
stations are indicated by letters corresponding to those in the
field book.

Use of Stadia Wires.—In tacheometric instruments having a
constant visual angle, two or more additional cross wires are
placed on the diaphragm of the telescope (see Fig. 200), and on
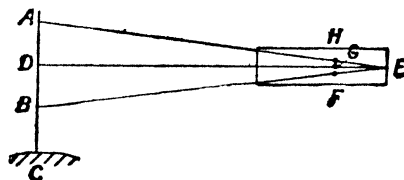
Fig. 200.

the assumption that light travels in straight lines, the angle
formed by the lines joining the eye of the observer to the addi-
tional cross wires is constant. These additional cross wires are
called " stadia wires."

The principle was first made use of by William Green in 1778.
He used a simple tube having three horizontal wires (F, G, and
H, Fig. 200). With this arrangement the distance from the staff
to the observer's eye varies directly as the length of the staff
apparently intercepted by the stadia wires; for the triangles
A B E and F E H (Fig. 200) are similar,

$$\therefore \quad \frac{HF}{AB} = \frac{EH}{EA} = \frac{EG}{ED},$$

or,

$$ED = \frac{AB . EG}{HF}$$

$$= \frac{S . r}{i},$$

where S is the staff reading intercepted by the stadia interval i, and r is the distance from the eye to the plane of the cross wires.

This primitive type of instrument is no longer employed, although an instrument of a similar form is occasionally used in Austria in levelling for railway sections. In this the tube is replaced by a pair of sights of the slit and window pattern. Crossing the window aperture is a pair of wires ·03 meter apart. The distance from one sight to the other is ·3 meter, thus the ratio of r to i is equal to 10, and the required distance is ten times the staff reading.

It is obvious that with instruments of this simple construction the determination of short distances only is possible; for long distances some form of telescope must be employed, in which case the simple distance formula given above must be modified to allow for the variation in the factor r, in focussing on staves at different distances.
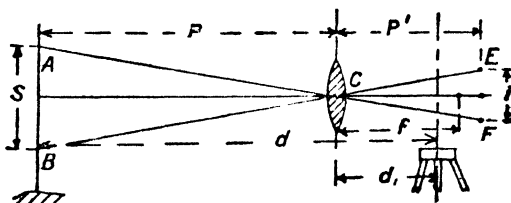
Fig. 201.

Theory of Application to Telescopes.—The theory is as follows: —First, when the line of collimation is truly horizontal.

Let d = the distance of the staff from the axis of the instrument,

P = the distance of the staff from the object glass of the telescope,

P' = distance of the image of the staff from the object glass,

f = focal length of the object glass,

i = stadia interval,

S = height of intersected portion of staff, and

d' = distance from the object glass to the vertical axis of the instrument.

Referring to Fig. 201, we have, from similar triangles A B C and C E F,

$$\frac{S}{P} = \frac{i}{P'},$$

or
$$\frac{P}{P'} = \frac{S}{i}.$$

Also, from the formula for lenses, we have

$$\frac{1}{P} + \frac{1}{P'} = \frac{1}{f},$$

∴
$$1 + \frac{P}{P'} = \frac{P}{f},$$

that is,
$$1 + \frac{S}{i} = \frac{P}{f}$$

∴
$$P = \frac{f}{i} \cdot S + f;$$

adding d' to both sides of the equation, we get

$$d = \frac{f}{i} \cdot S + f + d',$$

clearly a linear equation, since for the same instrument f, i, and d' are constants.

Line of Collimation Inclined to the Horizontal.—Secondly, when the line of collimation is inclined at an angle v to the horizontal, the staff being held vertically.

In this case the staff reading must be resolved perpendicular to the line of collimation A B, as shown at P R (Fig. 202).

Now the angle $C B P = B A j = v$, and considering the triangle C B P to be right-angled at P, we have $R P = D C \cos v$

$= S \cos v$, consequently $A B = \frac{f}{i} \cdot S \cos v + f + d'$, and $A j = d$

$= \frac{f}{i} \cdot S \cos^2 v + (f + d') \cos v.$

Since $(f + d')$ is a short distance, and v usually a small angle, this equation may be taken as equivalent to

$$d = \frac{f}{i} \cdot S \cos^2 v + f + d'.$$

The vertical component $H = A B \sin v$

$$= \frac{f}{i} S \cos v \sin v + (f + d') \sin v,$$

or H may be deduced from d, since $H = d \tan v$.

Necessity for Holding Staff Vertical.—Suppose the staff to be held at an angle θ to the vertical, then the distance becomes $d'' = \frac{f}{i} S \cos^2 v \cos \theta + f + d'$, and the error caused by the staff being out of the vertical is

$$d - d'' = \frac{f}{i} S \cos^2 v (1 - \cos \theta)$$

$$= \frac{f}{i} S \cos^2 v \frac{\theta^2}{2}$$

$$= d \frac{\theta^2}{2} \text{ nearly.}$$
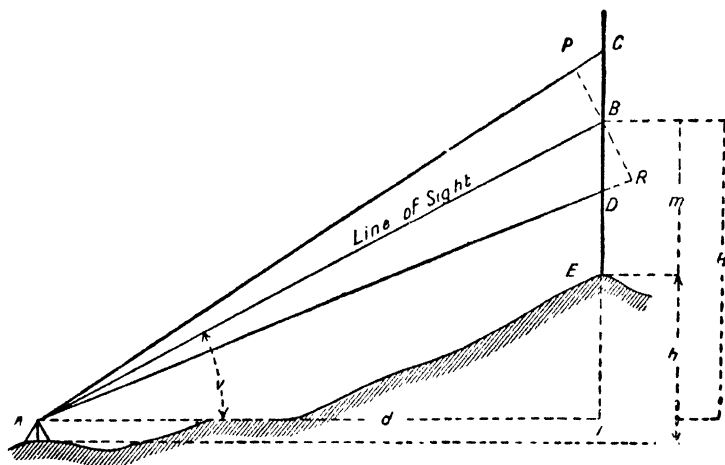
Fig. 202.

Let the inclination of the staff to the vertical be $1°$, then the error

$$= \frac{d}{2} \left(\frac{\pi}{180} \right)^2$$

$$= \frac{d}{6,600} = \cdot15 \text{ foot per 1,000 feet nearly,}$$

when $\theta = 2°$,

the error $= \dfrac{d}{1,640} = \cdot6$ „   „

when $\theta = 3°$,

the error = $\dfrac{D}{730}$ = 1·37 feet per 1,000 feet nearly.

These errors clearly show the necessity for using a plummet, or a spirit level (Fig. 66) on the staff, as the greatest deviation from the vertical should not be more than 2°.

Staff Perpendicular to Line of Sight.—Thirdly, when the line of collimation is inclined at an angle v to the horizontal and the staff is held perpendicular to the line of sight.
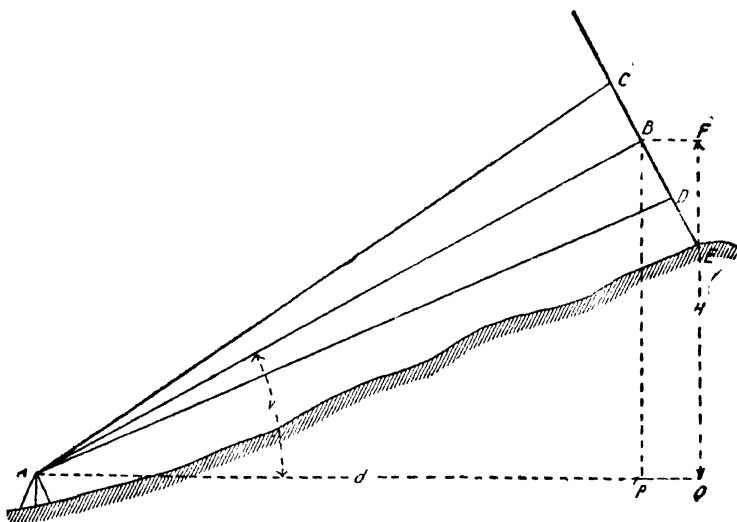
Fig. 203.

Most surveyors and engineers who use the tacheometer, do so with the staff held vertical; but some German engineers prefer to hold the staff at right angles to the line of sight. For this purpose, a pair of sights is fixed to the staff at right angles to its length, and in setting the staff in its proper position, the staff-man sights back to the instrument.

In this case the staff reading S requires no correction for the angular altitude, hence $d = \left\{ \dfrac{f}{i} S + f + d' \right\} \cos v$, and H = d tan v, as before.

These formulæ, however, introduce small errors, which must be allowed for. The horizontal distance given by the formula is equal to A P (Fig. 203); to obtain the horizontal distance to the foot of the staff, B F must be added, thus—

$$d = \left(\frac{f}{i} S + f + d' \right) \cos v + E B \sin v.$$

To find the height of the staff station relative to the axis of the instrument, ordinarily, we deduct from the vertical component H the reading given by the middle wire; but, in this case the reading (E B) given by the middle wire is too great, hence to determine the required level we must deduct E F, or E B cos v from the computed value of H.

Porro's Lenses.—In the general equation $d = \frac{f}{i} S + f + d'$. the term $\frac{f}{i}$ S is less than the distance between the axis of the instrument and the staff by the quantity $(f + d')$, and it appears as though the required distance were measured from an imaginary point $(f + d')$ feet in front of the instrument. It is, of course, impossible to set the instrument forward of the station by this amount, but it is possible to move back the position of the point until it coincides with the axis of the instrument by using an additional lens in the telescope. This additional lens was first used by Porro, a Piedmontese engineer, in 1823. The lens is placed between the object glass and the vertical axis of the instrument, and is constructed and arranged so that its focus coincides with that of the object glass. Consequently all rays after passing through this lens become parallel, while the sizes of objects subtending the same angle at the centre of " anallatism " or " unchangeableness " are directly proportional to their distances from the anallatic point. This point is arranged to fall in the vertical axis of the instrument, hence the necessity for adding the constant distance $(f + d')$ is obviated.

In addition to inserting the anallatic lens in his telescope, Porro used two large achromatic object glasses placed one behind the other, thereby reducing spherical aberration, and obtaining a clearer image. He also used three eyepieces, one to each wire, the wires being placed further apart than is possible with the ordinary eyepiece.

The final form assumed by Porro's instrument was so extremely delicate that it did not come into general use ; his anallatic lens is, however, frequently fitted to tacheometric instruments.

In all instruments fitted with this lens, its position in the telescope tube is made adjustable by a key. An example of an instrument fitted with an anallatic lens is shown in Fig. 204.

Porro's Anallatic Lens not Necessary.—The trouble involved in adding the constant distance $(f + d')$ to calculated distances is, however, more academic than real, hence we find that in many tacheometers the third lens is omitted. The only addition required to convert a good theodolite into a distance reading instrument is that of two extra horizontal cross wires on the diaphragm.

The conclusions arrived at as the result of exhaustive tests on the Schuykill topographical survey, and set forth in a paper read by Mr. Lyman, before the Franklin Institute in 1868, are (a) that "the additional complications introduced in the construction of the instrument by Porro's lenses are needless, as the inconvenience caused by adding the constant quantity $(f + d')$ is trifling"; (b) that "three fixed horizontal wires are sufficient for all purposes, the reading on the middle wire being a sufficient check on those obtained from the other two"; (c) that "the distance between the middle and outer wires should be $\frac{1}{100}$th of the focal length of the object glass, thus

Fig. 204.

avoiding calculation"; (d) that "the staff should be graduated to hundredths of a foot"; (e) that "a telescope magnifying 20 times and capable of reading to $\frac{1}{200}$th of a foot at a distance of 660 feet will give results as correct as those of Porro's larger instrument"; (f) and finally, "the errors due to spherical aberration may be neglected in angles of less than 10 degrees on each side of the focal axis."
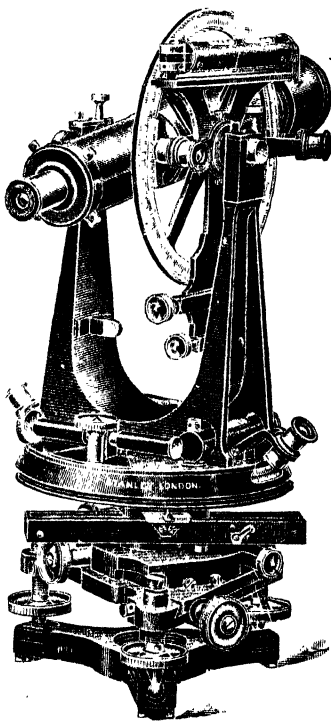
Accuracy of Distance Readings.—From these conclusions it appears that with a telescope reading to $\frac{1}{200}$th of a foot at a distance of 660 feet the error would be 50 $(13 \cdot 20 \mid \cdot 005) - 660$ $= \cdot 25$ foot, or about $\frac{1}{2600}$ of the measured distance. Using the same telescope at double the distance, and reading with half the above exactitude, the error would be about 6 inches in 1,320 feet. With a good telescope reading clearly to $\frac{1}{100}$th of a foot at 1,000 feet, the maximum error should not exceed $\frac{1}{2600}$th of the measured distance. From this it appears that distances obtained by telescope are quite as accurate as those obtained in good chain work over level ground, and on unfavourable ground, where the chainage error easily rises to 1 in 600 or 700, they are more accurate.

With regard to the displacement of the position of the staff caused by errors in angular measurement, it is well to note that at a distance of 1,000 feet, an error of 1′ causes a displacement of $\pm 0 \cdot 29$ foot, 30″ a displacement of $\pm 0 \cdot 145$ foot, and 20″ a displacement of $\pm 0 \cdot 097$ foot; at 500 feet the displacements are one-half of these amounts. A lateral displacement of $0 \cdot 29$ foot in 1,000 feet in the position of an object is not serious, as the error is too small to be shown on the plan; but, an error in levels of this amount is too great to be neglected. Great care is, therefore, necessary in reading the vertical angles, and the instrument employed should be a 5-inch or 6-inch instrument reading to 20″.

In order to obtain data for estimating the accuracy of tacheometric distances and levels, Mr. R. E. Middleton, M.I.C.E., conducted a series of experiments on a plot of ground near Wimbledon. He arranged a number of stations whose positions were fixed trigonometrically from a carefully measured base line, and obtained their levels by spirit levelling in the usual way. The stations were then surveyed and levelled tacheometrically on the tangential system of Eckhold, and also on the stadia system. Mr. Middleton calls the latter the "subtense" system.

The total length of the lines surveyed on the former system was 48,944·88 feet, the average length of line 479·88 feet, the average error $+ 0 \cdot 116$ foot, and the number of lines 102. On the subtense system the corresponding quantities were:— 56,047·72 feet, 452·00 feet, $- 0 \cdot 43$ foot, and 124.

From the results as given in Mr. Middleton's paper,[*] it appears that the limit of accuracy is reached with the tangential system,

[*] "Practical Observations in Tacheometry," *Proc. Inst.C.E.*, v 1. cxvi.

when the staff is about 1,000 feet from the instrument, as the average error in lines varying from 100 to 1,000 feet was + 0·116 foot per 1,000 feet, while the average error in lines more than 1,000 feet was + 2·44 feet. With the subtense system a distance of 800 feet appears to be the maximum for accurate reading, as up to 800 feet the error was 0·43 foot per 1,000 feet, while beyond that distance the error increased rapidly.

In levelling, the results given by the two systems are practically identical, and averaged 0·06 foot per mile on lines up to 800 feet in length, and 0·085 foot on lines above 1,000 feet.

The labour of reducing the observations appears to be about the same in both systems.

On the Mexican Boundary Survey, the average error in distance was 1 in 752, in level 0·282 foot per mile on fairly good ground, and 0·985 foot per mile on rough country.

On the United States Lake Survey, the permissible error was 1 in 300, the average error on 141 lines was 1 in 650.

On the St. Louis Topographical Survey, a traverse over 40 miles in length, the mean error in level on the whole distance was 0·64 foot, and the mean error in distance was 0·18 foot per mile.

Tacheometric surveying is now very largely used by engineers on the Continent of Europe, in the United States, and in the British Colonies. It is not much used in this country owing to the general use of ordnance maps for preliminary plans, but for the preliminary survey work for all engineering works of construction, its use is convenient, rapid, and of sufficient accuracy. The final setting out of a railway or road must, however, be done with the chain, theodolite and level, the centre line being marked out with pegs at every chain, as explained in Chap. XIV., Part II.

Effects of Refraction.— One source of error in stadia work is due to the unequal effects of refraction on the rays of light intercepted by the stadia wires. If both rays were equally affected no error would arise, but the ray intercepted by the upper wire passes nearer the ground than that intercepted by the lower wire, as the latter comes from the upper part of the staff. In consequence of this, the lower ray will suffer most from the effects of refraction, and the reading given by the upper wire will tend to be less than it ought to be. This will be most apparent when the staff and the instrument are on opposite sides of a summit, and the intervening ground is dry and hot.

The error due to this cause may be reduced by reading near the upper part of the staff, and by noting that the reading given

by the middle wire is the mean of the readings given by the stadia wires, at each observation.

Determination of Constants.—The constants f and d are easily found. To determine the focal length of the object glass, focus the instrument on the sun or moon, then measure the distance from the centre of the object glass to the diaphragm. The distance d is obtained by measuring the distance between the centre of the object glass and the vertical axis of the instrument.

The ratio of f to i must be determined with great accuracy, since it is the multiplier of the staff reading. As we have seen, the distance between the stadia wires should be $\frac{1}{50}$th or $\frac{1}{100}$th of the focal length of the object glass, thus making the ratio f to i 50 or 100 as the case may be, and although it is the duty of the maker of the instrument to set the wires at the required distance apart, his work should not be accepted until it has been checked. Some time ago the author had occasion to send a theodolite, to a London maker of repute, to be fitted with stadia wires having the ratio of f to $i = 100$; but, on checking the instrument its distance equation was found to be $d = 97 \cdot 88\,S + 1 \cdot 25$ feet, thus showing a considerable error on the maker's setting.

If the instrument is fitted with an anallatic lens the ratio of f to i is easily checked, since the reading on a staff 100 feet from the instrument should be exactly 1 foot; if at 200 feet, the staff reading should be 2 feet, and so on. When the instrument is not fitted with this lens the required ratio may be obtained as follows :—On a piece of level ground, chain off two distances d'' and d from the centre of the instrument, and obtain the staff readings at these points. Let the readings be S' and S; then from the general equation we have :—

$$d'' = \frac{f}{i} S' + f + d',$$

$$d = \frac{f}{i} S + f + d',$$

and
$$\frac{f}{i} = \frac{d'' - d}{S' - S}.$$

From the same data we have a check on the added constant, since by substitution we get,

$$(f + d') = d'' - \left(\frac{d'' - d}{S' - S}\right) S'.$$

The computed **values** of the constants should be checked from staff readings at two other distances, or, preferably, the distance equation of the instrument should be obtained from a graph, as explained on p. 138.

The Staff.— The staff should be made of some light **wood,** about 1 inch thick by 3 or 4 inches broad, and shod with brass. Its length should not be more than 14 to 16 feet. It is graduated to suit the purpose in hand, and the units employed, the gradu• ations are clearly painted on the face of the staff, the units being marked in bold figures. Either a plummet or a spirit level should be attached to the back of the staff to enable the staffman **to** keep it quite vertical during observation.

When the unit employed is the foot, the staff is usually divided to feet, tenths and hundredths, as in the case of the ordinary levelling staff (Fig. 64). In some cases the graduations are made to suit the stadia interval of the instrument. To do this, **a** point is fixed at a distance of $f + d$ in front of the instrument. From this point a length of 200 feet (or yards, or metres) is accurately measured. A mark is drawn across the blank face of the staff, which is then held vertical at the distant point, and the upper wire is set to this line. An assistant then draws a second mark on the staff at the point of intersection of the lower wire, as directed by the observer at the instrument. The observations are then repeated and the mean taken as the position of the lower mark. The space between the two marks is next divided into two equal parts, and each part is then further subdivided into tenths and hundredths. The objections to this form of staff are that (*a*) it can only be used with the instrument for which it has been prepared ; (*b*) should the stadia interval change, as when new wires are fitted, or from other causes, the staff must be regraduated ; and (*c*) the levels of the various points cannot be obtained without introducing laborious corrections.

Field Work.—The mode of procedure adopted in conducting **a** tacheometric survey will depend on its object ; but whatever the object of the survey may be, in this system, the responsible post is not at the instrument, and the chief engineer or surveyor must devote himself to directing the staffmen to the different staff stations, and to recording their positions on his sketch plan. The manipulation of the instrument and recording of observations must be left entirely to assistants specially deputed to this work.

As this system of surveying enables the engineer to determine levels as well as the relative positions of the topographical detail,

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for engineering purposes, many readings will be necessary for the former purpose alone. Such readings will be most useful for the determination of contours if they are taken on ridge and valley lines, as these lines indicate the points where the contours change in direction.

The number of assistants required will depend on the extent and nature of the ground to be surveyed, but the list will usually include the following:—One instrumentman; one clerk, to record the observations; three or more staffmen; one spare man to attend to rods, pegs, etc. If the ground to be passed over is wooded, two or more axemen must be added to the party.

If the object of the survey is the preparation of plans for the location of a railway, the operations will be in the nature of a traverse, and will proceed somewhat as follows:—The engineer in charge having selected the position of the first station, the instrumentman proceeds to set up his instrument over it, and he then measures the height of the trunnion axis above the ground. The staffman places his staff on the nearest bench mark (which has been previously placed in position by spirit levelling), and the instrumentman determines (a) the bearing of the staff; (b) the vertical angle; and (c) the staff readings of the bottom, middle, and upper wires. These observations, which are made at each staff station, are recorded by the clerk in their proper columns in the field book. While this is being done, the second staffman proceeds to hold his staff, with its back to the instrument, at the point indicated by the engineer, who records its position on his sketch plan. On receiving a pre-arranged signal from the instrumentman, the staffman turns his staff to face the instrument, and remains steadily in place with his staff quite vertical until he receives the signal that the instrumentman has finished his observations. Meanwhile the third staffman awaits his turn with his staff in position. As each staffman is liberated, he proceeds to set up his staff at the next vacant station in the series, the engineer, so far as possible, selecting the positions of the staff stations, so that the work goes continuously clockwise. When a pre-arranged number of staff stations have been dealt with, the instrumentman gives a special signal on a horn or a whistle, thus enabling the engineer, by checking the number of points on his sketch, to detect any which may have been omitted. The work continues in this way until all the points around the instrument, and within its range, have been surveyed, the last reading being taken with the staff at the next main station.

The positions of the staff and instrument are now interchanged, the height of the trunnion axis above the ground at the new station is measured and recorded, and the reading on the staff checks the bearing, distance, and level obtained from the preceding station. If these quantities differ within the limits of permissible error, the average value is used for plotting; but, if not, the staff readings as observed from the preceding station must be redetermined. The survey of the points around the second station is next dealt with, as described for the first station, any very important points fixed from that station are re-observed from the second station, and lastly, a reading is taken on a staff held on the next forward station. The positions of the staff and instrument are again interchanged, and the distance bearing and level checked as before; the survey of the points around the station commences, and so on to the end of the survey.

In distinguishing staff stations, it is convenient to denote the main stations by capital letters, and all the staff stations by corresponding small letters; thus all staff stations read from station A are denoted by a, a_1, a_2, a_3, etc., and similarly for the other stations.

If the survey covers a fairly extensive area, as in the case of a survey of a " gathering ground " and proposed reservoir for water supply, the survey may be conducted as described above, the traverse lines being arranged so that the whole of the topographical detail may be surveyed. A better plan, however, would be to arrange the stations to form triangles and polygons, as in a minor survey, the length of the sides of the triangles being kept within the range of the instrument. The topographical detail within each polygon could then be surveyed from the common vertex of the triangles forming it, and on the completion of the survey the lengths and bearings of the sides of all the triangles would be known. The work could be easily checked by chaining one or more sides of the triangles.

Form of Field Book.—The rulings and column headings of the field book for tacheometric surveying vary very much in the practice of different engineers, naturally each engineer thinks his own is the best, probably because he is most familiar with it. Some engineers prefer the entries to be made in a book, others prefer loose sheets, which are pinned on a thin pine board in the field, and are afterwards filed. The latter system is convenient in the field, but has the great disadvantage that the sheets are easily lost, thereby losing, maybe, the only record of the field work on several lines.

The field book should be about foolscap size, giving a broad column for remarks. Alternate pages only should be ruled, the opposite (blank) page being reserved for sketching-in the detail and positions of staff stations, referred to in the bookings on the (ruled) page opposite to it.

A form of field record which the author has found convenient in practice is given below. The entries are inserted in the book

TACHEOMETRIC

Survey, ...   Date,.....................

Instrument Station.	Height of Trunnion Axis above Station.	Staff Station.	Whole Circle Bearing.	Vertical Angle	Reading on Stadia Wires. Bottom Wire. / Top Wire.	Difference.	Reading on Centre Wire.
A	4·0'	a	65° 0'	+ 9° 15'	11·02 / 6·89	4·13	8·95
„		a_1	140° 15'	+ 8° 20'	7·52 / 4·27	3·25	5·89
„		a_2	175° 42'	+ 7° 30'	8·21 / 5·32	2·89	6·76
„		B	28° 30'	− 6° 12'	7·68 / 4·93	2·75	6·30
B	3·70	A	208° 32'	+ 6° 12'	2·78 / 0·03	2·75	1·40
		♭	150° 30'	+10° 42'	12·04 / 9·24	2·80	10·64
		b_1	182° 19'	+18° 30'	11·67 / 8·02	3·65	9·84
		b_2	320° 15'	+12° 18'	9·73 / 7·78	1·95	8·75

so that the student may have data for practice in computing distances and levels by the application of the rules already given.

Office Work.—The office work consists of the reduction of the field records of distances and levels, and the necessary plotting work in connection with the preparation of the plan.

Calculations.—Any good set of mathematical tables will suffice for the reduction of the distances and levels; a slide rule will

FIELD BOOK.

Multiplying constant = 100.   Unit of measurement = foot.

Collimation Distance.	Horizontal Distance, *d*.	Vertical Component, H.	Rise.	Fall.	Reduced Level.		Remarks.
					Of Trunnion Axis.	Of Staff Station.	
408·95	403·6	+ 65·75			43·20	100·00	On B.M.
322·85	319·5	+ 46·8		15·89		84·11	
287·75	285·3	+ 37·58		10·09		74·02	
274·65	273·1	− 29·66		66·78		7·24	
274·65	273·1	+ 29·66	31·96		10·94	39·20	
276·45	271·6	+ 51·36	12·46			51·66	
347·35	329·4	+110·2	59·64			111·30	
191·75	187·4	+ 40·8		68·31		42·99	
		Sums,	104·06	161·07 104·06			
		Diff. =	Fall =	57·01			

be found useful for the computation of the less important points, and also for checking purposes.

If the stadia interval on the instrument is 100th of the focal length of the object glass, the computation of distances (when the line of sight is horizontal) is very simple, we have only to move the decimal point of the staff reading two places to the right and add the constant. If the ratio is not 1 in 100, a table of distances for given staff readings should be prepared, as explained on p. 139, or if preferred, the graph of $\frac{f}{i}$ S may be plotted and the required quantities obtained by inspection.

When the line of sight is inclined to the horizontal, the number obtained from the table, or from the graph, must be multiplied by the square of the cosine of the inclination, and to the product the constant is added to give the required horizontal distance. In working out the products a slide rule will be found very useful.

If the circles on the instrument are divided on the centesimal system the labour of computation is much reduced.

Working out Reduced Levels.—When the line of sight is horizontal, the computation of the reduced levels proceeds as in ordinary levelling, but when the line of sight is inclined, the reduction of the levels presents more difficulty, and great care must be exercised in reducing the levels at the change points.

The staff being held vertical, the vertical component $H = d \tan v$ as already explained. This gives the vertical distance between the axis of the instrument and the apparent point of intersection of the middle wire and the staff.

Back-sights.—At a back-sight station, the staff will be held at the preceding station, whose level is known, or at a bench mark In either case, we must determine the reduced level of the axis of the instrument before proceeding to determine the reduced levels of the intermediates and fore-sights.

Let the reduced level of the axis of the instrument be x, the reading on the axial wire R, the vertical angle plus, and the level of the staff station B ; then, by inspection of Fig. 205, we see that
$$x = B + R - H.$$

When the angle of elevation is minus—
$$x = B + R + H.$$

Fore-Sights and Intermediates.—The reduced level (y) required at a fore-sight or intermediate reading is the reduced level of the

staff station. When the angle of elevation is plus (Fig. 206), $y = a + H - R$, a being the known level of the axis of the instrument.

When the angle of elevation is minus—

$$y = a - R - H.$$
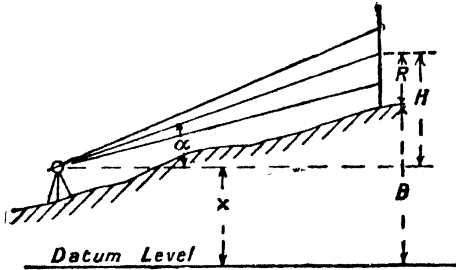
Fig. 205.

Plotting Work.—As in the chain surveying, the plotting of the plan should, so far as possible, keep pace with the field work, so that any errors or omissions may be speedily rectified. The contours obtained by joining spot levels (or by joining interpolated points) of equal altitude, should not be drawn in the
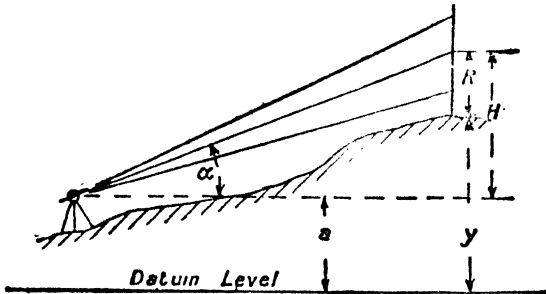
Fig. 206.

office ; either a tracing of the plan, or the plan itself, should be taken on the ground, and the contours joining the level points filled-in by inspection of the ground represented. In this way gross errors in joining the contour points are prevented.

The plotting of the main stations is carried out by the method of ' latitudes " and " departures," as explained on p 74, *et seq.*

The details are then plotted by lines radiating from each station, each line being, of course, drawn in the direction given by its bearing. On these lines the respective distances are scaled off, thus fixing the positions of the topographical details, or of spot levels, as the case may be.

An internal protractor (Fig. 54), constructed on thin horn paper, or celluloid, will be found very useful for plotting the survey points. The protractor is centred over the station with its zero line in the direction of the meridian, the plotting scale is then laid across the protractor in the direction given by the bearing, with a point of departure coincident with the station, and the survey point is pricked off in its proper position on the plan. This method avoids disfigurement of the plan by drawing the radiating lines at each station.

A special protractor has been devised with the same object in view. It is semi-circular in shape, with the scale of the plan engraved along the line of its diameter, the zero of the graduations being at the centre. The protractor is centred at the station and rotates around a fine needle, which is passed through a small hole at its centre. When the protractor is set with the bearing of a line in coincidence with a meridian line drawn through the station, the graduated diameter lies in the direction of the given bearing, and the position of the survey point is then plotted from the scale.

Micrometer Eye-piece.—A method of determining distances tacheometrically is sometimes used, in which the base on the vertical staff is a fixed length, and the stadia interval is variable. As in the subtense system, three horizontal wires are used, the middle wire (a, Fig. 207) being fixed in the optical axis of the telescope, and the outer wires b and c are movable towards or from the centre wire by screws S, S', of fine pitch. The distance between each of the outer wires and the centre wire may be obtained from the number of turns made by the screws, when the value of each turn is known. The whole turns made by the screws are read on the comb e, and the fractional parts of a turn are obtained from the divided drums D and D', which are fitted with verniers. The distance between the movable wires is the sum of the turns and parts of a turn made by the two screws. The scale on the drums may be divided into 60 or 100 equal parts.

The staff should be fitted with three vanes (A, B, and C, Fig. 208), at a known distance apart, the middle vane being exactly midway between the other two. By this arrangement

the outer vanes may be used for determining distance and the middle vane for levels; also, by setting the middle wire on the centre vane, a check is given on the distance readings, as the readings on the comb and drum-heads should be the same.

The general formula, $d = \dfrac{f}{i} S + f + d'$, applies to this instrument, f, S, and d' being constants, d and i the variables. As we
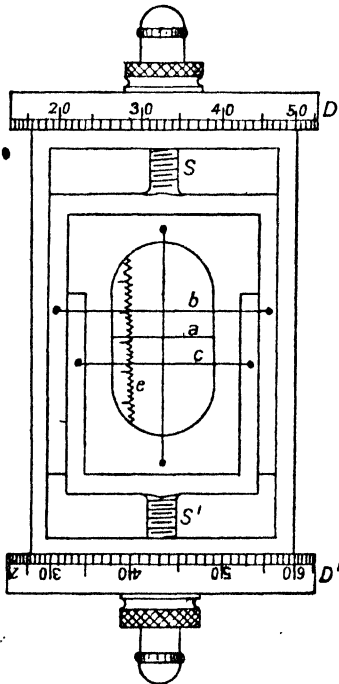
Fig. 207.
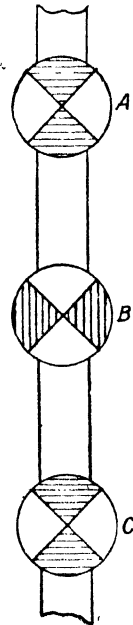
Fig. 208.

have already seen, the constants f and d' are easily obtained with sufficient accuracy by measurement on the instrument, and as in instruments with a fixed stadia interval, it is essential that the ratio of f to i be known with exactness. Before the variable i can be found, it is necessary to know the value (V) of the pitch of the screws in terms of the units employed. De-

noting $f + d'$ by C, and the sum of the turns made by the screws by N, the above formula becomes, $d = \dfrac{f}{V \cdot N} S + C$. N, of course, must be obtained at each staff reading. To determine the ratio of f to V, chain off accurately, on level ground, a distance d from the instrument, and note the number of turns made by the screws when the stadia wires are set to bisect the vanes on the staff held at the forward end of the measured base. Then, from the above formula, we have $\dfrac{f}{V} = \dfrac{N (d - C)}{S}$.

Example 1.—Let N = 20·050, d = 600 feet, S = 10 feet, and C = 1·5 feet.

Then, $$\frac{f}{V} = \frac{20 \cdot 050 \, (600 - 1 \cdot 5)}{10}$$

$$= 1,200 \text{ very nearly.}$$

Since S = 10 feet, the distance equation for the instrument is

$$d = \frac{12,000}{N} + 1 \cdot 5 \text{ feet.}$$

It is essential that the pitch of the screws be the same everywhere. To test this, the value of $\dfrac{f}{V}$ should be determined from observations on the staff held at different distances from the instrument. The result should be the same in all cases.

The computation of distance will be much simplified if the value of $\dfrac{f}{V}$ be made equal to 1,000. For an object glass of given focal length, this may be done by (*a*) cutting the screw thread to a suitable pitch, or (*b*) by altering the distance between the vanes. The former method is not easy to carry out in practice, as it involves the cutting of an exact screw to—in many cases—a very odd pitch. In the preceding case, if the focal length of the object glass be 12 inches, in order that $\dfrac{f}{V}$ may equal 1,000, the screws would have to be cut to a pitch of ·012 inch, or 83⅓ threads to 1 inch. The latter method is easily carried out; assuming S = 10 feet, the ratio of $\dfrac{f}{V}$ is obtained as described above, and the value of S′ which will make $\dfrac{f}{V}$ = 1,000 is computed from the distance equation; thus, in the above case,

$\dfrac{f}{V}$ S′ = 10,000, and S′ = $\dfrac{10,000}{1,200}$ = 8⅓ feet. The vanes being set to this distance apart, the distance of the staff from the instrument is then obtained by taking the value of N from a table of reciprocals moving the decimal point four figures to the right, and adding the constant C to the result. After alteration the staff must only be used with the instrument for which it has been adjusted.

Determination of the Index Error.—When the screws of the micrometer eye-piece are set to zero, the two movable wires should be in exact coincidence with the fixed centre wire. There are, however, mechanical difficulties in the way which prevent this coincidence being made with the necessary degree of exactness, and although there may be apparent coincidence when the screws are set to zero, index error may exist.

To determine the index error, set off a distance equal to the constant C in front of the instrument, and from this point chain off, on level ground, two distances A and B. It is convenient to make B double of A. Set the centre wire to cut one of the vanes of the staff held at (say) the nearer station, turn one of the screws until the corresponding wire cuts the other vane, and note the reading. Repeat this, using the same screw, with the staff held at the second station. If there be no index error, the second reading will be double the first, if there be an index error it will be constant and affect both readings equally. Let N and N′ be the number of turns made by the screw in the two cases, and x the index error, then

$$A(N + x) = B(N′ + x),$$

or

$$x = \dfrac{A.N - B.N′}{B - A}, \text{ but since } B = 2A,$$

$$x = N - 2N′.$$

The same mode of procedure will determine the index error for the other screw.

Example 2.—Let N = 20·435, and N′ = 10·206.

Then,
$$x = 20·435 - 2 \times 10·206$$
$$= 20·435 - 20·412$$
$$= + 0·023, \text{ which must be added.}$$

Let the readings obtained with the second screw be 20·584 and 10·312,

$$x = 20·584 - 2 \times 10·312$$
$$= - 0·040, \text{ which must be deducted.}$$

With the exception of the manipulation necessary to deter-
mine the stadia interval, an instrument fitted with a micrometer
eye-piece is used in precisely the same way as are instruments
in which the stadia interval is fixed. The formulæ given on
p. 401 for the reduction of inclined sights and for the computation
of the vertical interval also apply here, and, as the centre wire
of the telescope is set to bisect the middle vane or target, at each
reading, the staff reading for level will always be the same.

Reflecting Telemeters.—While in the determination of distance
by telescope the base of the measurement consists, on the one
hand, of a vertical staff carrying two simple marks exactly
10 feet apart, and on the other of a vertical staff graduated to
hundredths of a foot, from which the length of the base is obtained
by observation, in reflecting telemeters, the base is horizontal,
and is either contained in the instrument itself, or is measured
on the ground. The distant object whose distance is required
may be a plain vertical staff, or any vertical object such as the
vertical edge of a building.

In some instruments the angle between the reflecting surfaces
is fixed, and in others it is variable. In the former type, the fixed
angles are selected so that the required distance is some simple
multiple of the base, while in those having a variable angle the
required distance is either obtained from a scale on the instru-
ment, or by inspection from a table constructed to suit the
instrument, and the base used. The principle underlying the
construction of all telemeters is that the sides of a triangle may
be computed when the base and base angles are known.

Adie's Telemeter.—Of these instruments containing the base
of the measurement, that constructed by the optician Adie is
perhaps the best known. This instrument consists of a square
metal tube having two apertures placed in the side of the
tube near its ends ; the light entering each of these apertures
impinges on a reflecting prism, one of which is fixed, and the
other movable about its vertical axis by a tangent screw. The
rays of light are reflected from the two prisms in the direction
of the axis of the tube, in reverse directions ; these rays are
received on a pair of central reflecting prisms, which are reversed
in position, and direct the rays to the eye-piece, which is placed
in the centre of the side opposite to that carrying the apertures.
The usual length of the instrument is 36 inches.

As in other reflecting instruments, a horizontal line of separa-
tion of the field of view enables part of the reflected image of
the distant object to be seen in the lower and part in the upper

portion of the field. The exact alignment of the two parts of the image is obtained by turning the tangent screw to the movable prism, when the scale reading will give the angle required. The fixed scale is divided to read directly to two minutes, and this is further subdivided by the vernier to two seconds of arc.

When using the instrument, it is held as nearly as possible in a horizontal position, and the observer, looking through the eye-piece, turns the instrument round until he sees the distant object reflected in the fixed prism ; he then turns the tangent screw until the image of the same object as seen in the movable prism is brought into accurate alignment with the first image. The angle at the vernier is now noted, and the corresponding distance obtained from the table of distances supplied with the instrument.

The instrument is constructed so that the eye-piece and the various prisms are easily got at for cleaning purposes, and the end prisms are made adjustable for the purpose of taking-up any error in the position of the index.

Distances are obtainable on a 36 instrument with a probable error of about 1 in 1,000.

In all instruments of this type, the twice reflected images of distant objects tend to lose their sharpness of outline, and consequently the accurate alignment of the images requires the greatest care ; further, the long tube is liable to warp from the effects of solar heat, thus altering the angles of reflection and causing considerable error.

Weldon Range Finder.—This instrument, which is the invention of Colonel Weldon, R.E., consists of three prisms accurately ground to give angles 90°, 88° 51′ 15″, and 74° 53′ 15″. These angles are selected so that (1) the measured base may be set out at right angles to the direction of the required distance A D (Fig. 209) ; (2) the tangent of the base angle A B D may be 50 to 1 ; (3) the ratio of the sides A B and B E of the triangle A B E may be 4 to 1. The tangent of the angle 88° 51′ 15″ is 49·9989, and is taken as 50. This introduces an instrumental error of roughly 1 in 10,000, but this discrepancy is ignored, as it is a closer degree of approximation than can be obtained on distances with instruments of this class. The ratio of the sines of 74° 53′ 15″ and 13° 58′ is 3·99994, and is assumed to be 4.

Let A D (Fig. 209) be the distance required. Standing at A, and using the first prism, set up a rod at C in a direction at right angles to A D. Place a rod at A and move along the line C A until the image of the rod at A is exactly aligned with the image

of the object at D as viewed in the second prism. Suppose B
is the point at which alignment of the images takes place, then
the angle A B D will be 88° 51′ 15″, and A D is 50 times the length
of the base A B, which must be measured. A closer approxi-
mation is obtained by observing the distant object from both
ends of the base, as in Fig. 210, in which case the required dis-
tance is 25 times the length of the measured base C B.

When the distance to be determined is great, the time required
for the measurement of the base is reduced by using the third
prism. To do this, the point E is obtained, at which the images
of the points D and A—as viewed in the third prism—are accur-
ately superimposed. The point of intersection (B) of C A and E D
is found by lining through, and A B is measured. The angle
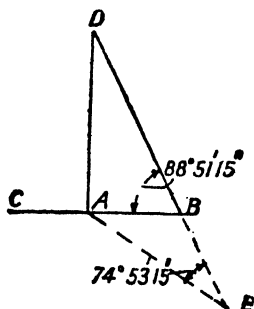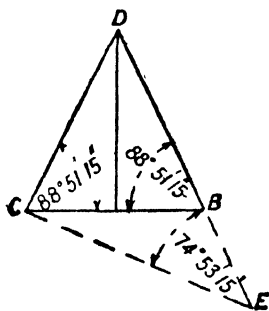
Fig. 209.　　　　　　　　　　　　　Fig. 210.

A E D will now be 74° 53′ 15″, and A D will be 200 times A B in
Fig. 209, or 100 times C B in Fig. 210.

The " Steward " Telemeter.—This is a small hand reflecting
instrument (shown about full size in Fig. 211), fitted with a
telescope of moderate power. The reflecting portion of the in-
strument consists of two small mirrors as in the sextant. One
of these mirrors is moved in azimuth by rotating a divided collar,
shown in Fig. 211, and the other by rotating a small toothed
wheel shown in the same figure at the forward end of the instru-
ment. When both the divided collar and the toothed wheel are
set to zero the angle between the mirrors is 45°

The instrument is designed for use on a normal base of 20 units,
but any base of not less than $\frac{1}{50}$th of the observed distance
may be used. When using a base other than 20 units long, the
distance given by the instrument must be corrected from the

proportion:—20 : units in measured base : : distance given by instrument : required distance.

The distance scale reads from zero to 1,000 units, each division corresponding to 10 units. The unit may be the yard, metre, pace, or any other measure.

To determine the distance of an object O (Fig. 212a) from a
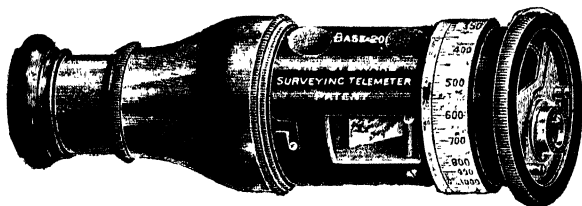
Fig. 211.

station A, the instrument is set to zero and the observer, holding the telemeter in his left hand, places it to his eye with the rectangular opening on his right towards O. By moving the instrument slightly in azimuth a position is found in which O is seen by reflection and at the same time forward objects are seen by direct vision. A well-defined forward object M, such as a church
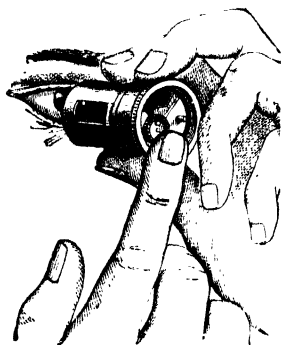
Fig. 212.
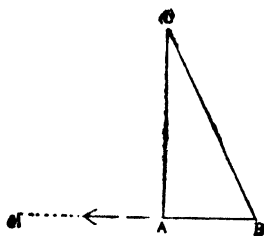
Fig. 212a.

spire, a tree, flagstaff, etc., is selected as a mark, and the observer then proceeds to move towards, or away, from O, until the reflected image of O is nearly coincident with M seen directly. Exact coincidence is then brought about by rotating the toothed wheel (as shown in Fig. 212). Care must be taken to make this coincidence exact, as it has a very important bearing on

the accuracy of the result. This preliminary operation estab-
lishes an approximate right angle (M A O), and before changing
his position, the observer must mark the point A vertically
under his eye by a picket, rod, or other handy object. The
observer now turns right about face, paces (or otherwise measures)
his base, and on arriving at its extremity (B) he carefully lines
himself in with A and M. Standing accurately over B and sighting
M directly, the divided collar is rotated until the exact coinci-
dence of the images of O and M is again obtained ; when this
occurs the reading on the scale will give either the required
distance (O A) or a number proportional to it, depending on
whether the base is of 20 or some other number of units ; thus,
if the reading on the scale is (say) 850, and the base 30 yards,
the required distance $= \dfrac{850 \times 30}{20} = 1{,}275$ yards. The result
may be checked by taking an observation of O at a point in the
base midway between A and B, when the scale reading should
be double that previously obtained.

For a full description of this instrument and the method of
using it under varying circumstances, the reader is referred to
the booklet issued by the maker, J. H. Steward, London.

The Bate Range Finder.—This instrument consists of a bi-
nocular field glass fitted with graduated limbs for measuring
angles. The angle subtended by the measured base is obtained
by placing a gauge between the graduated limbs, and the re-
quired distance is obtained by multiplying the length of the base
by the number given by the gauge. The instrument is used in
the same way as the Weldon and other range finders.

Reflecting telemeters are practically useless for accurate
surveying purposes, as distances cannot be obtained with them
to a sufficient degree of accuracy. They are, however, very
useful in carrying out surveys for military purposes, and for the
determination of ranges, for which they are chiefly intended.

EXAMPLES.

1. Compare the different systems of determining distances
by telescope.

2. Show how the horizontal distance between two stations
and their difference of level may be obtained by instrumental
observations on a staff carrying two vanes or marks at a fixed
distance apart. Briefly describe the construction of an instrument
which is based on this principle.

3. Suppose you were provided with a levelling staff and a level fitted with stadia wires. The constants of the instrument being unknown, how (without using a chain on the section) would you proceed to run a line of levels in a given direction?

4. What are the special advantages to be obtained by applying the principle of the tacheometer in surveying with the plane table? Illustrate your answer by sketches.

5. From the table given on p. 139, obtain the distance corresponding to a stadia reading of 9·43. (*Ans.* 924·34 feet.)

6. The data given in the accompanying table was obtained from observations made at one station. The reduced level of the station is 64 feet and the height of the axis of the instrument 4·5 feet. Find the reduced levels of the stations *c* to *i* inclusive.

Line.	Bearing.	Staff Station.	Horizontal Distance.	Vertical Angle.	Staff Reading.	Reduced Level.
			Feet.			Feet.
A B	54° 30′	*a*	150	6° 20′	3·6	81·55
,,	,,	*b*	200	,,	5·8	84·90
,,	,,	*c*	250	,,	9·7	..
A C	68° 34′	*d*	50	10° 13′	14·0	..
,,	,,	*e*	100	,,	8·2	..
,,	,,	*f*	150	,,	3·5	..
A D	120° 00′	*g*	75	0° 0′	0·5	..
,,	,,	*h*	120	,,	8·6	..
,,	,,	*i*	175	,,	6·7	..

7. From the data given in Example 6, prepare a plan showing the positions of the contours from 60 to 80 feet, rising in 5 feet vertical intervals. Obtain the contour points by interpolation.

8. If in Fig. 209 A D = 25 times A B, and A B = 4 times B E, find the angles D B A and A E B.
(*Ans.* 87° 42′ 33·9″ ; 73° 49′ 2·3″.)

9. The upper and lower vanes on a staff held vertically at a station B are observed from a station A. If the angular elevations of the vanes are 4° 20′ 30″, and 3° 10′ 15″ respectively, the distance between the vanes 10 feet, and the height of the lower vane above the foot of the staff 2 feet, find the distance between the stations and their difference of level. Assume the height of the instrument above the ground to be 5·2 feet.
(*Ans.* 487·27 feet, and 30·19 feet.)

10. An Eckhold telemeter having a base of 150,000 is used to determine the distance and height of a distant station. On

directing the telescope to the upper vane on the staff, the micro-scope reads 70·5, and the micrometer head 246, the corresponding readings on the lower vane being 68·0 and 448. The base on the staff is 10 feet, the height of the trunnion axis 4·82 feet, point of departure on the scale 50·012, and height of lower vane 2 feet. Find the distance between the two stations and their difference of level.   (*Ans.* 652·74 feet, and 83·046 feet.)

11. To determine the constants on an instrument having fixed stadia wires, readings were taken on a vertical staff at distances of 300 feet and 600 feet, and the observed staff readings were 2·91 and 5·84 feet respectively. Find the distance equation for the instrument.   (*Ans.* D = 102·389 S + 2·05 feet.)

12. Using the distance equation for horizontal sights given in answer to Question 11, determine the distance and reduced level of the staff station from the following data :—Stadia reading, 3·84 ; reading on axial wire, 5·69 ; angular altitude, + 2° 41′ ; reduced level of trunnion axis, 120·42 feet.

                    (*Ans.* 395·22 feet ; 133·25 feet.)

13. The screws in a micrometer eye-piece have 80 threads to 1 inch. Given $f = 1$ foot, $d = 0·5$ foot, and S = 10 feet, write down the distance equation for the instrument.

$$\left(\textit{Ans. } D = \frac{9,600}{N} + 1·5 \text{ feet.}\right)$$

14. What should be the distance between the staff vanes in Question 13, in order that the distance equation for the instru-ment may be $D = \dfrac{10,000}{N} + 1·5$.   (*Ans.* 10·417 feet.)

15. From the following readings determine the index errors of the screws of a micrometer eye-piece :—

          Staff distance (feet), 300,   .   .   600
          Screw reading on upper wire, 33·575,   .   16·723
          Screw reading on lower wire, 33·492,   .   16·764
                    (*Ans.* Upper screw error = + 0·129.
                         Lower screw error = — 0·036.)

CHAPTER XIV.

PRELIMINARY AND FINAL LOCATION.
SETTING-OUT.

Preliminary Remarks.—Setting-out or the location of the site of proposed works is the converse of surveying and levelling in their exact sense, as the object of the setter-out is the discovery of points on the ground corresponding to those indicated on the plan.

The greater portion of the setting-out required for engineering works on lines of communication consists of straight lines, as centre lines, with other transverse lines at right angles to them, required for the taking of cross-sections and setting-out of side widths, the straight lines being joined by curves at changes in direction.

Where good maps of the country exist, the preliminary location of a proposed line of communication may be decided upon from a careful study of the maps exhibiting the topographical detail between its termini. Obviously the problems which confront the engineer in locating the site of a common road are very different from those which confront him when locating the site of a railway, or a canal; but in all cases the chief factor to be considered will be the cost of construction. This may be kept down by (a) selecting the site so that the labour involved in the construction of cuttings and embankments may be a minimum; (b) avoiding so far as possible the construction of bridges, viaducts, and tunnels; and (c) avoiding the severance of important properties.

The engineer in charge of the construction of a common road has greater opportunities of keeping down the cost of construction in that he may use gradients which are inadmissible to the railway engineer, and also the curves at changes in direction may be to a much smaller radius.

Gradients.—As to gradients, the steepest ascent for wheeled vehicles that can be continued for any great distance without

causing animals extreme fatigue is 1 in 20. A gradient of 1 in 16 should not be continued for more than a quarter of a mile, nor one of 1 in 8 for more than 50 feet, this being the maximum for wheeled vehicles on roads in good condition.

Ruling Gradient.—In railway work, the maximum (or ruling) gradient depends on the greatest load per train, the least speed of ascent without obstructing traffic, and the construction and weight of the engine. From the results of experiments on train resistances conducted by D. K. Clark, the following formula [*] for determining the ruling gradient (i) has been obtained :—

$$i = \left\{ \frac{q\,\mathrm{E}}{7} - 0{\cdot}00268 \left(1 + \frac{\mathrm{V}^2}{1440} \right) \mathrm{T} \right\} \div (\mathrm{E} + \mathrm{T}), \quad . \quad . \quad (1)$$

where E denotes the weight of the engine, in tons,
   q E the part of that weight carried by the driving wheels,
   T the gross load of train and tender, in tons,
   V the least speed of train in miles per hour.

As a provision against the extra resistances caused by side winds, curves, and bad state of the roads, the factor 0·00402 may have occasionally to be substituted for the factor 0·00268.

Example 1.—Let the weight of the engine be 50 tons, and 70 per cent. of this fall on the driving wheels ; weight of train and tender, 400 tons ; and the minimum speed of ascent, 20 miles per hour ; find the ruling gradient.

$$\text{Here} \quad i = \frac{\dfrac{0{\cdot}7 \times 50}{7} - 0{\cdot}00268 \left(1 + \dfrac{20^2}{1440} \right) \times 400}{400 + 50}$$

$$= \frac{1}{123{\cdot}9}.$$

If we use the factor 0·00402,

$$i = \frac{\dfrac{0{\cdot}7 \times 50}{7} - 0{\cdot}00402 \left(1 + \dfrac{20^2}{1440} \right) 400}{450}$$

$$= \frac{1}{152{\cdot}8}.$$

The ruling gradient adopted in the construction of a railway is the maximum gradient permitted on the straight lengths only,

* Rankine's "Civil Engineering."

on a curve the gradient must be reduced to allow for the greater resistance. This additional resistance is allowed for in the factor 0·00402, in which it is taken at 20 per cent. above that on the level. In selecting a ruling gradient, the principal factors to be considered are the nature and amount of traffic on the line. On suburban lines the traffic will be much heavier at certain times of the day than it will be on (say) a branch line, hence the gradients to and from large centres of population should be smaller than the general ruling gradient, so that the traffic may be quickly dealt with.

Preliminary Location.—Having decided the ruling gradient, the proposed centre line is sketched on the maps of the district; the maps are then cut into strips about a foot wide for convenience in handling on the ground. The strips may be pasted together to form one continuous roll. The centre line is next marked out on the ground in straight lines, the direction being indicated by stakes, poles, marks at points of crossing of fences, walls, etc., the positions of these marks being as nearly as possible in agreement with their positions as shown on the plan. The centre line is then inspected on the ground along the whole of its course, and if the general direction is found to be satisfactory, the separate lengths are moved into the best possible positions that the local surface features will permit. During this process long straight reaches shown on the plan may be broken into several straights, in order to take advantage of local peculiarities in the surface of the ground, or to avoid obstacles; the slight movement of a line laterally may shorten the length of a bridge, or may entirely avoid its construction. Again, there may be two ways of connecting any two points, one which is straight and the work of construction heavy, the other may be more roundabout but the work of construction light, and, in deciding which course to follow, it is necessary to balance the cost of construction against the subsequent cost of maintenance and costs of running.

If the general direction of the proposed centre line is considered unsatisfactory after inspection on the ground, new routes are set out and studied *in situ*, until by trial and error the best possible route, under the given conditions, is obtained.

When the direction of the centre line has been finally decided upon, the separate lengths are carefully straightened out, either by ranging with the eye, or with a theodolite. The survey of the ground in the neighbourhood of the centre line then follows, with the object of bringing the plans up to date. The centre line is marked out with stakes at every 100 feet, or 66 feet, as

the case may be, and longitudinal and cross-sections are obtained along the whole of the route. From this data the preliminary plans, sections, and estimates are prepared. At this stage of the work, the centre line marked on the ground consists of a chain of straight lines. To obtain a gradual change of direction at their junctions, these lines are joined by curves—usually circular arcs—whose radii and positions are decided in the drawing office.

Final Location.—After the preliminary plans and estimates have been passed, before proceeding to the construction of the railway, the proposed site is again carefully inspected, to ensure that nothing has been overlooked which may improve its position, and where considered desirable, the line is again moved until no further improvement appears to be possible. The ground is then carefully levelled from end to end, curves are ranged in position, cross-sections are obtained at every 50 or 100 feet, or at greater distances as deemed necessary, bench marks are established at every 10 chains or thereabouts, and near the sites of proposed bridges, viaducts, tunnels, etc.

From the amended particulars the final plans, sections, and estimates are prepared.

Location of Canals.—The location of the site of a canal is carried out in a similar way to that of a railway, except that the canal engineer is more concerned in keeping the proposed route along a particular set of contours, than in taking the most direct course to his objective. The object of the canal engineer is to place the proposed site in the position which will have the longest horizontal lengths, the shape of these lengths on the plan is not of primary importance except in the case of canals intended for the passage of ships, when curves should be avoided wherever possible.

The curves joining the straight lengths do not require to be set out with the accuracy that railway work demands; they should, however, be of as large radius as the local circumstances combined with the necessity of keeping exactly to a pre-determined line of levels will permit.

In deciding the final location, care must be exercised to ensure a supply of water sufficient for all purposes, and if sufficient natural sources of supply (rivers, streams, etc.) are not available, artificial sources must be created by the construction of impounding reservoirs, lakes, etc., the sites of these additional works being selected as near as possible to the summits on the proposed routes.

Location in Unmapped Country.—Where reliable maps of the country to be passed through are not available, a reconnaisance and route survey of the country must be made, advantage being taken of all points of elevation from which more or less extensive views may be obtained. If there are trade routes between the termini of the proposed line, such routes will generally follow the line of least resistance offered by the country passed through, and will often be an invaluable guide in the preliminary operations. From the knowledge gained by the rough survey the preliminary location is set out, an accurate survey and section of the route then follow, and from these the preliminary plans and sections are prepared. Then follows the final location during which drastic changes may be made as the result of the experience gained from the preliminary operations. The plans and sections are then corrected, and final estimates prepared.

In moist tropical countries, owing to the rapid growth of vegetation, some difficulty is experienced in marking out the centre line so that it may be readily found at a later period. Wooden pegs, or stakes, are quickly overgrown, and are subject to rapid decay both from the attacks of insects and from climatic conditions. In passing through forests, the route will, as a rule, be sufficiently indicated by the passage created during the early operations by felling trees and cutting away of undergrowth; if this is not considered sufficient, other trees may be felled or marked, as thought desirable. In passing through jungle, the vegetation is cut away with axes, or billhooks, until a clear passage is obtained; this is then marked by any means, such as stones, trunks of trees, poles, etc., which may be available at the time. On arid plains, stones piled into heaps will form suitable marks.

After the final location, the whole of the centre line should be " nicked " out between the stakes by cutting a trench about a foot wide and 6 inches deep.

Setting-out Straight Lines.—When the preliminary reconnaisance and setting-out have been completed, it is necessary to straighten out the lines joining the points at each consecutive change of direction. An experienced person may do this, on favourable ground, by ranging poles in line with the unaided eye, but this method is not to be recommended. In setting-out a long straight line the greatest care should be taken to keep it quite straight, and the setting-out should be done with a theodolite, preferably with an instrument of the transit type.

If the extremities of the line are mutually visible, the marking

out of intermediate points is easily done, as the instrument may
be placed over the station at one end of the line and the telescope
directed to the forward signal; intermediate signals then bi-
sected by the cross wires will—if the instrument is in perfect
adjustment—be in the given line. It is seldom, however, that
the work is so simple; in most cases, the instrument will have
to be set up at several different points on the line before it is
completely delineated.

If the extremities of the line are not mutually visible, the line
may be set-out if its bearing, or its angular deviation from the
line joining the rear station to some permanent referring object,
is known. The mode of procedure is as follows :—Suppose P
(Fig. 213) is the given referring object, A and B the rear and
forward stations respectively. Set up the instrument at A,
send a chainman forward with a pole as far as it can conveniently
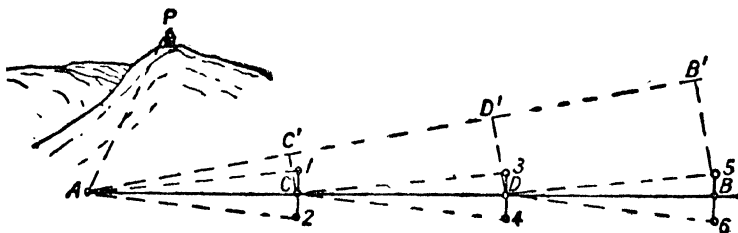
Fig. 213.

be seen, and turn off the given angle, or bearing, P A C, the
vertical circle being (say) on the left. If the instrument be in
perfect adjustment, when the forward pole is so placed that its
image is bisected by the cross wires, the pole will be exactly
in the line A B. It is seldom, however, that the instrument
will be in perfect adjustment, and the point marked will be a
little to the right or left of its true position. Suppose the pole
has been set up at the point 1 (Fig. 213); the angle is then
turned off with the circle on the right and the corresponding
point 2 is found. The chainman then drives a stout peg at the
point C, exactly midway between the points 1 and 2, and the
theodolite is moved forward and set up over it. The telescope
with (say) the vertical circle on the left is directed to the back
station A, transited, and the forward point 3 is marked. This
is repeated with the vertical circle on the right, and the position
of the point 4 is found. The middle point (D) of the line joining

the points 3 and 4 is marked, as before, with a stout peg, the theodolite is again moved forward and set up over the peg at D : these operations are then repeated until the forward extremity of the line is reached.

If the line has been correctly set-out, the station B will be midway between the last two points set-out ; if the closing error is only a few inches the signal may be moved and re-erected at the end of the line. When the closing error is large, the line must either be re-ranged, or if the setting-out is considered to be accurate and the forward signal must not be moved, the closing error B′ B (Fig. 213) is measured as an offset to the line set-out, and each of the pegs marking the successive positions of the instrument must be moved in the direction of B′ B by an amount proportional to its distance from the back station.

Thus, the peg at D′ is moved to D and $D' D = \dfrac{A D' \times B' B}{A B'}$.

The same mode of procedure may be adopted when the direction of the line at the rear station is only approximately known. The line is set-out by face-right and face-left repetitions as near as possible in the desired direction, and is afterwards moved to its proper position by moving the pegs in the direction of the closing error.

Obstacles, such as buildings, swamps, etc., are dealt with by the methods described in Chap. II., Part I., or a traverse may be run around them.

The method of setting-out side widths has already been dealt with (*vide* p. 159, *et seq*).

Setting-out Curves.—Curves are required to obtain a gradual change of direction at the intersection of the straight reaches. Circular arcs are generally used for this purpose ; but parabolic curves are frequently used on Continental railways. In this country, a curve is designated by its radius and the units employed, as a curve of so many chains, feet, or miles. In America a curve is designated by the angle subtended at its centre by a chord 100 feet long ; in this system the curves are spoken of as curves of 1°, 2°, 3°, etc. The radius of a degree curve is easily found, if we assume the length of the chord is equal to its subtended arc. Thus, if A B is the given chord, and R the radius of the curve, we have

$$A B : \pi R :: \theta° : 180°,$$

or

$$R = \dfrac{A B \times 180°}{\pi \theta°},$$

and for a 1° curve $R = \dfrac{100 \times 180}{3\cdot1416}$

$= 5,729\cdot6$ feet.

For a 2° curve, $R = \dfrac{5,729\cdot6}{2}$

$= 2,864\cdot8$ feet, and so on.

Strictly the radius, $R = \dfrac{100}{2 \sin \dfrac{\theta}{2}}$, and this value must be used

where it is necessary to allow for the disparity between the length of a chord and its subtended arc.
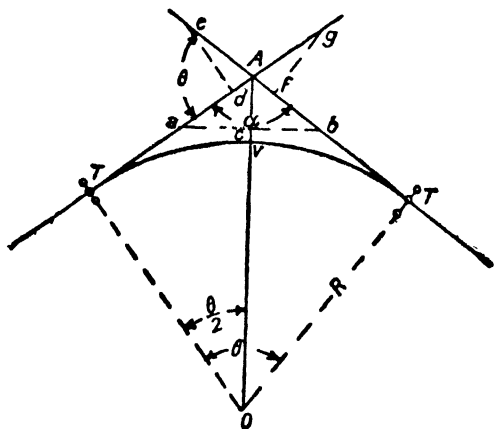
Fig. 214.

Length of Chord.—In setting-out curves, the lengths of the chord and arc are always assumed to be equal. This holds without sensible error, if the chord is not more than $\frac{1}{20}$th of the radius.

Different Systems of Setting-out Curves.—There are many methods for setting-out curves in common use, but all methods may be comprised in the following systems, depending on the instruments employed:—(1) Chain and tape systems; (2) theodolite and chain system; (3) the two theodolite system.

In the description of the methods of setting-out curves which

follow, it is assumed that (*a*) the point of intersection of the tangent lines is accessible ; (*b*) the ground is free from obstacles ; and (*c*) the radius of the curve is known.

Before a curve can be set-out, the angle of intersection of the two tangents must be known. This is generally observed with a theodolite centred over the intersection point. If an instrument for measuring angles is not available, the angle may be obtained by calculation from data furnished by chain measurements. To obtain the requisite data, chain off a length of 2, 3, or more chains from A (Fig. 214) along each of the tangent lines, thus obtaining the points *a* and *b* ; measure the length of the chord *a b*, then $\sin \frac{a}{2} = \frac{a\,b}{2\,A\,a}$, where *a* is the required intersection angle. If this method is not convenient, produce both the tangent lines beyond the intersection point. Mark off a length of 1, 2, or more chains along the tangents, thus obtaining the points *d* and *f*. At these points set out perpendiculars meeting the tangent lines produced in the points *e* and *g*. Measure *d e* and *f g*, then $\tan \theta = \frac{e\,d}{d\,A} = \frac{f\,g}{f\,A}$, the mean of the two results is used in the subsequent calculations.

To determine the points of tangence, chain from A along the tangent lines the distances A T and A T_1, such that $A\,T = A\,T_1$ $= R \tan \frac{\theta}{2}$.

It is often convenient to set-out the vertex V of the curve from the intersection point A. To find the position of V, chain along the bisector of the intersection angle the distance

$$A\,V = A\,O - V\,O$$

$$= R \left(\sec \frac{\theta}{2} - 1 \right).$$

The positions of three points on the curve are now determined, the method of setting-out the remaining points will depend on the system adopted.

Length of Curve.—Let L = length of the arc between the tangent points T and T_1, expressed in the same unit as the radius R,

I = intersection angle.

Then, the angle subtended at the centre of the arc T T = (180° — I), and L : π R :: (180° — I) : 180°,

$$\therefore \quad L = \frac{\pi R (180° - I)}{180°}, \qquad \bullet \quad \bullet \quad \bullet \quad (\beta).$$

$$= \cdot 01746 (180° - I) R$$

$$= \frac{\cdot 01746}{30} \left(\frac{180° - I}{2} \right) 60 \, R$$

$$= \cdot 000582 (5,400 - x) R, \qquad \bullet \quad \bullet \quad (2)$$

where x is the half-intersection angle in minutes.

Equation (β) may also be written in the form

$$L = \cdot 0002909 \, R \, \theta, \qquad \bullet \quad \bullet \quad \bullet \quad (3),$$

where θ is the supplement of the intersection angle in minutes.

Number of Chords.—If we assume the length of unit chord to be equal to its subtended arc, the above formulæ give the number of chords to be set out on the curve.

When setting out the curve by the method of deflection angles, if the deflection angle corresponding to unit chord be α, then the number of chords

$$= \frac{\theta}{\frac{2}{\alpha}} = \frac{(180 - I)}{\frac{2}{\alpha}}$$

$$= \frac{5,400 - x}{\alpha}, \qquad \bullet \quad \bullet \quad \bullet \quad \bullet \quad (4)$$

x, the half-intersection angle, and α being in minutes.

Various Systems of Ranging Curves.

I. To Set-out a Circular Arc by the Method of Offsets to a Chord Produced.—Having marked the points of tangence, the setting-out begins from the rear tangent point. The chain is pinned down at this point, the distance T B (Fig. 215) is marked on the tangent line, and the tape, or a special offset rod, is pinned down at B. The chain and tape are then swung around their fixed ends until they meet at the point C, which is so placed that

$$T B = T C, \text{ and the first offset } B C = \frac{T C^2}{2 R}$$

$$= \frac{\text{chord}^2}{2 R}.$$

The chain is then dragged forward, pinned down at C, and extended to the point D, in the chord T C produced, and the tape is pinned down at D. The chain and tape are next swung around the points C and D until they meet at the point E, the second offset D E being made equal to $\dfrac{T D \times C D}{2 R}$. If the chords T C and C E are equal, the second and succeeding offsets will be equal to $\dfrac{C D^2}{R} = \dfrac{\text{chord}^2}{R}$. The remaining points are set-out in the same manner.
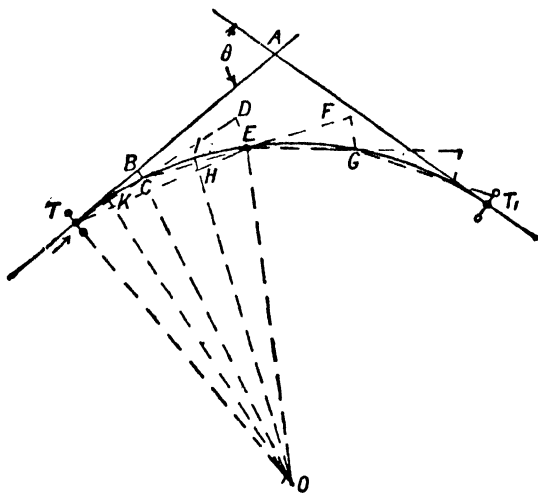
Fig. 215.

Owing to the necessity of keeping the permanent stakes at the standard distance apart, the first chord will usually be shorter than the others, as it will rarely happen that the rear tangent point is a whole number of chains from the commencement of the line. Suppose, for example, that the tangent point occurs at a distance of 546·40 chains from the commencement of the line, or "peg 0." The first peg on the curve will be peg 547, and the first chord T C will be 60 links. The remaining chords, with the exception of the last, will be of constant length, and after the point E (Fig. 215) has been set-out the offsets will be constant with the exception of that closing on the forward tangent point T".

The formulæ for the lengths of the offsets are obtained as follows :—Let T A (Fig. 215) be tangent to a circular arc of radius R and centre O. Draw T C any chord ; join C O, T O, and from C and O drop perpendiculars C B and O K on T A and T C re·spectively.

As may be seen by inspection, the triangles T B C and T K O are similar,

$$\therefore \qquad \frac{B C}{T C} = \frac{T K}{T O},$$

$$\text{or} \qquad B C = \frac{T C . T K}{T O},$$

$$\text{but} \qquad T K = \frac{T C}{2}, \quad .$$

$$\therefore \qquad B C = \frac{T C^2}{2 . T O}$$

$$= \frac{\text{chord}^2}{2 R}. \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad (5)$$

Produce T C to any point D ; make the chord C E equal to C D, join T E and D E. Bisect the arc C E at the point I, join I O, and let I O cut C E at H. Since, in practice, D E is small compared with O C or C D, T D will be nearly equal to T E ; also the angle E T D is equal to the angle C O H, therefore the triangles E T D and C O H are approximately similar, consequently

$$\frac{D E}{T D} = \frac{C H}{C O} \text{ very nearly,}$$

$$\therefore \qquad D E = \frac{T D . C H}{C O}$$

$$\text{that is,} \qquad D E = \frac{T D . C E}{2 C O}$$

$$= \frac{T D . C D}{2 R}. \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad (6)$$

When T C = C D, T D = 2 C D, and we get, for equal chords, the offset $D E = \dfrac{C E^2}{R} = \dfrac{\text{chord}^2}{R}$. $\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad$ (7)

One advantage of this method is that it may be used in confined positions, as all the work entailed in the setting-out is done in the immediate neighbourhood of the curve. It has, however, the great disadvantage that any point incorrectly placed will cause an error in the positions of all the subsequent points, the error increasing approximately as the square of the distance from the incorrect point. From this cause, the curve will rarely close accurately on the forward tangent point, and, if the closing error is large, the curve must be set-out again. When the final error is less than 10 links, it is divided among the points by moving them sideways by an amount which is proportional to the square of their distances from the commencement of the curve.

A thin lath should be used for marking off the offsets. A hole is bored through the lath near one end, and the offsets are laid off from the centre of the hole and marked by notches on one edge of the lath. When in use, the lath is pinned down by an arrow passed through the hole, and is swung around the arrow until the appropriate notch is in contact with the end of the chain.

II. To Set-out a Circular Curve by Offsets to a Tangent Line—Baker's Method.—In this method the curve is set-out by rectangular co-ordinates having a tangent line as axis and the tangent point as origin.

The preliminary data for setting-out the curve having been obtained, the equal distances, whether of 66 feet or 100 feet, or less, are chained along the tangent line from the tangent point, and at each distance offsets perpendicular to the tangent line are set-out. The length of any offset y at a distance x from the tangent point is equal to $R - \sqrt{R^2 - x^2}$.

$$\text{When } x = 1 \text{ chain, the offset } y = R - \sqrt{R^2 - 1^2}.$$
$$\text{,, } \quad x = 2 \text{ chains, } \quad \text{,, } \quad = R - \sqrt{R^2 - 2^2}.$$
$$\text{,, } \quad x = 3 \quad \text{,, } \quad \text{,, } \quad = R - \sqrt{R^2 - 3^2}.$$
$$\text{,, } \quad x = n \quad \text{,, } \quad \text{,, } \quad = R - \sqrt{R^2 - n^2}. \quad (8)$$

These quantities having been calculated are set-out along the corresponding offsets, thus determining the required points on the curve.

The equation $y = R - \sqrt{R^2 - n^2}$ is demonstrated as follows: —Let y be the offset to the tangent line from the point at the extremity of n chains measured from the tangent point T, and

A be the corresponding point on the curve. From A (Fig. 216) draw A B parallel to the tangent line to meet the radius T O at B. Join A O. Then,

$$T B = y = T O - B O$$
$$= R \pm \sqrt{A O^2 - n^2}$$
$$= R \pm \sqrt{R^2 - n^2}.$$

The plus sign gives the positions of points on the other side of the vertex, and as such offsets are inconveniently long, the minus sign only is made use of.
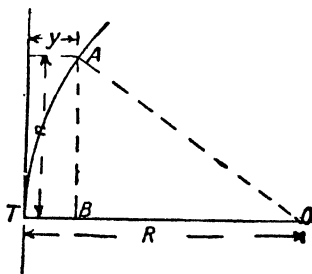
Fig. 216.

Baker's Method.—The general formula

$$y = R - \sqrt{R^2 - n^2}$$

may be written in the form

$$y = R - R \left(1 - \frac{n^2}{R^2} \right)^{\frac{1}{2}}. \qquad \bullet \qquad \bullet \qquad \bullet \quad (a)$$

Expanding the factor $\left(1 - \dfrac{n^2}{R^2} \right)^{\frac{1}{2}}$, we get,

$$\left(1 - \frac{n^2}{R^2} \right)^{\frac{1}{2}} = 1 - \tfrac{1}{2} \frac{n^2}{R^2} - \tfrac{1}{8} \frac{n^4}{R^4}, \text{ etc.}$$

Neglecting powers of R higher than the second, we obtain

$$\left(1 - \frac{n^2}{R^2} \right)^{\frac{1}{2}} = 1 - \tfrac{1}{2} \frac{n^2}{R^2} \text{ nearly.}$$

Substituting this value in equation (a), we get

$$y = R - R \left(1 - \tfrac{1}{2} \frac{n^2}{R^2} \right)$$
$$= \frac{n^2}{2R}. \qquad \bullet \qquad \bullet \qquad \bullet \qquad , \qquad \bullet \qquad (9)$$

From this equation we observe, if the offsets are equidistant and the first offset is y,

The offset at the end of the 2nd chain $= 2^2 y$.

,, ,, 3rd ,, $= 3^2 y$.

,, ,, Nth ,, $= N^2 y$. . . (10)

This system of curve ranging is known as Baker's system, and was formerly much used for railway curves on account of its general convenience, since very few figures are required in the field; the curve set-out is, however, not a circular arc but a parabola, as is evident from its equation. This is not a serious
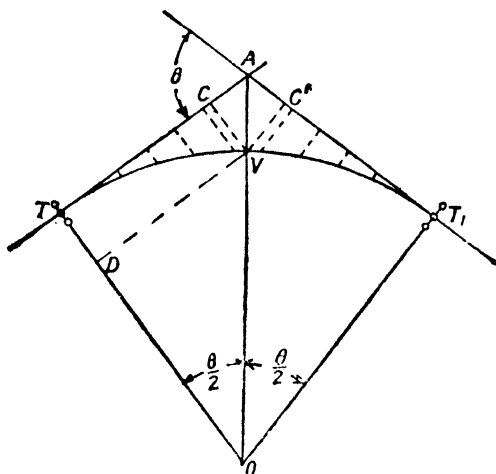
Fig. 217.

objection if the versine of the curve is not more than one-eighth of its chord, as, with this condition satisfied, a parabola approximates very closely to a circular arc. The curve set-out by offsets calculated from equation (8) is a true circle. Both cases have the objection that the offsets become too long when nearing the vertex; on a 40-chain curve subtending an angle of 60°, the offset at the vertex is 5·36 chains, and the error caused by faulty alignment or by incorrectly setting out the perpendicular may be easily imagined. A further objection to both systems is that the distances between the points on the curve are not equal, the discrepancy increasing with the number of offsets; thus,

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the distance between the 4th and 5th pegs on a 10-chain curve will be 1·15 chains, giving an error of 15 links ; between the same pair of pegs the error will be ·94 link on a 40-chain curve, the error varying inversely as the square of the radius. When the length of the curve is not more than one-quarter of the radius, the curve is set-out from one tangent until the vertex is reached. If the vertex has been set-out from the intersection point A (Fig. 217), the foot of the offset C is obtained with the
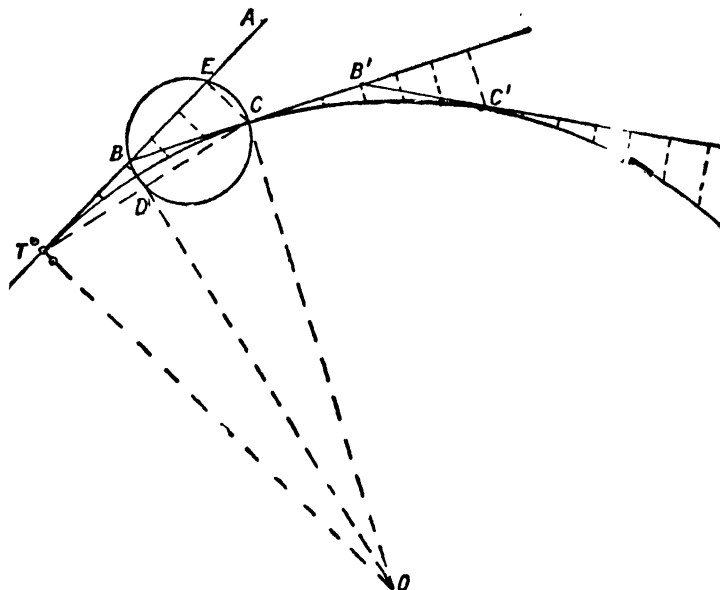
Fig 218.

cross staff or from calculation, thus $T C = V D = R \sin \dfrac{\theta}{2}$, and $C V = T D = R \left(1 - \cos \dfrac{\theta}{2}\right)$. A C is now known by difference, since $A C = A T - T C$.

The point C′ is next obtained by chaining along A T_1 a distance equal to A C, and the setting-out is continued from C′, the offsets being taken in the inverse order. When the curve is long, five or six points are set-out from the original tangent line A T (Fig. 218), at the last point set-out a new tangent line B C is ranged in position, and the work commences *de novo*

from the new tangent point C, the operations being repeated until the end of the curve is reached.

If the distances between the offsets on the first tangent are all equal, the same set of offsets is used on the remaining tangents ; but if the first chord from the rear tangent point is an odd length, a new set of offsets will be required for use on the second tangent ; they will, however, repeat on the remaining tangents.

To set-out the direction of a new tangent at any point C on the curve, chain along the preceding tangent a distance $T B$ such that $T B = \dfrac{T E^2 + E C^2}{2 T E}$; the line joining the points B and C will be tangent to the curve at C. To show that B C is tangent to the curve at C, join $T C$; from O (Fig. 218) drop a perpendicular on the chord $T C$ cutting the chord at D, and the tangent at B. Then, since the angles $T E C$ and $B D C$ are right angles, a circle may be drawn through the four points $B D C E$, hence $T E . T B = T D . T C$;

but
$$T D = \frac{T C}{2},$$

∴
$$T E . T B = \frac{T C^2}{2},$$

and
$$T C^2 = T E^2 + E C^2 ;$$

consequently,
$$T B = \frac{T E^2 + E C^2}{2 T E}. \qquad . \qquad . \qquad . \quad \textbf{(11)}$$

Further, since $T D = D C$ and $\angle T D B = \angle B D C =$ a right angle, $T B = B C$ and $\angle B T D = \angle B C D$; therefore, B C is tangent to the curve at C.

The tangent to the curve at the last point set-out may also be ranged in position by producing the chord A B, joining the last two pegs a distance $B C$ equal to the length of the chord, and from C marking a new point D on the curve by the method of offsets to a chord produced. Thus, $C D = \dfrac{B D^2}{R}$; the line joining the point B to the middle point (E) of the oblique offset C D will be tangent to the curve at B.

III. To Set-out a Circular Arc by the Method of Offsets to a Chord.—Let T V T′ be the curve to be set-out. The length of the chord joining the points of tangence $T T' = 2 R \sin \dfrac{\theta}{2}$, and $T C = R \sin \dfrac{\theta}{2}$. The offset at any point D (Fig. 219) on the chord

is calculated as follows :—Join B O, draw the line B E **parallel** to the chord, and meeting the line A O at the point E. Then

$$B E = D C,$$

and $$\qquad B D = E C,$$

also $$\qquad E C = E O - C O$$

$$= \sqrt{R^2 - B\,E^2} - \sqrt{R^2 - T\,C^2}.$$

If the distances are measured from T and not from C, then, since

$$D C = T C - T D = B E,$$

$$B D = \sqrt{R^2 - (T C - T D)^2} - \sqrt{R^2 - T C^2}. \qquad . \qquad (12)$$

Fig. 219.

It is obvious from the figure that the offsets to symmetrically placed points will be the same. At the centre, where

$$T D = T C,$$

the offset

$$V C = R - \sqrt{R^2 - T C^2}.$$

The simplest way to set-out a curve by this method is to divide the chord into an even number of equal parts, calculate the offsets at each of the points of division for one-half of the

curve ; the same offsets will apply to the other half if taken in the reverse order.

Example 2.—Let the radius of the curve be 300 feet, and the intersection angle 120°.

The length of the half-chord

$$T C = 300 \sin 30°$$
$$= 150 \text{ feet.}$$

The offset at the first 50 feet from

$$T = \sqrt{300^2 - (150 - 50)^2} - \sqrt{300^2 - 150^2}$$
$$= 282 \cdot 8 - 259 \cdot 8$$
$$= 23 \cdot 0 \text{ feet.}$$

The offset at the second 50 feet from

$$T = \sqrt{300^2 - (150 - 100)^2} - 259 \cdot 8$$
$$= 36 \cdot 0 \text{ feet.}$$

The offset at the vertex

$$V = 300 - \sqrt{300^2 - 150^2}$$
$$= 40 \cdot 2 \text{ feet.}$$

The offset from the fourth 50 feet from

$$T = 36 \cdot 0 \text{ feet.}$$

The offset from the fifth 50 feet from

$$T = 23 \cdot 0 \text{ feet.}$$

When the offsets to the chord joining the tangent points are too long, the curves should be set-out from the chords joining the vertex to the tangent points. The offsets are calculated from equation (12) as before, the semi-chord $V C'$ (Fig. 219) will now be equal to $R \sin \dfrac{\theta}{4}$.

This method is of most service in setting-out short curves on roads or canals ; it is not very suitable for setting-out the long curves required on railways, as the ground is seldom sufficiently free from obstacles. It is, however, less laborious than the following method.

IV. To Set-out a Circular Curve by Successive Bisections of Arcs.—Let A T and A T_1 (Fig. 220) be the tangents to the curve

at the points T and T_1; bisect the chord T T_1 at C, and set-out the offset C V such that C V $= R \left(1 - \cos\dfrac{\theta}{2} \right)$, V is the middle point of the arc T V T. Bisect the chords T V and T_1 V, and at the points of section E and F set-out the offsets E G, F H each equal to $R \left(1 - \cos\dfrac{\theta}{4} \right)$. At the middle points of the chords T G, G V, V H, and H T_1 set-out offsets each equal to $R \left(1 - \cos\dfrac{\theta}{8} \right)$; this process is repeated until sufficient points have been set-out.
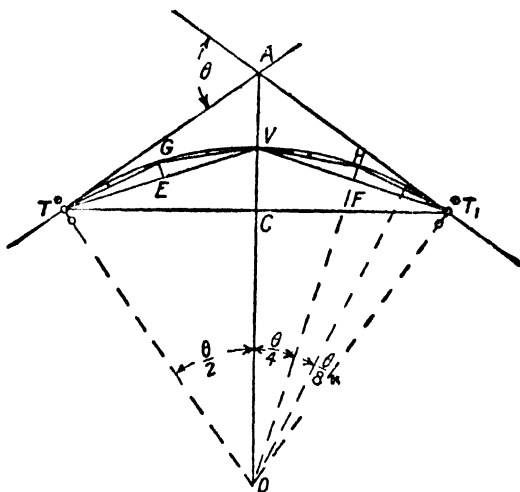
Fig. 220.

In methods III. and IV., it will be necessary to adjust the positions of the pegs along the curve to the standard distance apart.

V. To Set-out a Circular Curve with the Theodolite and Chain, by the Method of Deflection Angles.—This is the method most employed in setting-out circular curves on railways. The procedure is simple, quick, and accurate; few calculations are required, and as the work entailed is done in the immediate neighbourhood of the curve, this method is well adapted for use in confined positions, such as occur in setting-out tunnels and curves on viaducts.

The mode of procedure is as follows :—The instrument is erected over the rear tangent point, the plates are clamped at zero, and the telescope is directed to bisect the signal at the intersection point. The plates are then unclamped and the vernier set to give the first deflection angle as calculated from the formula, $a = 1,718 \cdot 9 \dfrac{\text{arc}}{\text{radius}}$, where a is the required deflection angle in minutes. The telescope now points in the direction of the first point on the curve, beyond the tangent point. The chainman, having pinned down one handle of the chain at the tangent point, stretches it in the direction (T B, Fig. 221) given by the telescope, and holds an arrow upright at a distance on the chain corresponding to the length of the arc used in calculating the deflection angle. The chain is then swung around its
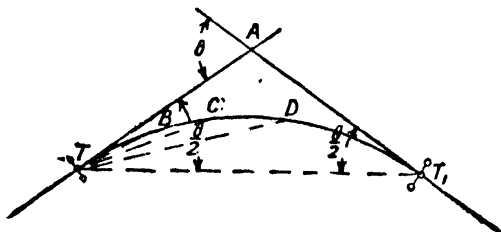
Fig. 221.

fixed end until the arrow is bisected by the cross wires ; the point thus obtained is on the curve, and is marked with a distance peg (B). The chainman releases the rear end of his chain, drags it forward, pins down the handle at B, and stretches the chain in the direction of the second point on the curve ; the instrument-man sets the vernier to read the second deflection angle as given by the above formula. The position of the arrow, on the chain, when bisected by the cross wires, gives the position of the next distance peg ; these operations are repeated for the remaining points on the curve.

It should be noted that the second, third, etc., deflection angles will be double, treble, etc., the first, if the same length of chord is used throughout ; also, the sum of the deflection angles is equal to $\dfrac{\theta}{2}$. This gives a check on the setting-out, for if the chord joining the last peg to the forward tangent point be

measured, and the corresponding deflection angle calculated, this angle added to the last deflection angle set out should equal $\frac{\theta}{2}$, and, if the vernier be set to this angle, the cross wires should bisect the signal at the forward tangent point.

Deflection Angles for Degree Curves.—If the curve to be set out is specified by degrees, on the American system, and not by its radius, the first deflection angle (for a 100-feet chord) is half the number of degrees specified. Thus, the first deflection angle on a 1° curve is 30′, on a 2° curve 1°, and so on. When the first chord is shorter than the standard distance, the corresponding deflection angle will be proportional to the length of the chord, and is given by the following proportion :—Required deflection angle : degree of curve : : length of chord : 200. The deflection angle for a 25-feet chord on a 1° curve will be equal to $\frac{25}{200} \times 1°$, or 7′ 30″, and for a 30-feet chord on a 2° curve, $\frac{30}{200} \times 2°$, or 18′ 00″.

Example 3.—Two straight reaches intersect at an angle of 150° 34′, and are to be connected by a curve of 40 chains radius. The intersection point is at 248·75 chains from peg 0. Find the chainage at the tangent points, and the first four deflection angles.

Here, $\dfrac{\theta}{2} = \dfrac{180° - 150° \ 34'}{2}$

$\qquad = 14° \ 43'.$

Distance from intersection point to tangent point
$\qquad = 40 \ . \ \tan 14° \ 43'$
$\qquad = 10·504 \ \text{chains}.$

Chainage at initial tangent point $= 248·75 - 10·50$
$\qquad\qquad\qquad\qquad\qquad = 238·25 \ \text{chains}.$

The length of the curve $= ·000582 \ (5,400 - 4,517) \times 40$
$\qquad\qquad\qquad\qquad = 20·56 \ \text{chains},$

and the chainage at the forward tangent point
$\qquad\qquad\qquad\qquad = 238·25 + 20·56$
$\qquad\qquad\qquad\qquad = 258·81 \ \text{chains}.$

First chord $= 239 - 238·25$
$\qquad\qquad = 75 \ \text{links}.$

$$\text{First deflection angle} = \frac{1,718 \cdot 9 \times \cdot 75}{40}$$

$$= 32' \ 13 \cdot 2'' \qquad \qquad \mathbf{32' \ 13 \cdot 2'}$$

$$\text{Deflection angle for 1-chain chord} = \frac{1,718 \cdot 9 \times 1}{40}$$

$$= 42' \ 57 \cdot 6'' \qquad \qquad \mathbf{42' \ 57 \cdot 6''}$$

$$\text{Second deflection angle} = \qquad \qquad \begin{matrix} \mathbf{1° \ 15' \ 10 \cdot 8''} \\ \mathbf{42' \ 57 \cdot 6''} \end{matrix}$$

$$\text{Third deflection angle} = \qquad \qquad \begin{matrix} \mathbf{1° \ 58' \ 08 \cdot 4''} \\ \mathbf{42' \ 57 \cdot 6''} \end{matrix}$$

$$\begin{matrix} \text{Fourth deflection angle} = \\ \text{Etc.,} \end{matrix} \qquad \qquad \begin{matrix} \mathbf{2° \ 41' \ 06 \cdot 0''} \\ \text{etc.} \end{matrix}$$

Example 4.—Assuming the given distances in example 3 to be in feet, calculate the corresponding quantities for a **2°** curve.

$$\text{In this case the radius R} = \frac{180° \times 100}{2 \pi}$$

$$= 2,865 \text{ feet.}$$

$$\text{Distance to the tangent point} = 2,865 \times \tan 14° \ 43'$$

$$= 752 \cdot 3 \text{ feet.}$$

$$\text{Chainage at initial tangent point} = 24,875 - 752 \cdot 3$$

$$= 24,122 \cdot 7 \text{ feet.}$$

$$\text{Length of curve} = \cdot 000582(5,400 - 4,517) \times 2,865$$

$$= 1,472 \text{ feet.}$$

$$\text{Chainage at forward point tangent} = 24,122 \cdot 7 + 1,472$$

$$= 25,594 \cdot 7 \text{ feet.}$$

$$\text{First chord} = 24,200 - 24,122 \cdot 7$$

$$= 77 \cdot 3 \text{ feet.}$$

$$\text{First deflection angle} = \frac{77 \cdot 3 \times 2 \times 60}{200}$$

$$= 46' \ 22 \cdot 8''.$$

Second deflection angle $= 46' 22\cdot8'' + 1°$
$$= 1° 46' 22\cdot8''.$$

Third deflection angle $= 1° 46' 22\cdot8'' + 1°$
$$= 2° 46' 22\cdot8''.$$

Fourth deflection angle $= 2° 46' 22\cdot8'' + 1°$
$$= 3° 46' 22\cdot8''.$$

Etc.,   etc.

The formula $= \dfrac{1,718\cdot9 \times \text{arc}}{\text{radius}}$, is easily demonstrated, for, in any circle the angle between the tangent and a chord drawn from the point of contact is half the angle subtended by the chord at the centre of the circle ; therefore,

$$2\,a \,:\, 360° \,::\, \text{arc} \,:\, 2\,\pi\,\text{R},$$

or,
$$a = \frac{360 \times \text{arc}}{4\,\pi\,\text{R}} \text{ in degrees}$$

$$= \frac{360 \times 60}{4\,\pi}\frac{\text{arc}}{\text{R}} \text{ in minutes}$$

$$= 1,718\cdot873\,\frac{\text{arc}}{\text{R}} \text{ minutes.} \qquad \bullet \quad (13$$

VI. To Set-out a Circular Arc by the Method of Deflection Angles, using Two Theodolites.

—In this system, a theodolite is set-up over each of the tangent points ; the instrument at the rear tangent point T (Fig. 222) is directed to bisect the signal at the intersection point A, with the plates clamped at zero, and on unclamping the plates, the vernier is set to the first deflection angle a.

The second instrument, at the forward tangent point T_1, is clamped at zero, and the telescope is directed to the forward tangent point ; the plates are unclamped and the vernier is also set to the first deflection angle ; for it is obvious that the angle $T\,T_1\,B$ (Fig. 222) is equal to the angle $A\,T\,B$. The point of intersection of the lines of sight of the two instruments is determined by an assistant, who moves a pole, under the directions of the two observers at the instruments, until its image is bisected by the cross wires of both instruments, the point thus obtained being the first point on the curve. To determine the second and succeeding points, both instruments are set to the

second, third, etc., deflection angles, and the corresponding points on the ground are obtained as before.

If it is impossible for the observer at the forward tangent point to see the signal at the rear tangent point, he may sight some forward signal P in the second tangent line, and on turning off the angle $180 - \dfrac{\theta}{2}$ from P, his telescope will point in the direction of the rear tangent point, thereafter the angle at the vernier is increased by the first, second, etc., deflection angles, to give the corresponding points on the curve. If the signal at the intersection point A is used, the angle $360 - \dfrac{\theta}{2}$ must be turned off to set the telescope in the direction of the rear tangent point.

As the chain is not used in determining points on the curve, the error caused by assuming the length of the chord to be equal to its subtended arc is eliminated, hence this method gives the
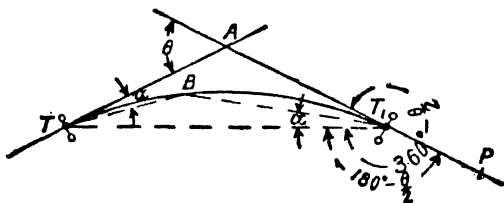
Fig. 222.

most correct results. Unfortunately, this method cannot often be used in practice, as it is seldom that the ground is sufficiently free from obstructions.

Methods of Dealing with Obstacles.

VII. To Determine the Intersection Angle when the Point of Intersection of the Tangent Lines is Inaccessible.—Since one of the chief objects sought by the use of curves in railway or road construction is to avoid difficult ground on which the cost of construction would be heavy, it often happens that the point of intersection of the tangent lines falls in a position which is difficult or impossible of access, as, for example, when the intersection point falls in a wood, a river, or the sea. If the chain is being used for setting out the curve, and a theodolite is not available, equal perpendiculars, B C, D E, F G, and H I (Fig. 223)

m ıy be set out from the tangent lines, the length of the per-
p ndiculars being such that the lines joining their extremities
shall (a) meet at an accessible point A_1, and (b) cut the tangent
lines at the points K and L, which are also accessible. The lines
K I and L C, being parallel to the tangent lines, the angle at
A_1 is equal to the angle at A, and its magnitude may be deter-
mined with the chain and tape, as described on p. 435. Further,
the figure A K A_1 L is a rhombus, and A_1 L is equal to the other
sides, and is, therefore, equal to A K or A L. On chaining A_1 L
or A_1 K the distances of the points K and L from A are known,
and the positions of the tangent points T and T_1 may be fixed
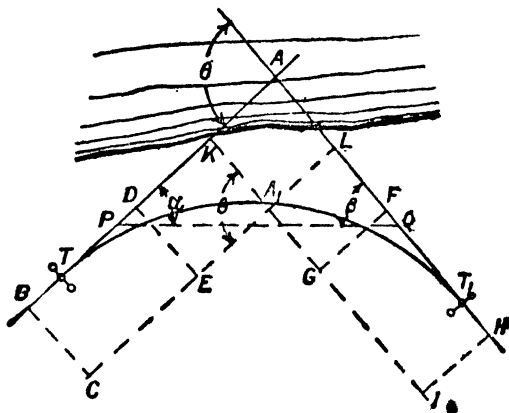
Fig. 223.

from K and L by chaining off a distance K T or L T_1, equal to
R . $\tan \dfrac{\theta}{2}$ — A K.

Having fixed the tangent points, the curve may be set out
by any of the preceding methods.

If a theodolite is available, a straight line P Q (Fig. 223) is set
out from one tangent to the other, in any convenient direction
and, the base angles α and β of the triangle A P Q are obtained
with the instrument. The angle θ is now known, since $\theta = \alpha + \beta$;
also, the intersection angle P A Q $= 180 - (\alpha + \beta)$. To deter-
mine the distances A P and A Q, the base P Q of the triangle
A P Q is chained, and by the sine rule, we have :—

$$\frac{A P}{P Q} = \frac{\sin \beta}{\sin (180 - \alpha + \beta)},$$

or, $$A P = \frac{P Q \sin \beta}{\sin (180 - \alpha + \beta)}.$$

Similarly, $$A Q = \frac{P Q \sin \alpha}{\sin (180 - \alpha + \beta)};$$

also, $$A T = A T_1 = R \tan \frac{\theta}{2} = R \tan \frac{\alpha + \beta}{2},$$

∴ $$P T = R \tan \frac{\alpha + \beta}{2} - \frac{P Q \sin \beta}{\sin (180 - \alpha + \beta)}, \qquad . \quad (14)$$

and $$Q T_1 = R \tan \frac{\alpha + \beta}{2} - \frac{P Q \sin \alpha}{\sin (180 - \alpha + \beta)}. \qquad . \quad (15)$$

Fig. 224.

The positions of the tangent points are then obtained by chaining off their calculated distances from the points P and Q.

VIII. To Continue a Curve when an Obstacle to the Use of the Chain occurs.—Let the curve have been set out from T to H (Fig. 224), and an obstacle occur on the next chain. Through H set out a transversal B H F K C. Compute the lengths of B T and C T_1, as in the preceding problem. Let F be the middle point of the chord H K. Then if B F be known, the point K can be found, since H F is equal to F K, and B H may be measured on the ground.

To find B F, we observe that

$$T B^2 - T_1 C^2 = B K . B H - C H . C K$$
$$= B H (B C - C K) - (B C - B H) C K$$
$$= B C (B H - C K)$$
$$= B C (B H + H F - \overline{C K + F K})$$
$$= B C (B F - F C)$$
$$= 2 B F . B C - B C^2,$$

$$\therefore \quad B F = \frac{T B^2 - T_1 C^2}{2 B C} + \frac{B C}{2}. \quad . \quad . \quad . \quad (16)$$

Similarly, $\quad F C = \frac{B C}{2} - \frac{T B^2 - T_1 C^2}{2 B C}. \quad . \quad . \quad . \quad (17)$

The point K is found by chaining off B F equal to its calculated distance and making F K equal to H F, the point H being already marked on the ground. The position of the point K on the curve is next determined from the angle (φ) subtended at the centre by the arc T K, thus,

$$\varphi = \angle T O F + \angle F O K$$
$$= 180° - \angle T B C + \angle F O K$$
$$= 180° - \angle T B C + \sin^{-1} \frac{F K}{R}.$$

and the arc $T K : \pi R :: \varphi : 180°$,

$$\therefore \text{ arc } T K = \frac{\pi R \varphi}{180°} = \cdot 0002909 \, R \, \varphi,$$

if φ be expressed in minutes.

The position of K on the chainage from peg 0 is now known, and the curve may be set out backwards from K to the obstacle, and continued forwards from the same point. Suppose, for example, that the chainage at the point T is 450·62, and the peg H is at 455. Let the arc T K as computed from the above equation be 8·86 chains; then the chainage at K is 450·62 + 8·86 = 459·48. The pegs 456, 457, 458, and 459 will lie between H and K. The peg 459 will be 48 links back from K, and the peg 460, 52 links forward from the same point.

If the position of the point H is not known, its distance from

the point B may be found ; for, since F is midway between H and K, we have

$$H F^2 = B F^2 - B K . B H$$

$$= B F^2 - B T^2,$$

$\therefore$

$$H F = \sqrt{B F^2 - B T^2}$$

$$= \sqrt{C F^2 - C T_1{}^2} = F K,$$

and

$$B H = B F - H F.$$

Similarly,

$$C K = C F - F K.$$

Setting-out New Tangent.—To continue the curve at the point K, it will be necessary to set out a new tangent line at this point. To do this with the chain and tape, at any convenient point in the line T K erect a perpendicular M N (Fig. 224) meeting the tangent T A in N. Measure T M and M N. From K chain off a distance K Q equal to T M, and at Q erect a perpendicular Q P, make Q P equal to M N. The line joining K to P will be tangent to the curve at K.

If a theodolite is being used, the tangent at K is set out as follows :—With the instrument at T, determine the deflection angle A T K. Remove the instrument to K, set the vernier to read $360° - \angle$ A T K, and sight back to T ; on bringing the vernier to zero, the telescope will be in the plane containing the new tangent at K.

IX. To Continue a Curve with a Theodolite when an Obstacle to Vision Occurs.—It seldom happens that all the points on a long curve are visible from either tangent point ; this may be due to undulations in the surface of the ground, or to buildings, trees, etc. This trouble will arise most frequently when setting out a curve by the method of deflection angles ; but, whatever be the nature of the obstacle, the curve is set out as far as possible from the tangent point, and the deflection angle to the last point set out (H, Fig. 224) is carefully determined. The instrument is then removed and erected over the peg at H, the vernier is set to $360°$ — the last deflection angle, and the telescope is sighted back to T. On unclamping the plates and swinging the telescope back to zero, it will point in the direction of the tangent line. For forward working, the telescope must be transited (or turned through $180°$). The setting-out is continued from H as though it were the original tangent point.

If the pegs between T and H are one chain apart, the same set of deflection angles is used in continuing the curve from H :

but, if the first chord from T is an odd length, a new set of deflection angles, for chords of 1, 2, 3, etc., chains, will be required.

Left-Hand Curves.—When a curve turns towards the left, it becomes necessary to set out the deflection angles in the same direction—*i.e.*, in a direction opposite to that in which the instrument is divided. In this case, the angles to be set out must either be subtracted from 360° and the vernier set to the angles thus obtained, or the vernier must be used backwards and each angle set off directly. To do this, the end division of the vernier is set to the zero of the main scale, and not the zero of the vernier, as is usually done, and the vernier is read backwards. The numbered divisions on the horizontal circle are ignored, and the degrees are counted off as required; thus, 10° will be at 350°, 20° at 340°, and so on. Care must be exercised to overcome the natural confusion which arises, due to using the instrument in an unfamiliar way; but with a little practice it will become as easy to set-out angles one way as the other.

X. To Determine a Serpentine or " S " Curve.—The serpentine curve consists of two arcs in contact, having their centres on opposite sides of the line joining the points of tangence on the straight lines united by the curve. The curve is used on railways for joining two parallel or nearly parallel straight reaches, or when obstructions render its adoption preferable. On main lines, when the radius is small, the two parts of the curve should be joined by a straight length, in order that the change in curvature may be more gradual. Usually the tangent points are decided from local considerations, and in determining the curve it is necessary to know the radius of one part and to find the radius of the other, or to know the ratio of the radii. The best conditions are obtained when the two parts of the curve are of equal radius.

1. To Determine the Common Radius of the Two Parts of a Serpentine Curve. the Tangential Points and their Distance Apart being given.—To find the common radius geometrically, we proceed as follows :—At the given tangent points B and C (Fig. 225) erect equal perpendiculars B E, C F, any convenient length; through E draw E G parallel to B C; with F as centre and radius equal to twice B E or twice C F, strike an arc cutting E G at G; join C G and produce the line to meet B E at J. Through J draw J K parallel to G F meeting C F produced in the point K. Then J and K are the required centres, and J B ⇒ K C = the required radius. For the triangles C F G and

C K J are similar, and the triangles J E G and J B C are also similar, therefore

$$C K : C F :: C J : C G,$$
$$:: B J : B E, \text{ but } B E = C F,$$

∴ $$C K = B J.$$

Again, $$\mathbf{C K} : K J :: C F : F G$$
$$:: 1 : 2,$$

∴ $$K J = 2 . C K$$
$$= 2 . B J,$$

hence the arcs will be tangent at the middle point P of the line J K.
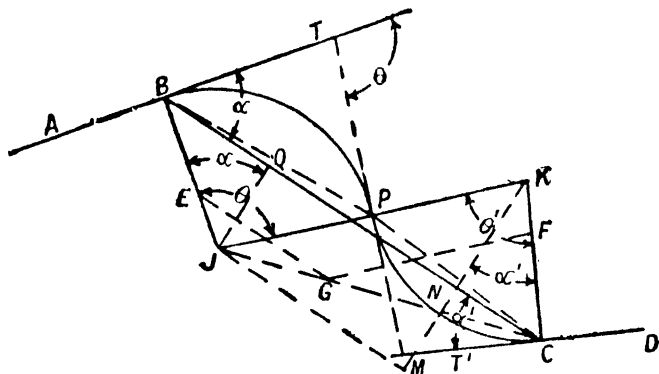
Fig. 225.

To compute the radius; let the angle T B C = α, and the angle T' C B = α'. Draw J Q, K N perpendicular to B C; through J draw J M parallel to B C meeting K N produced in M. Then B Q = R sin α; C N = R sin α'; Q J = R cos α; K N = R cos α'.

Also, K M = K N + M N = K N + Q J,

and $$\frac{K M}{2 R} = \sin \angle K J M,$$

∴ $$\angle K J M = \sin^{-1} \frac{(\cos \alpha + \cos \alpha')}{2}.$$

Again, $J M = 2 R \cos \angle K J M$

$$= 2 R \cos \left\{ \sin^{-1} \frac{(\cos \alpha + \cos \alpha')}{2} \right\} ;$$

but, $B C = B Q + Q N + N C$

$$= B Q + J M + N C$$

$$= R \sin \alpha + 2 R \cos \left\{ \sin^{-1} \frac{(\cos \alpha + \cos \alpha')}{2} \right\} + R \sin \alpha',$$

or $$R = \frac{B C}{\sin \alpha + 2 \cos \left\{ \sin^{-1} \frac{(\cos \alpha + \cos \alpha')}{2} \right\} + \sin \alpha'}. \qquad (18)$$

Example 5.—Let $\alpha = 27°$, $\alpha' = 50°$, and $B C = 200$ chains; then

$\sin \alpha = \sin 27° = \cdot 45399$; $\cos \alpha = \cos 27° = \cdot 89100$

$\sin \alpha' = \sin 50° = \cdot 76604$; $\cos \alpha' = \cos 50° = \cdot 64279$

$$
\begin{aligned}
\text{sum} &= 1 \cdot 53380 \\
\tfrac{1}{2} \text{ sum} &= \cdot 76690 \\
&= \sin 50° 5' \text{ nearly} \\
2 \cos 50° 5' = 2 \times \cdot 64167 \\
&= 1 \cdot 28334,
\end{aligned}
$$

$$R = \frac{200}{\cdot 45399 + 1 \cdot 28334 + \cdot 76604}$$

$$= 79 \cdot 89 \text{ chains.}$$

2. Case of Unequal Radii.—When the radii are unequal and one radius (say, $K C = r$) is known, then $J K = R + r$, $B Q = R \sin \alpha$; $J Q = R \cos \alpha$; $N C = r \sin \alpha'$; $N K = r \cos \alpha'$.

Also, $J M = Q N = \sqrt{J K^2 - K M^2}$

$$= \sqrt{(R + r)^2 - (R \cos \alpha + r \cos \alpha')^2},$$

and $B C = R \sin \alpha + r \sin \alpha'$

$$+ \sqrt{(R + r)^2 - (R \cos \alpha + r \cos \alpha')^2}.$$

By transposing and squaring, we obtain :

$B C^2 - 2 B C (R \sin \alpha + r \sin \alpha') + (R \sin \alpha + r \sin \alpha')^2$

$$= (R + r)^2 - (R \cos \alpha + r \cos \alpha')^2,$$

which gives on reduction,

$$B\,C^2 - 2\,B\,C\,(R\sin\alpha + r\sin\alpha') = 2\,R\,r\,(1 - \cos\overline{\alpha - \alpha'})$$

$$= 4\,R\,r\sin^2\frac{\alpha - \alpha'}{2}.$$

Solving for R, we get

$$R = \frac{B\,C\,(B\,C - 2\,r\sin\alpha')}{2\left(B\,C\sin\alpha + 2\,r\sin^2\dfrac{\alpha - \alpha'}{2}\right)} \quad . \quad . \quad . \quad (19)$$

Example 6.—Let $B\,C = 200$ chains; $r = 100$ chains; $\alpha = 50°$; $\alpha' = 30°$.
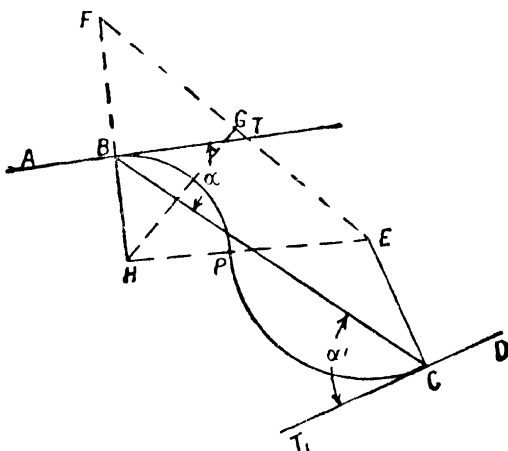
Fig. 226.

Then $\sin\alpha = \sin 50° = \cdot 76604$; $\sin\alpha' = \sin 30° = \cdot 50000$

$$\sin\frac{\alpha - \alpha'}{2} = \sin\frac{50° - 30°}{2} = \sin 10° = \cdot 17365,$$

and by equation (19), $R = \dfrac{200\,(200 - 200 \times \cdot 5)}{2\,(200 \times \cdot 76604 + 2 \times 100 \times \cdot 17365^2)}$

$$= 62 \cdot 81 \text{ chains.}$$

To determine the required radius geometrically, we proceed as follows :—At C and B (Fig. 226) the given tangent points erect perpendiculars C E, B F, each equal to the given radius. Join F E, draw the perpendicular bisector G H of the line F E, and

let G H meet F B produced at H. Then B H is the required radius; for from the construction

$$H F = H E, \text{ or } H B + B F = H B + C E$$
$$= H P + P E,$$

the point P being the point of contrary flexure.

Length of Serpentine Curve.—The length of a circular arc may be found (*vide* p 436) when its radius and the angle the arc subtends at the centre are known. The angles (θ and θ') at the centres of the two parts of a serpentine curve are easily determined, for, referring to Fig. 225, we see that the angle

$$\theta = \angle B J Q + \angle Q J P$$
$$= a + 90° - \angle K J M$$
$$= a + 90° - \sin^{-1}\left\{\frac{\cos a + \cos a'}{2}\right\}$$

Similarly, $$\theta' = a' + 90° - \sin^{-1}\left\{\frac{\cos a + \cos a'}{2}\right\}$$

The length of each curve is obtained by introducing half the supplement of these angles in minutes in equation (2), or by introducing the angles in minutes in equation (3).

Example 7.—From the data given in Example 5, determine the length of each portion of the curve.

Here, $a = 27°$; $a' = 50°$; $R = 79·89$ chains.

$$\theta = 27° + 90° - \sin^{-1}\left\{\frac{\cos 27° + \cos 50°}{2}\right\}$$
$$= 27° + 90° - 50° 5'$$
$$= 66° 55'$$
$$= 4,015 \text{ minutes.}$$
$$\theta' = 50° + 90° - 50° 5'$$
$$= 89° 55'$$
$$= 5,395 \text{ minutes.}$$

Length of curve B P = $0·0002909 \times 79·89 \times 4,015$
$$= 93·28 \text{ chains.}$$

Length of curve P C = $0·0002909 \times 79·89 \times 5,395$
$$= 125·4 \text{ chains.}$$

Length of Chords.—If θ is the angle subtended by the curve, the length of the cord joining the tangent points (on the same curve) is equal to $2 \cdot R \sin\dfrac{\theta}{2}$.

The deflection angle to the point of contrary flexure is equal to $\dfrac{\theta}{2}$, hence the position of this point, if not already known, may now be found.

The setting-out of the curve should present no difficulty.

3. **To Determine the Tangent Points on a given Deviation made up of Three Curves of equal Radius, the Amount of the Deviation and the Common Radius being known.**—This problem
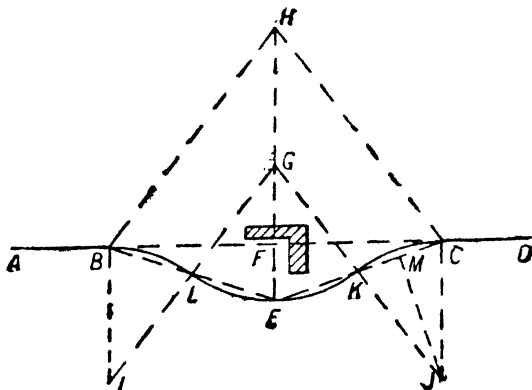
Fig. 227.

arises when it is desired to alter the direction of a straight line in order to avoid an obstruction such as a block of buildings, a bend of a river, or an arm of the sea.

To determine the points B and C (Fig. 227) geometrically, from F, the point of deviation, set off F E, the given deviation, perpendicular to the line A D ; prolong E F to G and H such that E G = G H = given radius, R. With H as centre, and a radius equal to twice R, strike an arc cutting A D in the required points B and C. At B and C erect perpendiculars B I and C J ; through G draw G I, G J parallel to H B and H C respectively, then I, J, and G are the centres of the arcs forming the required deviation. The points of contrary flexure L and K are given by the intersection of the lines joining E B, E C, and the lines

G I and G J ; the chords B L, L E, E K, K C are all equal.
The proof of the construction is obvious.

To obtain the distances F B and F C by calculation, from J
drop a perpendicular J M on the chord K C. Then C M = $\frac{1}{2}$ C K
= $\frac{1}{4}$ C E, and since the triangles F E C and J M C are similar,
we have F E : E C : : $\frac{1}{4}$ E C : C J

$$: : E C : 4 R,$$

∴ $$4 R . F E = E C^2 \quad . \quad . \quad . \quad . \quad (a)$$
$$= F E^2 + F C^2,$$

that is, $$F C = \sqrt{F E (4 R - F E)}.$$

Again, from equation (a), E C = $2 \sqrt{R . F E,}$

and, $$C K = \frac{1}{2} E C$$
$$= \sqrt{R . F E.}$$

Resistances on a Curve.—In addition to the resistances to the
motion of a vehicle on a straight rail, with cylindrical wheels,
on a curve on which both rails are at the same level, other
resistances arising from the following sources are met with :—
(a) The friction caused by the slipping of the wheels due to one
wheel travelling a greater distance than the other ; (b) the friction
of the flanges on the outer rail, caused by centrifugal force ; (c)
the resistance due to the obliquity of the moving effort ; and
(d) the resistance due to the axles being parallel instead of con-
vergent.

These resistances are reduced in practice by (a) the use of
running tyres ; (b) by raising the outer rail to a higher level
than the inner rail—i.e., by superelevation of the outer rail ;
(c) is partly reduced by the action of centrifugal force, and is
small ; and (d) placing the axles closer together, as in the use
of " bogey " trucks.

According to experiments conducted by Mr. Latrobe, in cases
where the superelevation is given, the increase in resistance due
to each degree of deflection subtending an arc of 100 feet is
equal to an ascent of 2·5 feet per mile.

**To Determine the Superelevation or Cant of the Rails on a
Curve.**—The object of superelevation on a curve is to counteract
the effect of the centrifugal force acting on a vehicle moving on
the curve. The ideal condition is arrived at when the resultant
force acting on the vehicle is perpendicular to the plane on which
the vehicle is moving at each instant. Unfortunately, this con-
dition is rarely obtained in practice, as in a particular case the

superelevation varies with the square of the speed, and any departure from the speed for which the superelevation is intended causes the resultant force to be oblique to the plane of the rails.

To find the centrifugal force tending to drive the vehicle off the rails :—

Let W = the weight of the moving vehicle,

V = velocity of the vehicle in feet per second,

R = radius of the curve in feet,

g = acceleration due to gravity

= 32·2 feet per second, per second,

F = centrifugal force acting on the vehicle through its centre of gravity.
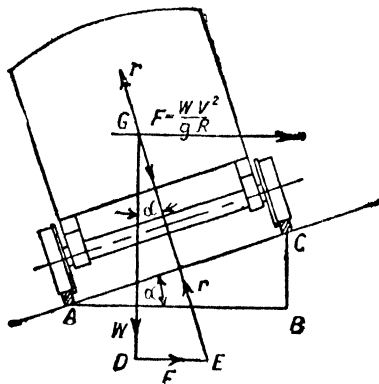
Fig. 228.

The axis about which rotation takes place is vertical, and since F always acts at right angles to the axis of rotation, F acts horizontally. Its magnitude is given by the equation

$$F = \frac{W \cdot V^2}{g \cdot R}.$$

Referring to Fig. 228, let B C be the required superelevation, A C the gauge, G the centre of gravity of the vehicle. Then for equilibrium in the plane at right angles to its motion, the vehicle must be acted on by three balanced forces—viz., the centrifugal force F, the weight W, and the reaction r, acting through the centre of gravity G. The force r is the resultant reaction of the ground and must be equal and opposite to the

resultant G E of the forces W and F. As may be seen by in-
spection, the triangle of forces G D E is similar to the triangle
A B C, hence

$$F : W :: B C : A B$$
$$:: B C : A C \text{ nearly,}$$

∴
$$\frac{W V^2}{g R} : W :: B C : A C,$$

or
$$B C = \frac{A C . V^2}{g R}$$
$$= \frac{b V^2}{g R}, \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad (\beta)$$

where b is the gauge of the rails.

On railways, speeds (S) are estimated in miles per hour. In
these units equation (β) becomes

$$B C = \frac{b \left(\frac{5280}{60 \times 60} S \right)^2}{32 \cdot 2 R}$$
$$= \frac{b S^2}{15 R} \text{ very nearly.} \qquad \cdot \qquad \cdot \qquad (20)$$

Example 8.—Find the superelevation on a curve 500 feet
radius, gauge 4′ 8½″, speed 50 miles per hour.

Here,
$$B C = \frac{4 \cdot 71 \times 50 \times 50}{15 \times 500}$$
$$= 1 \cdot 57 \text{ feet.}$$

It is usual to put the cant on a rail by raising the outer rail
and depressing the inner by half the calculated amount.

Danger Points on a Curve.—The danger points on a curve are
the points at which the direction suddenly changes, and occur
at the tangent points and points of contrary flexure. At these
points the wheels tend to climb the outer rail, and to reduce
this tendency, the outer rail is raised to throw more weight
on the inner rail : some engineers also increase the gauge so as
to give the wheels more play between the rails. As the cant
must be put on the rails gradually, in some cases the straight
rails are gradually canted so that the full cant is attained at the
tangent points ; a better plan, however, is to make the curve
of greater curvature at the tangent points as in the compound
circular curve, the parabolic curves on French railways, or in
the transition curve.

Compound Circular Curves.—Compound circular curves consist of short tangential arcs of varying radii, the arcs of largest radius being at the tangent points, and that of least radius at the vertex of the curve. The increment of the radius may be gradual or rapid, according to local circumstances, and may be simply additive or made to follow some mathematical law. In practice it is usual to increase or decrease the length of the radius by a constant amount at each change of curvature, and to make
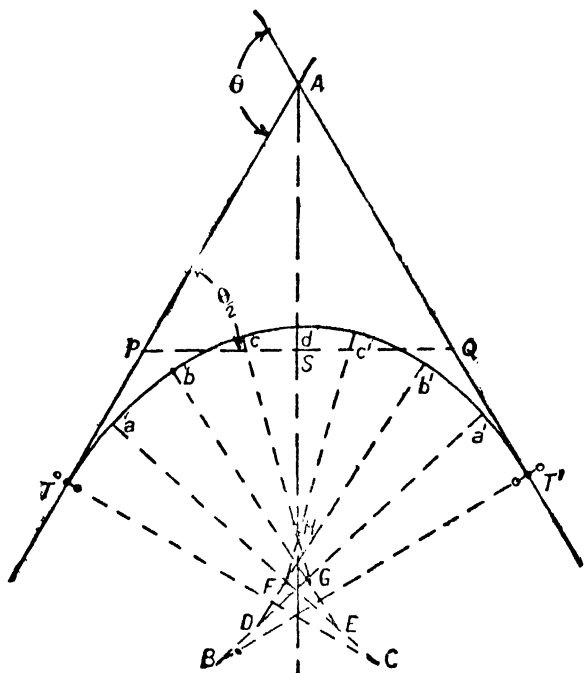
Fig. 229.

the arcs forming the curve of equal length, symmetrically placed above the vertex.

Before commencing the setting-out, a scale drawing of the curve, showing the lengths and radii of the component arcs, should be made, for use on the ground.

In fixing the shape of the curve, the initial tangent point may be chosen to suit the local conditions, and it is convenient to

make the tangent point at one of the distance pegs. The position of the forward tangent point then follows as the curve is symmetrical about the line bisecting the intersection angle. The maximum radius, the increment of the radius, and lengths of the component arcs are next decided on, and the curve is drawn to scale as shown in Fig. 229, in which the construction is obvious. The last centre used (H) must be on the line bisecting the inter-section angle ; to obtain this condition, it will usually be found that the arc at the vertex is an odd length, and the last increment in the radius F H is greater or less than the increment decided on. An odd length of the curve at the vertex is not objectionable, but a considerable increase in the increment of the radius should not be permitted, and if it occurs, further trials should be made with other data, until a satisfactory outline is obtained. The curve should also be drawn on the plan, in order to be certain it runs clear of obstacles.

In setting-out the curve, the tangent points are first determined, the points on the curve may then be found either with the chain and tape or by the method of deflection angles, but in either case it will be necessary to set-out a new tangent line at each of the points a, b, c, etc., where the curvature changes. If the curve is set-out forwards and backwards from the tangent points, the two parts of the curve should join at the vertex d. If set-out in the forward direction it will be necessary to determine the point d so that the curve $d\,c'$ may be made equal to $c\,d$. The point d is, of course, the point of intersection of the curve and the bisector of the intersection angle, and should be determined on the ground. When the intersection point A is inconveniently situated, the bisector of the intersection angle may be set-out by erecting a perpendicular at the middle point S of a symmetrically placed transversal, such as P Q.

Errors of closure may be adjusted by moving each of the pegs sideways by an amount proportional to the error to be allowed for, and the square of the distance of the peg from the commencement of the curve

As shown by equation (20), each component arc of a compound curve should have a cant inversely proportional to its radius ; but, as sudden changes in cant are not permissible, in practice the cant is made to change gradually from the tangent points to the vertex.

Transition Curves.—Like the compound curve, the transition curve has for its object the enlargement of the radius of curvature near the tangent points, thus making the transition

from the straight line to the curve more gradual. This is done by replacing the circular curve near both tangent points by a curve having a radius of curvature increasing rapidly and continuously from its point of contact with the circular curve to the tangent points. At the point of contact with the circular curve both curves have the same radius of curvature.

Transition curves are largely used on the Continent and in America; they have not been much used by English engineers on home railways in the past, but of late years they have been adopted to a small extent.

Various types of curves have been proposed, the two most commonly used being the cubic parabola and Searles' spiral. The latter has been much used in the United States and in Canada, and is set-out from tabular quantities worked out by the author of this system, and published in his book * on railway curves.

The cubic parabola is set-out by perpendicular offsets (y) from distances (x) measured along the tangent line from the tangent point. The equation to the curve is of the form $y = c x^3$ where c is a constant. By differentiating y with respect to x, we obtain the slope of the curve, thus $\dfrac{d y}{d x} = 3 c x^2$, and the second differential $\dfrac{d y^2}{d x^2} = \dfrac{1}{r} = 6 c x$, where r is the radius of curvature at the point x. From the last equation we have $r = \dfrac{1}{6 c x}$, hence, when $x = 0$, r is infinite, and when $x = l$ $r = R$, the radius of the circular curve.

As the curve is very flat, it is usual to assume its length to be equal to x. The length of the curve is quite arbitrary, it may be decided upon to suit local conditions, or made to depend on the cant, as in Froude's transition curve, where the length is made equal to 300 times the cant. Knowing the length and the radius of the circular curve (for at the point at which the curves join, the radius is the same), we may find the value of the constant c, and from this the offset to the point of junction. For example, suppose the length of the curve is 300 feet, and the radius of the circular curve is 1,200 feet; then $c = \dfrac{1}{6 \times 1,200 \times 300}$ and the offset to the end of the curve $= y = \dfrac{300^3}{6 \times 1,200 \times 300}$ $= 12 \cdot 5$ feet.

* " The Railroad Spiral," by W. H. Searles, C.E.

The offsets at intermediate points are proportional to the cubes of their distances from the tangent points, thus the offset at 100 feet will be equal to $\dfrac{100^3}{300^3} \times 12{\cdot}5 = {\cdot}46$ feet. At 200 feet, the offset $= \dfrac{200^3}{300^3} \times 12{\cdot}5 = 3{\cdot}70$ feet, and so on.

In order to continue the curve as a circular arc from the point of junction, it will be necessary to set-out a tangent to the two curves at this point. The slope of the curve is

$$\frac{d\,y}{d\,x} = 3\,c\,x^2 = \frac{3\,c\,x^3}{x} = 3\,\frac{y}{x}.$$

At the end of the curve, where $x = l$, we have

$$\tan \beta \ (\text{Fig. 230}) = \frac{3 \cdot \text{B C}}{l} = 3 \cdot \tan \alpha.$$
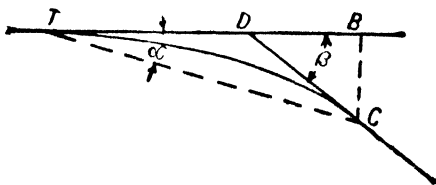
Fig. 230.

Also, the angle $\text{T C D} = \beta - \alpha$. To set-out the tangent at C, the instrument, with the plates clamped at zero, is set up over the point, the telescope is sighted back to T, and the angle $\beta - \alpha$ is turned off. The line of collimation will now be tangential to both the circular curve and the cubic parabola.

To determine the positions of the tangent points T and T_1 (Fig. 231), let B C be the circular curve, centre O; join B O, and let fall the perpendicular O E on the line A T. From B drop perpendiculars B F, B D on the lines A T and O E respectively. Draw the tangent B G at the point B. Then

$$\text{A T} = \text{T F} + \text{F A}$$

$$= \text{T F} + \text{E A} - \text{E F}$$

$$= \text{T F} + (\text{O D} + \text{D E}) \tan \frac{\theta}{2} - \text{E F},$$

and since the angle $\text{B G F} =$ the angle $\text{E O B} = \beta,$

$$\mathbf{A}\, T \,=\, T\, F + (R \cos \beta + B\, F) \tan \frac{\theta}{2} - R \sin \beta$$

$$= X + (R \cos \beta + Y) \tan \frac{\theta}{2} - R \sin \beta, \quad . \quad (21)$$

Clearly, the length of the whole curve $= 2l + \cdot0002909\, R$ $(\theta - 2\,\beta)$, the angle $(\theta - 2\,\beta)$ being expressed in minutes.

Example 9.—Determine the necessary quantities for setting-out a circular curve 1,000 feet radius joining by two transition curves two straight lines which intersect at an angle of 100°.
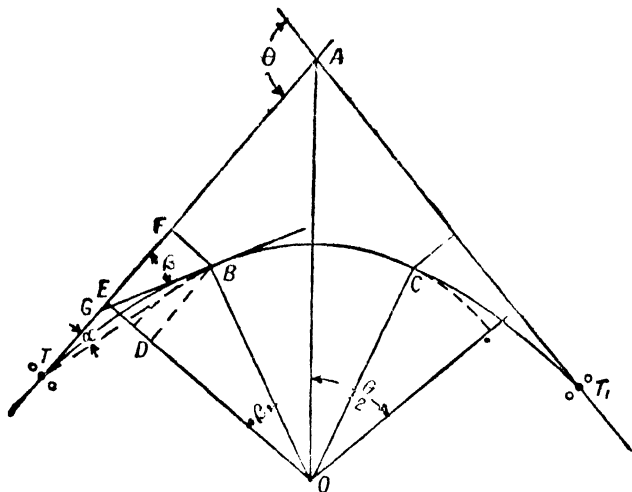
Fig. 231.

The length of each transition curve to be 300 feet, and the chainage at the intersection point 9,845 feet.

In this case,

$$c = \frac{1}{6 \times 1,000 \times 300},$$

and

$$Y = \frac{300^3}{6 \times 1,000 \times 300} = 15 \text{ feet.}$$

$$\text{Tan } \beta = \frac{3 \times 15}{300} = \cdot15 = \tan 8° \, 31\cdot6'.$$

$$\text{Tan } \alpha = \frac{15}{300} = \cdot 05 = \tan 2° \, 51 \cdot 6'.$$

$$\therefore \ \beta - \alpha = 5° \, 40'.$$

$$\mathbf{A\,T} = \mathbf{A\,T_1} = 300 + (1,000 \cos 8° \, 31 \cdot 6' + 15) \tan \frac{80°}{2}$$
$$- \, 1,000 \sin 8° \, 31 \cdot 6'$$
$$= 300 + (1,000 \times \cdot 9889 + 15) \cdot 8391 - 1,000 \times \cdot 1484$$
$$= 994 \cdot 4 \text{ feet.}$$

Length of curve
$$= 2 \times 300 + \cdot 0002909 \, (4,800 - 1,023 \cdot 2) \times 1,000$$
$$= 1,698 \text{ feet.}$$

Chainage at initial tangent point
$$= 9,845 - 994 \cdot 4 = 8,850 \cdot 6 \text{ feet.}$$

Chainage at forward tangent point
$$= 8,850 \cdot 6 + 1,698 = 10,548 \cdot 6 \textbf{ feet.}$$

First chord $= 8,900 - 8,850 \cdot 6 = 49 \cdot 4$ feet.

1st offset, at 8,900 feet $= \dfrac{49 \cdot 4^3 \times 15}{300^3} = \cdot 067$ **foot.**

2nd offset, at 9,000 feet $= \dfrac{149 \cdot 4^3 \times 15}{300^3} = 1 \cdot 85$ feet.

3rd offset, at 9,100 feet $= \dfrac{249 \cdot 4^3 \times 15}{300^3} = 8 \cdot 62$ feet.

Offset, at 9,150·6 feet $= 15$ feet, end of transition **curve.**

1st deflection angle from tangent at B (Fig. 231) **to** peg 9,200
$$= \frac{1,718 \cdot 9 \times 49 \cdot 4}{1,000} = 1° \, 24' \, 53 \cdot 4''.$$

2nd deflection angle to peg 9,300
$$= \frac{1,718 \cdot 9 \times 100}{1,000} + 1° \, 24' \, 53 \cdot 4''$$
$$= 2° \, 51' \, 53 \cdot 4'' + 1° \, 24' \, 53 \cdot 4''$$
$$= 4° \, 16' \, 46 \cdot 8''.$$

3rd deflection angle to peg 9,400
$$= 2° \, 51' \, 53 \cdot 4'' + 4° \, 16' \, 46 \cdot 8''$$
$$= 7° \, 8' \, 40 \cdot 2''$$

4th deflection angle to peg 9,500

$$= 2° 51' 53 \cdot 4'' + 7° 8' 40 \cdot 2''$$

$$= 10° 00' 33 \cdot 6'', \text{ etc., etc.}$$

The last deflection angle to peg 10,248·6

$$= \frac{1,718 \cdot 9 \times 1,098}{1,000}$$

$$= 31° 27' 21'', \text{ commencement of transition curve.}$$

1st offset = 15 feet, checking the setting-out of peg at 10,248·6.

2nd offset, at 10,300 $= \dfrac{(300 - 51 \cdot 4)^3 \times 15}{300^3}$

$$= 8 \cdot 53 \text{ feet.}$$

3rd offset, at 10,400 $= \dfrac{(300 - 151 \cdot 4)^3 \times 15}{300^3}$

$$= 1 \cdot 82 \text{ feet.}$$

4th offset, at 10,500 $= \dfrac{(300 - 251 \cdot 4)^3 \times 15}{300^3}$

$$= \cdot 064 \text{ foot.}$$

Froude's Transition Curve.—The offset on a cubic parabola at its point of contact with a circular arc of radius R is $Y = \dfrac{X^2}{6 \, R}$, the offset at the middle point, where $x = \dfrac{X}{2}$ is $\dfrac{X^2}{48 \, R}$, in Froude's transition curve twice this quantity, i.e., $\dfrac{X^2}{24 \, R}$, is taken, and is called the " shift " (S), and denotes the distance the tangents, and with them the circular arc, are supposed to be shifted towards the centre of the curve. The tangent points are computed on the assumption that the radius of the curve is (R + S). Thus A T = A T′ (Fig. 232) = (R + S) $\tan \dfrac{\theta}{2}$. The length (l) of the transition curve is made equal to 300 times the *change* in cant. If the curve joins a straight line, the change in cant is the cant simply ; where two curves join, the change in cant is the sum or difference of the cants according as the curves turn in the opposite or the same direction. The maximum gradient at which the cant comes on the rail is $\dfrac{1}{300}$.

The " shift " S may now be determined, since

$$S = \frac{l^2}{24\,R} = \frac{300^2 \times \text{change in cant}^2}{24\,R}.$$

The offset (Y) to the end of the transition curve is given by the equation

$$Y = \frac{300^2 \times \text{change in cant}^2}{6\,R} = 4 \cdot S,$$

the remaining offsets are proportional to the cubes of their distances from the commencement of the curve. In setting-out
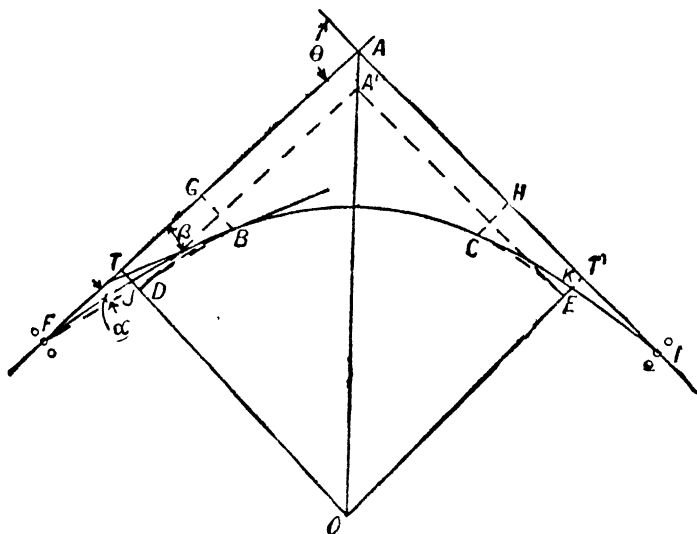
Fig. 232.

a curve by this system, the tangent point T and T′ (Fig. 232) are first located, and from these points the distances T F, T G, T′ I, T′ H are laid off along the tangent lines, each equal to ½ l, thus obtaining the points at which the curves begin, and those from which the last offsets are set-out. At the tangent points, perpendiculars T D and T′ E are erected and made equal to $\frac{l^2}{24\,R}$, thus obtaining the points of contact of the circular arc and the lines A′ D and A′ E. The middle points J and K of the transition curve are obtained by bisecting the offsets T D and

T′ E ; the remaining points on the curve then follow by making the offsets proportional to the cubes of their distances from the points F and I. The circular arc B C may be set-out in the usual way from the tangent points D and E, working from the tangents D A′ and E A′. If preferred, a new tangent line may be set-out at B from the calculated value of the angle $\beta - \alpha$, or from the observed value of the deflection angle A′ D B, and the curve may then be set-out from B.

Application to the Serpentine Curve.—At the point of contrary flexure on a serpentine curve, the curvature suddenly changes from a plus to a minus value, the cant changing in like manner ; hence, the two portions of the curve should always be joined by a curve of adjustment.

In applying Froude's method, the two circular arcs are shifted towards their respective centres the proper amount A B and A C (Fig. 233). The total length of the transition curve, in this
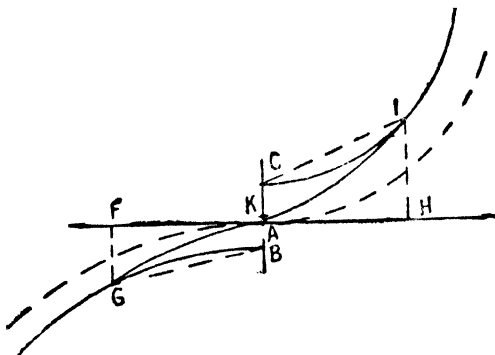
Fig. 233.

case, is equal to 300 × sum of cants. The two parts of the curve may be, set-out from the common tangent F A H through A, making the length of each part equal to 300 × the appropriate cant, the offsets being made proportional to the cubes of their distances from A.

The more usual practice is, however, to make the middle point K of the total shift B C the middle point of the whole curve, thus making G K equal to $K I = \dfrac{300 \times \text{sum of cants}}{2}$

The offsets are set-out from the chords G B and C I, making

31

each proportional to the cube of its distance from the points G and 1.

When the two parts of a serpentine curve are joined by a transition curve, it will be necessary to make use of similar curves of adjustment to fill in the gaps between the circular arcs and the straight lines, since the latter may not be moved.

Junctions and Crossings.—The laying and alignment of the rails on a railway are carried out by the plate layers working from the staked centre line with the proper gauge. Where super-elevation is necessary a few extra levels are put in for their guidance, but at junctions and crossings further assistance from the setter-out is necessary.

The usual method of turning off a main track is by a pair of switch rails and a curve tangent, both to the switch rails and the first straight reach on the branch line. This necessitates the severance of the main track at its point of intersection with the outer rail of the branch curve ; this point is called a " crossing." The ordinary length of switches is about 14 feet ; where the speed is low, as in sidings and goods yards, the switches are made shorter, but should not be less than 7 feet. Switch rails are not usually cambered, and it is obvious that little or no supereleva-tion can be given to the outer rail, hence the speed should be low in diverging from the main line.

In the case of a simple turn-out, where both the main and branch lines consist of a single pair of rails, there is only one crossing. Crossings are specified by their leads, such as 1 in 8, 1 in 10, etc., the lead being half the cosecant of half the angle of crossing. If the main line is straight, the angle of crossing is the angle between the main line and the tangent to the curve at the point of crossing ; if the main line is curved, the angle of crossing is the angle between the tangents to the two curves at their point of intersection. A simple way of approximately deter-mining the lead from a plan is to apply a scale across the lines E F, E G (Fig. 234), so that the chord F G is equal to one unit on the scale, the length of E F or E G measured in the same units gives the required lead, for it is evident that

$$\mathrm{E\,F} = \tfrac{1}{2}\,\mathrm{cosec}\,\frac{\beta}{2},$$

where β is the angle of crossing.

Near stations, and at points where it is necessary to clear the main track quickly, the angle of crossing is made fairly large and the radius of the branch curve small, 500 feet or less. In

other cases the radius of the branch curve is made as large as
local conditions will permit.

It must be remembered that the angle of crossing is the angle
between the centre lines of the actual rails, and the point of
crossing is the point of intersection of these lines. When the
crossings are constructed in cast steel, the parts of the rails
that meet at a point are, for a few feet, cast in one piece. The
forms and sizes of these crossings vary according to the maker.
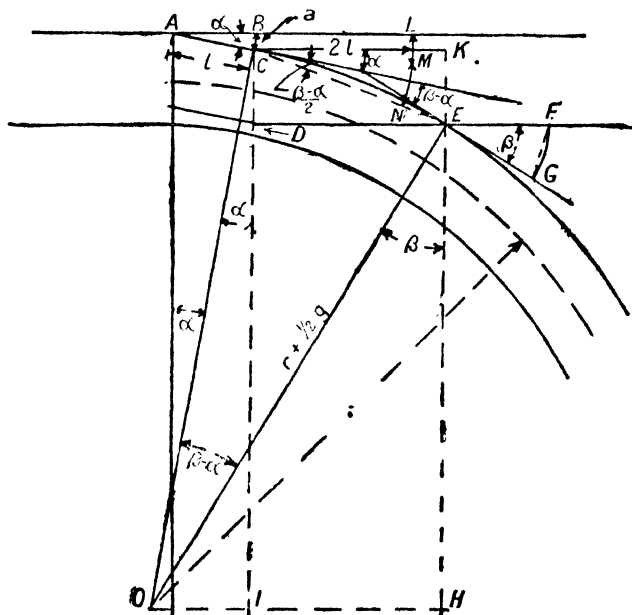
Fig. 234.

To Determine the Necessary Data for Setting-out a Simple Turn-out.

Case a. *When the Main Track is Straight.*—The data neces-
sary for setting-out the turn-out consists of (1) the switch angle
a ; (2) the angle of crossing β ; (3) the chord distance C E (Fig.
234) ; (4) the radius of the branch curve ; and (5) the gauge
of the rails. Usually the quantities (1), (2), and (5) are known,
and (3) and (4) have to be determined. In some cases the radius

of the branch r is predetermined, in which case β must be found.

The switch angle α is known from the length of the perpendicular offset to the heel of the switch and the length (l) of the point rail, for $\alpha = \sin^{-1} \dfrac{a}{l}$, where a is the length of the offset.

Referring to Fig. 234, we have :—

$$\cos \beta = \frac{E\,H}{O\,E} = \frac{D\,I}{O\,E}$$

$$= \frac{C\,I - C\,D}{O\,E}$$

$$= \frac{C\,I}{O\,C} - \frac{B\,D - B\,C}{O\,E}$$

$$= \cos \alpha - \frac{g^* - a}{r + \frac{1}{2}\,g}. \qquad . \qquad . \qquad . \quad (22)$$

Again, $\dfrac{C\,E}{E\,K} = \operatorname{cosec} \dfrac{\alpha + \beta}{2}$,

and the chord $C\,E = E\,K \operatorname{cosec} \dfrac{\alpha + \beta}{2}$

$$= (g - a) \operatorname{cosec} \frac{\alpha + \beta}{2}. \qquad . \qquad . \quad (23)$$

If β is a predetermined angle and the radius of the branch is required, it may be found from the computed length of $C\,E$ and angles, thus :—

$$C\,E = 2\,(r + \tfrac{1}{2}\,g) \sin \frac{\beta - \alpha}{2}.$$

Solving for r, we get,

$$r = \tfrac{1}{2} \left(C\,E \operatorname{cosec} \frac{\beta - \alpha}{2} - g \right). \quad . \quad . \quad (24)$$

To Set-out the Crossing.—The point E may be set-out from the known value of the lead $\left(\text{say } \dfrac{1}{p}\right)$ in the following way :—Mark the position of the point C from the known values of l and clearance, a. With C as centre, radius equal to $2\,l$, strike the arc L M N (Fig. 234) cutting the rail at the point L. From L lay

---

* g is measured to the centres of the rails.

off the chord L M = 2 a, and from M lay off the chord M N, such that M N : C M :: F G : 2 E F ;

i.e., $$M N : 2 l :: 1 : 2 p,$$

or $$M N = \frac{l}{p}.$$
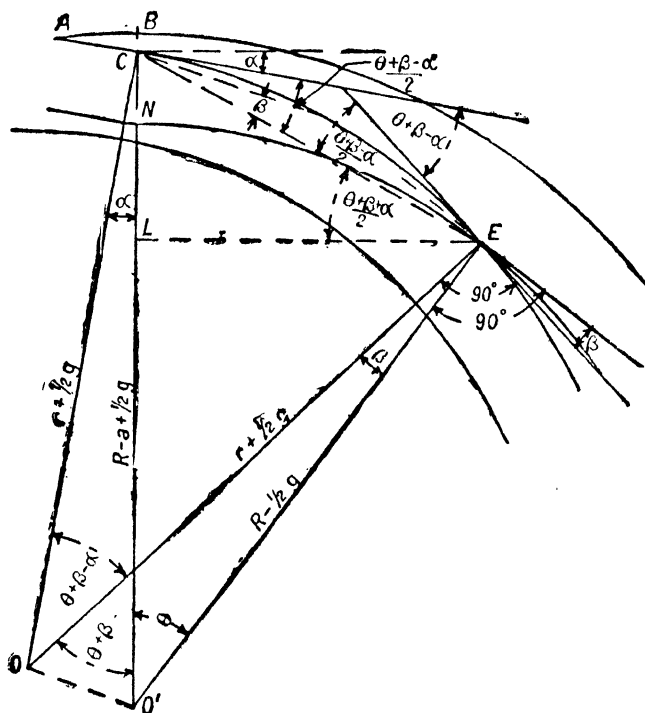
Fig. 235.

As may be seen from the figure, the angle K C E = $\dfrac{\alpha + \beta}{2}$, and the line C N produced will cut the rail D F at the required point E.

Case b. *When the Main Track and the Branch both curve in the same Direction.*—Let R be the radius of the centre line of the main track, and θ the angle subtended at its centre O′ by the chord C E (Fig. 235).

Before we can obtain the necessary data for setting-out the crossing, the value of θ must be determined. Referring to the figure, the angle

$$\mathrm{O'\ C\ E} = 90° - \left(\frac{\theta + \beta + \alpha}{2}\right)$$

and $$\angle\ \mathrm{O'\ E\ C} = 90° - \left(\frac{\theta - \beta - \alpha}{2}\right)$$

$$\therefore \quad \angle\ \mathrm{O'\ E\ C} - \angle\ \mathrm{O'\ C\ E} = \alpha + \beta.$$

Also, $$\tan\left(\frac{\angle\ \mathrm{O\ E\ C} - \angle\ \mathrm{O'\ C\ E}}{2}\right) = \tan\frac{\alpha + \beta}{2}$$

$$= \frac{\mathrm{R} - a + \frac{1}{2}g - (\mathrm{R} - \frac{1}{2}g)}{\mathrm{R} - a + \frac{1}{2}g + \mathrm{R} - \frac{1}{2}g}\cot\frac{\theta}{2}$$

or, $$\tan\frac{\theta}{2} = \frac{g - a}{2\ \mathrm{R} - a}\cot\frac{\alpha + \beta}{2}. \quad \textbf{.}\quad\textbf{.}\quad (25)$$

The chord

$$\mathrm{C\ E} = \mathrm{C\ L}\ \mathrm{cosec}\ \frac{\alpha + \beta + \theta}{2}$$

$$= (\mathrm{C\ O'} - \mathrm{L\ O'})\ \mathrm{cosec}\ \frac{\alpha + \beta + \theta}{2}$$

$$= \{\mathrm{R} - a + \frac{1}{2}g - (\mathrm{R} - \frac{1}{2}g)\cos\theta\}\mathrm{cosec}\ \frac{\alpha + \beta + \theta}{2}. \quad . \ (26)$$

If β is a predetermined angle and r is required, it may easily be found from the triangle C O E, since $\frac{\mathrm{C\ E}}{2\ (r + \frac{1}{2}g)} = \sin\frac{\theta + \beta - \alpha}{2}$. Solving for r, we get,

$$r = \frac{1}{2}\ (\mathrm{C\ E}\ \mathrm{cosec}\ \frac{\theta + \beta - \alpha}{2} - g).$$

If r be the predetermined quantity and β be required, the side O O' of the triangle C O O' is first computed, and the value of β then follows from the known lengths of the sides of the triangle O O' E. Thus,

$$\mathrm{O\ O'} = \sqrt{(\mathrm{R} - a + \frac{1}{2}g)^2 + (r + \frac{1}{2}g)^2 - 2(\mathrm{R} - a + \frac{1}{2}g)\ (r + \frac{1}{2}g)\cos a}, \quad (27)$$

and $$\tan\frac{\beta}{2} = \sqrt{\frac{(\mathrm{S} - \mathrm{O\ E})\ (\mathrm{S} - \mathrm{O'\ E})}{\mathrm{S}\ (\mathrm{S} - \mathrm{O\ O'})}},$$

where S is half the sum of the sides of the triangle O O' E. When a is small, O O' may be taken equal to $(\mathrm{R} - a + \frac{1}{2}g) - (r + \frac{1}{2}g)$; *i.e.*, equal to $\mathrm{R} - r - a$.

Case c. *When the Main and Branch Lines curve in opposite Directions.*—In this case, referring to Fig. 236, we observe that the angle
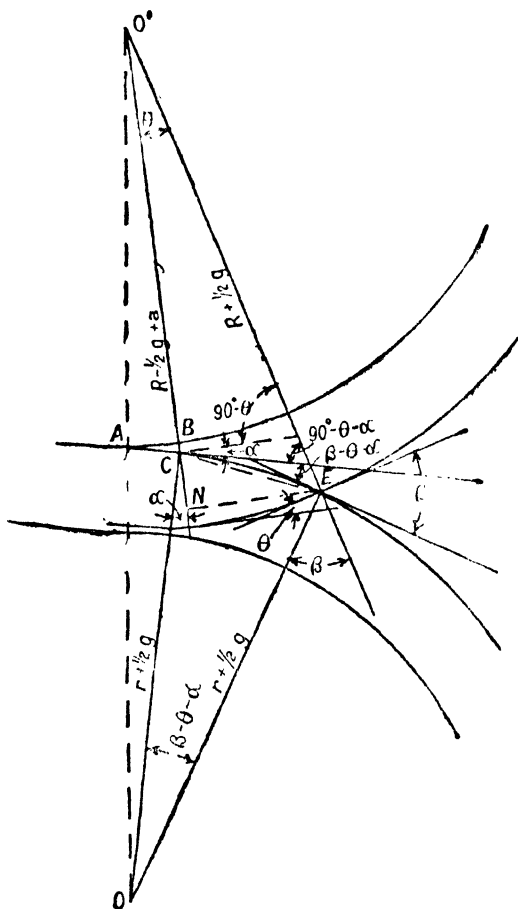
Fig. 236

$$O'C E = 90° + \left(\frac{\beta + \alpha - \theta}{2}\right), \text{ and } \angle O' E C = 90° - \left(\frac{\beta + \alpha + \theta}{2}\right)$$

$$\therefore \angle O' C E - \angle O' E C = \alpha + \beta, \text{ as in previous case.}$$

Also,
$$\tan \frac{\theta}{2} = \frac{R + \frac{1}{2}g - (R - \frac{1}{2}g + a)}{R + \frac{1}{2}g + R - \frac{1}{2}g + a} \cot \frac{\alpha + \beta}{2}$$

$$= \frac{g - a}{2R + a} \cot \frac{\alpha + \beta}{2}. \qquad . \qquad . \quad (28)$$

The chord

$$C E = C N \operatorname{cosec} C E N = (N O' - C O') \operatorname{cosec} \frac{\alpha + \beta - \theta}{2}$$

$$= \{(R + \tfrac{1}{2}g) \cos \theta - R - \tfrac{1}{2}g + a\} \operatorname{cosec} \frac{\alpha + \beta - \theta}{2}. \quad (29)$$

If β be known, and r is required, we have, from the triangle

O C E, $\dfrac{C E}{2(r + \frac{1}{2}g)} = \sin \dfrac{\beta - \alpha - \theta}{2}$

or, $\qquad r = \frac{1}{2} (C E \operatorname{cosec} \dfrac{\beta - \alpha - \theta}{2} - g). \qquad . \quad . \quad (30)$

If r is predetermined and β is required, we proceed as in the previous case to find O O' from the triangle O' C O, and the angle β from the triangle O' E O. Thus,

$$O O' = \sqrt{(R - \tfrac{1}{2}g + a)^2 + (r + \tfrac{1}{2}g)^2 + 2(R - \tfrac{1}{2}g + a)(r + \tfrac{1}{2}g) \cos \alpha}. \quad (31)$$

When α is a small angle, this may be taken as

$$= (R - \tfrac{1}{2}g + a) + (r + \tfrac{1}{2}g)$$

that is, $= R + r + a.$

Tan $\dfrac{O' E O}{2} = \tan \dfrac{(180° - \beta)}{2}$

$$= \cot \frac{\beta}{2}, \text{ and}$$

$$\cot \frac{\beta}{2} = \sqrt{\frac{(S - O E)(S - O' E)}{S(S - O O')}}, \qquad . \qquad . \qquad . \quad (32)$$

where S is half the sum of the sides of the triangle O E O'.

Example 10.—A branch line diverges from a straight main track. The length of the switch rail is 14 feet; the clearance at the heel of the switch is $4\frac{1}{2}$ inches, the lead of the crossing 1 in

8, and the gauge of the rails 4′ 11″. Find the radius of the branch and the chord distance.

Here, $\sin \alpha = \dfrac{4\frac{1}{2}}{14 \times 12} = 0 \cdot 026786.$

Also,

$\log 0 \cdot 026786 - 4 \cdot 6856 = \log$ arc in secs.

$\overline{2} \cdot 4279 - 4 \cdot 6856 = \log$ arc in secs.

$3 \cdot 7423 = \log 5,525'',$

and $\alpha = 1° 32′ 5″.$

Again, $8 = \frac{1}{2} \operatorname{cosec} \dfrac{\beta}{2},$

or, $\operatorname{cosec} \dfrac{\beta}{2} = 16$

$= \operatorname{cosec} 3° 35′,$

∴. $\beta = 7° 10′.$

Chord distance, C E $= (g - a) \operatorname{cosec} \dfrac{\alpha + \beta}{2}$

$= (4′ 11″ - 4\frac{1}{2}″) \operatorname{cosec} \dfrac{1° 32′ 5″ + 7° 10′ 00″}{2}$

$= 59 \cdot 87$ feet.

The radius of the branch,

$r = \frac{1}{2} \left(\text{C E} \operatorname{cosec} \dfrac{\beta - \alpha}{2} - g \right)$

$= \frac{1}{2} \left(59 \cdot 87 \operatorname{cosec} \dfrac{7° 10′ 00″ - 1° 32′ 5″}{2} - 4′ 11″ \right)$

$= 606 \cdot 91$ feet.

Example 11.—A branch line turns off a main track, the radius of which is 2,500 feet. The length of the switch rail is 18 feet, clearance 5″, gauge 4′ 8½″, and the lead of the crossing 1 in 15. If both curves turn in the same direction, find the chord distance and the radius of the branch line.

In this case, $\sin \alpha = \dfrac{5}{18 \times 12} = \sin 1° 19′ 34″.$

∴. $\alpha = 1° 19′ 34″.$

The lead $= 1$ in 15,

$\therefore$ $\qquad 15 = \frac{1}{2} \operatorname{cosec} \dfrac{\beta}{2},$

and $\qquad \beta = 3° \ 49' \ 14''.$

$$\alpha = 1° \ 19' \ 34''$$
$$\beta = 3° \ 49' \ 14''$$
$$\overline{\alpha + \beta = 5° \ \ 8' \ 48''}$$
$$\frac{\alpha + \beta}{2} = 2° \ 34' \ 24''$$

$$\operatorname{Tan} \frac{\theta}{2} = \frac{g - a}{2 \, \mathrm{R} - a} \cot \frac{\alpha + \beta}{2}$$

$$= \frac{4' \ 11'' - 5''}{2 \times 2{,}500 - 5''} \cot 2° \ 34' \ 24''$$

$$= \tan 1° \ 8' \ 50 \cdot 5'',$$

$\therefore$ $\qquad \theta = 2° \ 17' \ 41''.$

$$\frac{\alpha + \beta + \theta}{2} = 2° \ 34' \ 24'' + 1° \ 8' \ 50 \cdot 5''$$

$$= 3° \ 43' \ 14 \cdot 5''.$$

Chord distance

$$\mathrm{C\,E} = \{ \mathrm{R} + \tfrac{1}{2} g - a - (\mathrm{R} - \tfrac{1}{2} g) \cos \theta \} \operatorname{cosec} \frac{\alpha + \beta + \theta}{2}$$

$$= \left\{ 2{,}500 + \frac{4' \ 11''}{2} - 5'' - \left(2{,}500 - \frac{4' \ 11''}{2} \right) \right.$$
$$\left. \cos 2° \ 17' \ 41'' \right\} \operatorname{cosec} 3° \ 43' \ 14 \cdot 5''$$

$$= 100 \cdot 32 \text{ feet.}$$

$$\theta = 2° \ 17' \ 41''$$
$$\beta = 3° \ 49' \ 14''$$
$$\overline{\theta + \beta = 6° \ 06' \ 55''}$$
$$\alpha = 1° \ 19' \ 34''$$
$$\overline{\theta + \beta - \alpha = 4° \ 47' \ 21''}$$
$$\underline{\underline{\frac{\theta + \beta - \alpha}{2} = 2° \ 23' \ 40 \cdot 5''.}}$$

Required radius,

$$r = \tfrac{1}{2} \; (\text{C E cosec} \; \frac{\theta + \beta - \alpha}{2} - g)$$

$$= \tfrac{1}{2}(100 \cdot 32 \; \text{cosec} \; 2° \, 23' \, 40 \cdot 5'' - 4' \, 11'')$$

$$= 1,198 \cdot 1 \; \text{feet.}$$

While the above formulæ have been demonstrated for simple turn-outs, it is evident they may be applied to more complicated cases by inserting the appropriate data, since any formulæ for setting-out junctions and crossings can only be modifications of those already given.

When the turn-out forms part of a cross-over, or of a series of cross-overs, special designs must be made and the setting-out done in accordance with them.

Setting Back a Switch Point.—When a design shows the positions of tangent points without reference to the length of the switch rail, the setter-out may allow for this either by cambering the " stock rail," or by setting back the switch point. The stock rail is the fixed rail which joins up to the heel of the switch.

Let the distance the switch rail is set back from the tangent point A (Fig. 237), be x, then

$$x = \sqrt{\text{B D}^2 - \text{D C}^2} - \text{D E}$$

$$= \sqrt{l^2 - a^2} - \sqrt{(\text{R} + \tfrac{1}{2} \, g)^2 - (\text{R} + \tfrac{1}{2}g - a)^2}. \qquad (33)$$

If the radius of the curve be such that the value of x is negative, the switch point should be placed at the exact tangent point of the design, and the stock rail cambered so as to fit up to the heel of the switch, thus making the curve flatter at the tangent point.

To Determine the Radius of the Centre Line of a Constructed Railway.—For various reasons, it is often convenient to determine the radius of the centre line of a constructed railway, in the field. This may be done in the case of a single track by measuring the length of the chord of the outer rail tangential to the inner rail, the measurements being taken to the centre of the metals. Let F H (Fig. 237) be this chord, then, as may be seen from the figure,

$$\frac{\text{F H}^2}{4} = (\text{R} + \tfrac{1}{2} \, g + \tfrac{1}{2} \, \text{W})^2 - (\text{R} - \tfrac{1}{2} \, g + \tfrac{1}{2} \, \text{W})^2,$$

or,

$$\text{R} = \frac{\text{F H}^2}{8 \, (g + \text{W})} = \frac{\text{chord}^2}{8 \, (g + \text{W})}. \qquad \cdot \qquad \cdot \qquad (34)$$

For a double track, the chord of the inner rail of the outer track is measured tangential to the outer rail of the inner track in the six-foot interval, and in this case,

$$R = \frac{chord^2}{8\,(S + W)}. \quad . \quad . \quad . \quad (35)$$

S being the width of the six-foot interval, which varies, and should be measured, and W the width of the top flange of the rails.

The radii of the curves formed by the actual rails are equal to

$R \pm \frac{1}{2}\,g$, for a single track,

$R \pm \frac{1}{2}\,S$, for the inner rails of a double track,

and $R \pm \frac{1}{2}\,S \pm g$, for the corresponding outer rails.
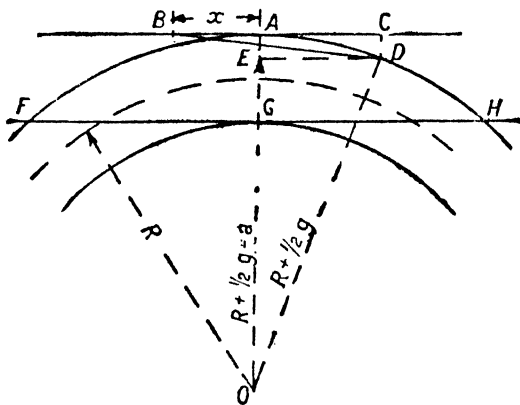
Fig. 237.

Vertical Curves at Changes of Gradient.—These have been fully dealt with on p. 168, *et seq.*

Setting-out Fence Widths.—The fence width is usually pegged out at every chain, and at such intermediate points as may be necessary. When the ground is irregular, the widths are scaled off the plan, but when the ground is straight on cross-section, the widths may be obtained by calculation as explained in Chap. VII., Part I. The fence pegs are laid off from the centre pegs on both sides of the railway before the actual construction begins, and the fences are erected on the lines marked out by them.

Re-determination of Curves.—As the work of construction of a railway progresses, the pegs marking out the centre line are

removed in cuttings and covered up by embankments, and as the formation is nearing completion it will be necessary to re-determine the position of the centre line, for which purpose provision must be made before the work of construction com-mences. To do this, pegs are placed in suitable positions, near all important points, such as tangent points, ends of transition curves, points of junction, etc., clear of the works, so that the positions of the points may be readily found either by direct measurement from the pegs, or by lining through. For a point in a cutting, two pegs may be placed at right angles to the centre line equidistant from the centre peg, and near the top of the cutting. On stretching a steel tape, carrying a plumb-bob suspended from its middle point, over the pegs, the position of the centre peg is at once determined by the position of the plumb-bob.

To determine the position of a point on an embankment, two pegs should be placed on each side of the embankment, in lines making an angle of about 45° with the centre line and intersecting at the centre peg. The position of the latter may then be found either by lining through rods erected at the pegs, or with a theodolite, or by steel tape measurements from the pegs.

On straight reaches it is only necessary to re-establish points in this way at the end of every 5 or 10 chains, the intermediate pegs may afterwards be replaced by chaining between those which have already been fixed.

In the case of curves, it will be necessary to re-determine their position, working from the tangent points on the formation.

To guide the plate layers in their work of laying the rails on the finished road surface, it is usual to mark out all curves by pegs at intervals of 5 feet. These pegs are set out from the chords joining the points already fixed, by ordinates to the chord. The lengths of the ordinates may be found from equation (12) in the manner illustrated by Example 2.

Setting-out Bridge and Culvert Foundations.—These are best set out from a drawing giving the co-ordinates of the corners of the abutments and wing walls, with reference to two lines at right angles to each other and intersecting at the centre of the bridge. Thus, in Fig. 238, if R Q and S T are lines at right angles passing through the centre P of the bridge, and for any corner such as A we lay off the known distances A b, A b' at right angles to R Q and S T, the point A will be located. To set out the foundation, set up the theodolite over the point P, and carefully line in a number of chain arrows or perforated

pegs along the line S T. A cord passed through the eyes of the arrows or through the holes in the pegs will now mark out the line. Two or three arrows will be sufficient if the ground is fairly flat, but a fair number may be needed if the ground is irregular. Next, set out the line R Q at right angles to S T, and line in sufficient arrows or pegs to keep the cord stretched in this line, quite straight. The distances P 5, P 6, P 7, P 8, and P b', P 5', P 7', P 8', are laid off along their respective lines, and the points marked with chain arrows. Now take two tapes, and putting
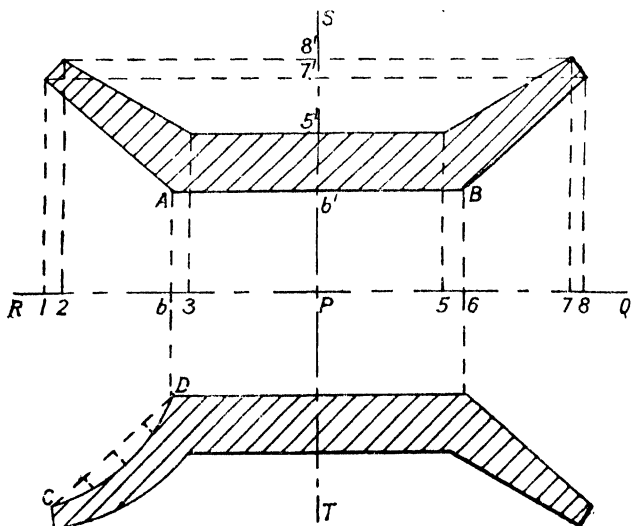
Fig. 238.

the rings together, direct the chainman to pull them tight, while distances B 6 and B b' on the tapes are held on the lines R Q and S T by the engineer and his assistant. Thus point B is fixed and its position is marked by driving in a peg. In a similar way the positions of the other points on the outline of the foundation are found and marked. The foundation for the second abutment is set out in a similar manner.

When all the points have been marked out, a cord should be passed around the periphery of each abutment, and the work checked by comparison with the plan. While the cord marking the periphery of the abutment is in position the outline of the

foundations should be " nicked " out, so that there can be no mistake when the work of excavation commences.

If the wing walls are curved, points on the curve may be set out by offsets to the chord, as at C D (Fig. 238), both offsets and distances along the chord may be scaled off the plan.

The levels at the pegs should be obtained for the double purpose of determining the quantity of earth to be excavated and the depth of the foundation below the surface of the ground.

The mode of procedure in the case of a skew bridge is the same as that already described, except that the angle between the
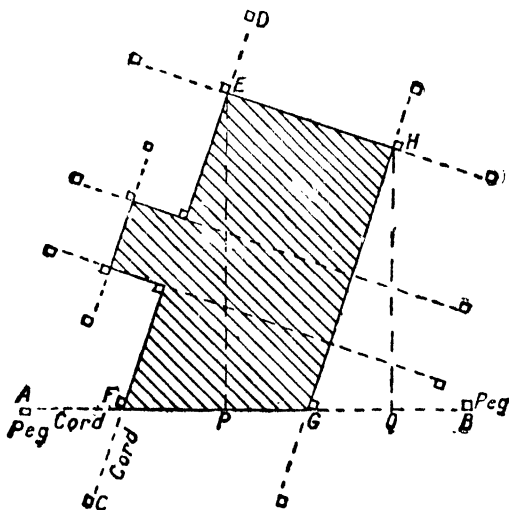
Fig. 239.

lines R Q and S T will be equal to that between the centre lines of the two roads.

When the foundations of the bridge have been laid, the corners of one abutment are laid down with a steel tape from co-ordinates as described above, and the corners of the second abutment are marked from the centre line of the bridge and the corners of the first abutment, thus ensuring the correctness of the span.

Setting-out Buildings.—In marking-out the positions of the corners of proposed buildings, it must be remembered that if the exact position of each corner is pegged, the pegs will be lost as the work of excavating the foundation proceeds. Pegs for

this purpose must be placed well back from the work, clear of all timbering, in positions where they will not be disturbed. They may be protected by standing a drain pipe over each of them.

The pegs are first put in at the sites of the actual corners of the building, and reference pegs are then put in position by lining through with cords, as shown in Fig. 239. The reverse process will determine the positions of the corners of the building, for it is evident that the point of intersection of cords stretched between A B and C D is the point F. Instead of lining through with two cords in this way, the corners of the building may be fixed by one cord and measured distances from the nearest pegs.

Usually the right angles at the corners of buildings are set out with the tape, as described on p. 30. The angles at the corners of important buildings should be set out with the theodolite or a box sextant. If the angles to be set out are not right angles, and an instrument for setting-out angles is not available, the sides of the building may be set out from the co-ordinates F P, P E ; F Q, Q H measured from the plan. The comparison of the measured length of E H and its length given on the plan will form a sufficient check on the setting-out.

Bench Marks on Building Sites.—For setting-out buildings which have several sets of floor, engine, and machine bed levels, bench marks are established in convenient places around the site. A convenient form of bench mark for this purpose consists of a stout peg set in the centre of a block of concrete about 2 feet square, and protected by a piece of pipe bedded in the concrete. The level point is that of the head of a round-headed nail driven into the top of the peg. In some cases it is convenient to have all the bench marks on the site set to the same level. This may be done by slowly driving in the peg until the desired level is obtained, while the concrete is still wet.

The bench marks should be set a few yards away from the site, so that they may remain undisturbed until the work is completed.

Setting-out Sewers.—In the setting-out of sewers and drains, the engineer is, as a rule, more concerned about his levels than the direction of the centre line. In many cases the sewer passes down the centres (or at a known distance from the centres) of streets which are already constructed, and the centre line is marked out as required by measurements from the curbs. Where the centre line passes across fields, it may be set-out with

a theodolite or with poles, and is marked with pegs in the usual way. If the sewer is to be constructed by tunnelling, as is now usual in the centres of large towns and cities, the direction of the centre line is transferred underground by suspending weighted wires down the shafts, as described in Chap. XV., Part II. Underground curves are best set-out by the theodolite and chain system, allowance being made for the difference between the arc and its chord.

The level of the invert of the sewer is given on the longitudinal section at all points of change in gradient, and great care is necessary to see that the given levels are adhered to. The
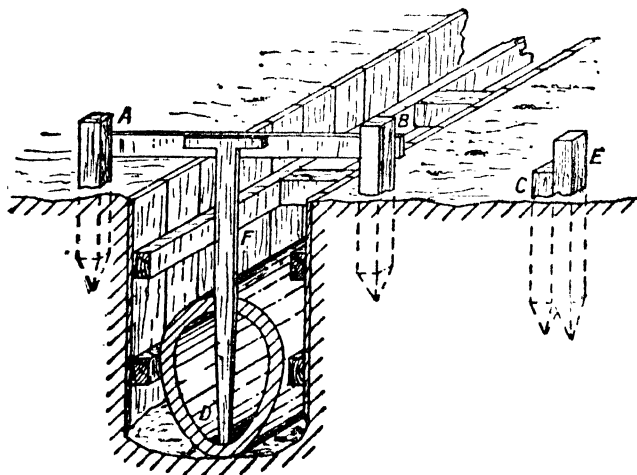
Fig. 240.

gradients on the site are marked out by sight rails (A B, Fig. 240), which are placed at each change of gradient, or at intermediate points as required. The line joining the tops of a pair of sight rails is made parallel to the invert (D), and at a given height above it, which may be fixed in either of two ways, (a) as a definite height above D, or (b) at a given level relative to a bench mark peg C. The direction of the gradient given by the sight rails is transferred to the invert by means of a boning rod (F), the top of which is exactly in line with the tops of the sight rails when the foot of the rod is held on the invert. The sight rails should be white-washed, and the top of the boning rod

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blackened, so that it may be more readily seen against the white background of the rails. The bench mark peg C should be in a safe place, and be protected by a guard post E (Fig. 240).

The sight rails are set in position by levelling ; thus, suppose the reduced level of the invert D is given on the section as 72·50, the reduced level of C 78·05, and the length of the boning rod is 7 feet. Obviously, the reduced level of the sight rail is 72·50 + 7 = 79·50, and its height above C is 79·50 − 78·05 = 1·45 feet. To place the sight rail in position, set up the level anywhere convenient and take the staff reading on C ; let this be (say) 6·81. As the sight rail is 1·45 feet higher than C, the staff reading will be 6·81 − 1·45 or 5·36, when the staff is held against the rail post with its foot at the height at which the top of the sight rail must be placed. Now scribe a line on the rail post, at the foot of the staff, when the reading is 5·36 ; determine and mark a line at the same level on the other post. If the sight rail be nailed to the posts with its top edge in coincidence with the lines, it will be at the required level.

The other sight rails are set in position in precisely the same way.

EXAMPLES.

1. Find the ruling gradient for a railway on which the maximum load on the driving wheels of the engine is 40 tons, maximum weight of tender and train 450 tons, and least speed 25 miles an hour. Weight of engine 65 tons.   (*Ans.* 1 in 130, or 1 in 165.)

2. A long, straight line has to be set-out from a point A. The angle between the line at A and a referring object P is known. How would you set-out the line with a theodolite ?

3. Determine the radius of a 4° curve, (*a*) assuming the length of the chord is equal to its subtended arc ; and (*b*) allowing for the difference between the arc and its chord.
                                   (*Ans.* 1,432 feet ; 1,432·62 feet.)

4. Two straight reaches which intersect at an angle of 120° 30′ are joined by a curve of 1,000 feet radius. The chainage at the intersection point is 2,895 feet ; find the chainage at the tangent points.   (*Ans.* 2,323·50 feet ; 3,362·0 feet.)

5. Compare the different systems of ranging-in a curve by the chain and tape.

6. If the curve in Question 4 is set-out in 100-feet chords by the " method of offsets to a chord produced," find the lengths of the first three offsets. (*Ans.* 2·93 feet, 8·82 feet, and 10 feet.)

7. Show that the length of the Nth offset in Baker's system of ranging a curve is approximately equal to $\dfrac{N^2}{2\,R}$, N being the number of chains set-out on the tangent line from the tangent point, and R the radius of the curve. From the data given in Question 4, determine the lengths of the first four offsets to the curve, and the offset to the vertex. Peg distance 100 feet.

(*Ans.* 2·93 feet, 15·58 feet, 38·21 ft, 70·89 feet, and 123·1 feet.)

8. Show that the actual distance (in links or feet) between the pegs N and N + 1 set-out by Baker's system, is given by the equation $D = 100 \left\{ 1 + \tfrac{1}{2} \left(\dfrac{2\,N + 1}{2\,R} \right)^2 \right\}$ very nearly; hence, find the consecutive distances between the first four pegs in Question 7.

(*Ans.* 76·57 feet, 100·8 feet, 102·5 feet, and 105·2 feet.)

9. Two straight lengths on the centre line of a proposed road meet at an angle of 110° 30′, and are to be joined by a curve of 500 feet radius. The curve is to be set-out by ten equi-distant offsets from the chord joining the tangent points. Determine the lengths of the offsets.

(*Ans.* 0 ; 34·1 ; 59·0 ; 76·0 ; 85·9 ; 89·2 ; 85·9 ; 76·0 ; 59·0 ; 34·1 ; 0 feet.)

10. It is proposed to connect a pithead (P) with a main line of railway by a road consisting of a straight length and a curve of 40 chains radius. The main line is straight, and runs due N. and S. The straight length from P has a bearing of S. 30° 30′ E., and meets the main line at a distance of 45·52 chains. Describe fully how you would mark out the centre line of the road. What length would the road be when finished ?

(*Ans.* If T and T_1 are the tangent points, and A the intersection point, A T = A T_1 = 10·91 chains ; P T = 34·61 chains ; length of road = 55·9 chains.)

11. Find the first four deflection angles from the data given in Question 10. The pegs are to be 1 chain apart.

(*Ans.* 16·76′ ; 59·73′ ; 1° 42·7′ ; 2° 25·67′.)

12. If the first chord on a 2° curve is 45 feet long, and the pegs are 100 feet apart, find the first four deflection angles.

(*Ans.* 27′ ; 1° 27′ ; 2° 27′ ; 3° 27′.)

13. A 2° curve is to be set-out in 100-feet lengths between two straights, which meet at an angle of 150°. The ground

is such that all points on the curve after the sixth cannot be seen from the initial tangent point. The chainage at the intersection point is 8,945 feet. Determine the radius of the curve and the necessary deflection angles. Explain how you would proceed to range the curve.

> (*Ans.* R = 2,865 feet ; chainage at tangent points, 8,177·2 and 9,677·2 feet ; 1st chord, 22·8 feet ; deflection angles, 0° 13·68′ ; 1° 13·68′, etc.)

14. The point of intersection (A) of two straight reaches is inaccessible. To determine the intersection angle a straight line B C is set-out between the tangent lines. The radius of the curve is 1,500 feet, and chainage at B, 8,475 feet. If B C is 642 feet, the angles A B C and A C B are 14° 20′ and 20° 10′ respectively, find the chainage at the tangent points (T and T_1) and the distances B T and C T_1.

> (*Ans.* 8,400 feet, and 9,303·4 feet ; 75 feet, and 184·9 feet.)

15. In Question 14, let the line B C cut the curve at the points H and K. Find the distances of these points from the point B.

> (*Ans.* B H = 9·54 feet ; B K = 587·94 feet.)

16. Two straight lengths A T B and C T_1 D are to be joined by a reverse curve, the two parts of the curve having the same radius. If the length of the chord joining the tangent points T and T_1 is 160 chains, and the angles B T T_1 and C T_1 T are 25° 30′ and 30° 15′, find the radius.   (*Ans.* 85·46 chains.)

17. If in Question 16 one part of the curve has a radius of 90 chains, find the radius of the other part. (*Ans.* 80·15 chains.)

18. To pass an obstacle, a straight line is given a deviation of 300 feet ; the gap in the line is filled-in by three reverse curves, each having a radius of 2,500 feet, the curves being placed symmetrically about the point of deviation. Find the distance of the tangent points from the deviation point.

> (*Ans.* $\pm$ 1,706 feet.)

19. Find the superelevation on a curve of 2,500 feet radius for a speed of 40 miles an hour, and gauge 4′ 8½″.

> (*Ans.* 0·2 foot.)

20. The equation to a transition curve is of the form $y = c\,x^3$, y being the offset at a distance x, measured from the tangent point along the tangent line. If the radius of the circular curve to be adjusted is 2,000 feet, and the length of the transition curve 400 feet, find (*a*) the lengths of eight equidistant offsets on the transition curve, and (*b*) the angle between the common

tangent at the end of the transition curve, and the chord to the initial tangent point.

(*Ans.* 0·03 ; 0·21 ; 0·70 ; 1·66 ; 3·26 ; 5·63 ; 8·95 ; 13·33 feet. 3° 48′ 05″.)

21. If in Question 20 the intersection point is 12,642 feet from peg 0, what will be the distance at the tangent point ? The intersection angle is 95° 30′. (*Ans.* 10,621·8 feet.)

22. A branch line turns off a straight track by a curve 1,000 feet radius. The length of the switch rail is 15 feet, and the clearance 5 inches. If the gauge is 4′ 11″, find the chord distance to the point of crossing, and the lead. (*Ans.* 71·16 feet ; 1 in 10·12.)

23. If the lead of the crossing in Question 22 is 1 in 8, find the radius of the branch and the chord distance to the point of crossing. (*Ans.* 603·5 feet ; 58·94 feet.)

24. A branch turns off a curved main track, the radius of which is 3,000 feet ; both lines curve in the same direction. The lead of the crossing is 1 in 10, length of switch rail 14 feet, clearance 5 inches, find the chord distance to the point of crossing, and the radius of the branch. $g = 4′ 11″$.

(*Ans.* 69·64 feet ; 743·9 feet.)

25. If in Question 24 the main and branch lines curve in opposite directions, find the radius of the branch line and the chord distance. (*Ans.* 1,375·9 feet ; 66·26 feet.)

26. From the following data determine the radius of the centre line of a constructed curve :—Length of chord tangential to centre of inner rail 300 feet, gauge 4′ 8½″, width of top flange of rails 2⅛″. (*Ans.* 2,280·0 feet.)

CHAPTER XV.

SETTING-OUT TUNNELS.

General Considerations.—Tunnels are rarely used in the construction of common roads, they are occasionally constructed in mountainous districts when a section of the road is subject to avalanches, and they not only protect the road, but serve as places of refuge for travellers. Tunnels are also sometimes constructed under rivers, where the construction of a bridge is considered undesirable, or impracticable.

On railways, the question of connecting two points by a tunnel, a cutting, or a bridge is decided mainly from the point of view of capital expenditure. When a cutting reaches a depth of 60 feet, and the ground rises rapidly afterwards for a considerable distance, it will usually be more economical to construct a tunnel than an open cutting. Other cases arise when (1) a cutting would cause an expensive severance of property; (2) the construction of a surface road would be too costly, as in the tube railways under London; and (3) a tunnel is the only practical means of connecting two points, as in the case of the tunnel under the Severn, or that under the Mersey at Liverpool.

Of necessity, tunnels are entered either on the level or by inclines. In the construction of a tunnel, vertical shafts are often made use of, both for the purpose of facilitating the construction of the work, and for checking the accuracy of the alignment and levels. Such shafts are usually lined with brickwork, and form part of the permanent works, being useful as ventilating shafts.

The formation level of a tunnel is always set at a slight inclination to the horizontal, for drainage purposes. If the tunnel is short the gradient may be in one direction only; but, if long, the formation is made to slope to both ends.

Surface Alignment.—Wherever possible, the whole of the centre line of a proposed tunnel should be accurately marked out on the surface of the ground. If this cannot be done, the

centre line must at least be set-out over the contiguous shafts near the ends of the tunnel. In some cases the centre line cannot be set-out on the surface, as, for example, when the tunnel passes under high snow-clad mountain peaks. In such a case, the ends of the tunnel must be connected either by a chain of triangles, or by a traverse.

The method described by Mr. Baggallay* of setting-out the chief tunnel in a series of tunnels forming an almost continuous tunnel 10 miles in length, is of interest. The tunnels are at an almost continuous gradient of 1 in 12½, and it was necessary to determine the length of each section with great accuracy " In setting-out the chief tunnel at an elevation of 10,000 feet above the sea, it was necessary to pass the top of the mountain 3,000 feet higher. A straight line was set-out as nearly as possible over the direction of the tunnel, and another about 2,000 yards from it, that one over the tunnel being about 6,000 yards long Between these two lines a series of large triangles, as nearly equilateral as possible, was set-out, the base line measured in one valley being 2,000 metres long. From this the triangles were set-out. They were checked by a carefully measured base line in the other valley, and the error was only one metre."

The simplest case of surface alignment occurs when the centre line of the tunnel is straight in plan, and a suitable point can be found which commands both extremities. The line is first set-out with a 6-inch theodolite, and at the selected station an observatory is erected, in which the instrument is mounted, accurately over the centre line. Two points in the preliminary setting-out are taken as fixed, one at the observatory, the other being some conveniently situated point in the line; and from the main station at the observatory, the line is set on suitable permanent objects at the ends of the tunnel, and near to each shaft.

Setting-out Totley Tunnel.—In setting-out the surface alignment of the Totley Tunnel on the Dore and Chinley Railway, observatories were built at the highest point on the surface (the Summit Observatory),* at the extreme points east (the Bradway Observatory), and west (Sir William Observatory), and at each of the commanding positions on the surface of the ground (Fig. 241). An observatory (No. 3 West) was also built beyond the western entrance of the tunnel, and a station (No. 4 West) at the foot of the hill beyond the River Derwent, to enable these points to be seen from the western heading, whenever necessary.

In setting-out the line, points set by the 6-inch theodolite at the summit and No. 1 West were taken as fixed ; and the line was set from the summit observatory to the extreme observatories east and west, and on No. 1 East. The instrument was then removed and set-up on No. 1 West, and the line set on No. 2 West, with Sir William Observatory as the fixed point. The instrument was next removed to No. 2 West and the line set on No. 3 West. In a similar way the line was set-out on the eastern side. To check the line, the instrument was subsequently set-up at the extreme observatories and the centre lines of No. 4 East and No. 3 West were re-observed. No. 4 West was set-out from No. 3 West and checked from No. 2 West.
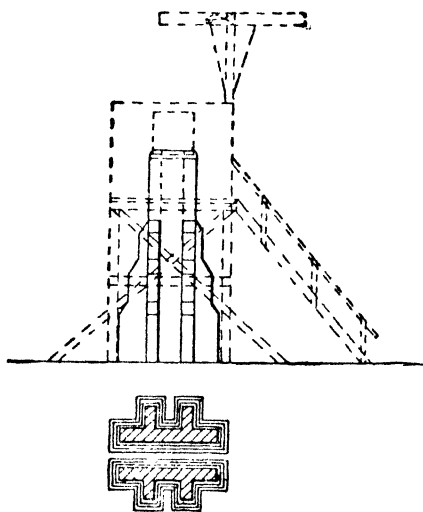
Fig. 242.

With the exception of the Severn Tunnel, the Totley Tunnel is the longest tunnel in this country, being over $3\frac{1}{2}$ miles long. When the headings met, the error in alignment was found to be $4\frac{1}{2}$ inches.

Observatories.—Observatories (Fig. 242) * are simply brick, stone, or concrete piers, connected by a stone or concrete cap to carry the instrument ; the piers are surrounded by a wooden hut, in which the observer works. The hut is carried on its own

* By permission of the Council of the Inst. C.E., vol cxvi.

foundation, and the platform on which the observer stands is not connected to the piers. In this way the vibration caused by the movements of the observer, or by the action of the wind, is not communicated to the instrument. The observer's instructions to his assistants marking stations may be communicated either by a semaphore or by a field telephone.

Marking Surface Line.—In the country the position of the centre line may be marked by stout pickets, and the exact position of the point in the centre line by a centre punch or file mark in the head of a large brass nail driven into the top of the picket. The position of the mark on the nail is obtained as described in the previous chapter, by circle right and circle left observations with the theodolite.

In towns the centre line is marked by driving iron spikes between the sets or other road covering, the exact point on the line being marked with a centre punch. To locate the spikes, measurements must be made to fixed points on the nearest buildings, or other permanent objects.

Curves in Tunnels.—When a tunnel is curved, it is desirable, although not absolutely necessary, to set-out the curve on the surface. If this is impracticable, sufficient measurements must be made on the surface to fix the positions of the tangent points exactly, and the curve is set out underground usually by the chain and theodolite system. If the curve is sharp, an allowance must be made for the difference in length of the arc and its chord.

Use of Bore Holes.—If the line follows the centre of a street or road, and is at no great depth below the surface, bore holes made may be from the surface at intervals; the underground centre line may then be checked by plumb lines suspended in the bore holes. This method of checking the position of the underground lines cannot be relied on unless the suspended wire can actually be seen in free suspension from the surface point; the observation may be made by slowly moving a lamp around the wire underground, if the light can be seen in all positions from the surface, the wire is in free suspension. Bore holes are seldom quite vertical, and even when tube-lined, we have no guarantee except by visual examination that a plumb line hung down the hole may not be touching the side of the bore, or be caught by some accidental projection; thus causing the line to deviate from its true position. With proper precautions, bore holes are a valuable aid in checking the underground alignment, particularly when the tunnel is curved.

Measurements.—All surface measurements should be made with a steel band, or a steel tape. Where the nature of the ground permits, as in the case of streets or land having an even surface, the measurements are best made on the surface, the measurements being afterwards reduced to the horizontal from data obtained either by levelling, or with the theodolite. The measurements should be made at least twice, and should agree within a small fraction of an inch.

When the ground is uneven, but otherwise free from obstacles, the required distance may be measured by means of wooden rods, set horizontal with a hand level, the position of the end of one rod relative to the next in the series being fixed with a plumb line. This method was used in fixing the position of a point at which an angle was turned in the tunnel under Dunmail Raise on the line of aqueduct from Thirlmere to Manchester,* it being necessary, in order to fix the point, to measure a length of 1,715 yards, which fell 250 feet in that distance.
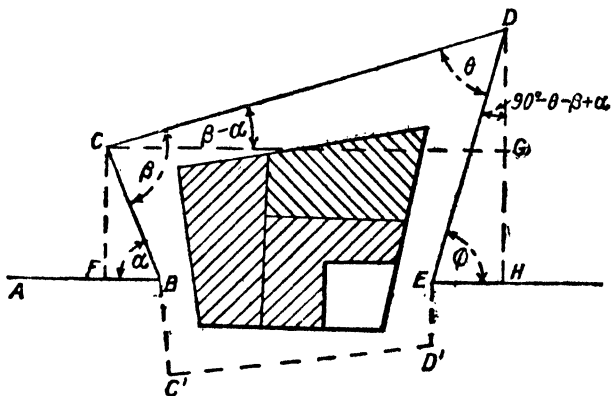
Fig. 243.

Obstacles on the Line.—If an obstruction such as a block of buildings occurs when setting-out the line, the most convenient way of passing the obstacle is by a traverse around it. Suppose, for example, the line A B (Fig. 243) intersects an obstacle as shown, and the traverse B C D E is thrown around it. The line may be continued from E, if we know the length of D E and the angle D E H (φ). By measuring the angles α, β, and θ, and the sides B C and C D of the traverse, the values of D E and φ may

* Minutes, *Proceedings Inst. C.E.*, vol. cxvi.

be found. Probably the easiest way to compute the required quantities is by the method of " latitudes " and " departures " with reference to A B, thus : by inspection of Fig. 243 we observe that $\varphi = \theta + \beta - \alpha$.

" Latitude " of E = B E = $- \text{B C} \cdot \cos \alpha + \text{C D} \cdot \cos (\beta - \alpha)$
$$- \text{D E} \cdot \cos (\theta + \beta - \alpha).$$

" Departure " of E = B C $\cdot \sin \alpha + \text{C D} \cdot \sin (\beta - \alpha)$
$$- \text{D E} \cdot \sin (\theta + \beta - \alpha)$$
$$= 0.$$

$\therefore$
$$\text{D E} = \frac{\text{B C} \cdot \sin \alpha + \text{C D} \cdot \sin (\beta - \alpha)}{\sin (\theta + \beta - \alpha)}.$$

To continue the line beyond E, make D E equal to its calculated length, set-up the instrument at E, and turn off the angle D E H equal to its computed value. The telescope will now point in the forward direction of the line. The position of E and the forward direction of E H should be checked by a similar traverse, such as B C' D' E (Fig. 243) run in a new direction.

Example 1.—Let B C = 240 feet 8 inches ; C D = 960 feet 9½ inches ; $\alpha = 62° 30' 15''$; $\beta = 84° 15' 24''$; $\theta = 50° 22' 24''$; find the angle φ and the lengths of B E and D E.

Here,
$$\beta = 84° 15' 24''$$
$$\alpha = 62° 30' 15''$$
$$\overline{\beta - \alpha = 21° 45' 09''}$$
$$\theta = 50° 22' 24''$$
$$\overline{\theta + \beta - \alpha = 72° 07' 33'' = \varphi.}$$

$$\text{D E} = \frac{240 \cdot 66 \cdot \sin 62° 30' 15'' + 960 \cdot 79 \cdot \sin 21° 45' 09''}{\sin 72° 07' 33''}$$
$$= 598 \cdot 43 \text{ feet.}$$

B E $= -240 \cdot 66 \cos 62° 30' 15'' + 960 \cdot 79 \cos 21° 45' 09''$
$$- 603 \cdot 652 \cos 72° 07' 33''$$
$$= 597 \cdot 59 \text{ feet.}$$

The same mode of calculation will enable us to determine the closing side, no matter how many sides the traverse polygon may have.

Instruments for Setting-out Tunnels.—Many of the longest tunnels in this country have been set-out with 6-inch theodolites reading to 20 seconds ; but, the 7- or 8-inch theodolite reading to 5 or 10 seconds is more suitable for larger work. When long sights of 3 or 4 miles are necessary, transit instruments of the type shown in Fig. 244 are often used for the alignment. These instruments are not intended for measuring horizontal angles ; when set in position, the only possible movement of the telescope
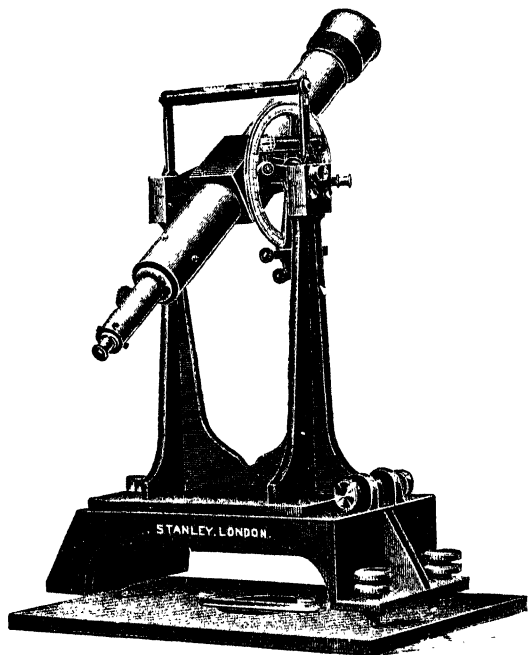
Fig. 244.

is in a vertical plane. The base of the instrument is substantially made in cast iron, is fitted with levelling screws, and is provided with vertical standards on the upper extremities of which are the bearings of the trunnion axis of the telescope. The axis is fitted with a vertical circle reading by verniers either to minutes, or to 20 seconds, the adjustment of the circle is made in the usual way, by a clamp and tangent screw. A fine adjustment in azimuth is provided by the adjusting screws shown at

the bottom of the right-hand standard. One of the trunnions is perforated for illumination of the cross wires at night, by means of a lamp mounted on the standard. In instruments of this class, the telescope is fairly large, the size of the object glass varies from $1\frac{3}{4}$ inches to $3\frac{1}{2}$ inches, and the focal length from 24 inches to 36 inches. The power of the eye-piece is usually about 30 or 40. Levelling of the transverse axis is done with a delicate striding level, which rests on the ends of the trunnions.

Face-right and face-left observations are made with the instrument by reversing (end for end) the axis in its bearings.

An instrument of this type, having a telescope 30 inches long and an object glass 3 inches in diameter, was used in setting-out the Totley Tunnel, the terminal stations being each over 3 miles from the Summit Observatory.

Transferring the Line Underground.—Where vertical shafts are used in the mining operations, the usual practice in transferring the line underground is to place two points in the centre line near the mouth of each shaft in a position clear of the works in connection with the sinking operations. When the shaft has been sunk to the desired level, and it becomes necessary to set-out the direction of the line underground, the theodolite is centred over one of the marks, and the telescope is set to bisect the other mark. The line is then set on two baulks of timber, spanning the shaft and near its edge, and from these points plumb lines are suspended down the shaft. The plumb lines are usually fine wires, stretched tight by a weight of 20 to 30 lbs. at their lower ends, the stretching weight being freely suspended in a vessel of water (thickened with mud if necessary) to still its vibrations. When the wires are in position they are examined along the whole of their length, to ensure that they are hanging in free suspension and are not caught on any accidental projection, on the side of the shaft. The line joining the two wires gives the direction of the line in the underground working. To continue the line, the instrument is removed and set up underground exactly in line with the two wires (Fig. 245). The line is then set, with the instrument on nails or dogs driven into convenient byats of timber, from which plumb-bobs or lamps may be suspended ; the exact point in the line is marked on the nails or dogs with a centre punch, or a file.

When tunnels are iron-lined, wooden wedges may be driven between the joints of the segments, and the line set on nails or staples driven into the wedges. The same method may be

used in brick lined tunnels, or cross byats may be temporarily
built in the brickwork for the purpose of carrying the mark.
Where the nature of the work will permit, the marks may be
set on brass nails in the heads of stout stakes driven into the
formation, or the invert of the tunnel. The stakes should be
surrounded with brickwork parged over with cement flush with
the top of the peg. The marking of the point on the head of
the nail should not be done until the cement has set. Under-
ground marks of this description are very convenient, as the
instrument can be more quickly set over them than placed in
line with two plumb lines.
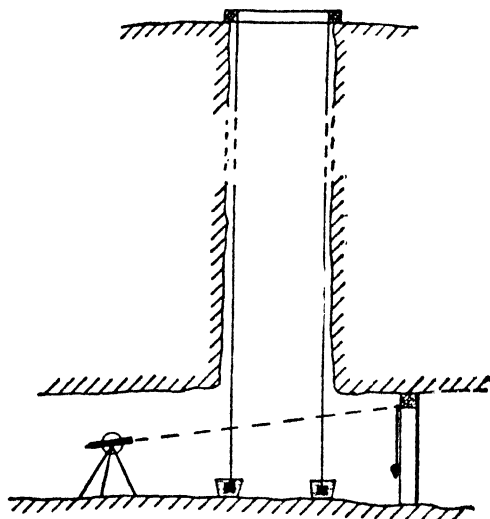
Fig. 245.

The operation of setting-up the instrument exactly in line
with the plumb lines requires great nicety in manipulation;
the suspended wires are rarely quite free from vibration, and
the observer is working under bad conditions in semi-darkness.
The instrument should be provided with a movable substage,
as described on p. 229, this device being especially useful in
underground work; and some arrangement (*vide* p. 237) must
be provided for illuminating the cross wires.

Illumination of Plumb Lines.—The suspended wires may be
illuminated from behind by a hand lamp. If the light from the

lamp passes through a piece of ground glass or a sheet of oiled paper, the wire behind which it is placed will be seen as a black mark on a white ground, and will be more clearly visible. The lamps should be placed at slightly different levels to reduce the difficulty of distinguishing between the suspended wires. This difficulty may be obviated by using the arrangement devised by Mr. E. W. Watkins, and used by him on the Croton Aqueduct, New York. In this arrangement, two vertical sharp-edged pieces of sheet brass are fitted in guides and connected by a screw, so that the vertical slit between their edges may be made broader or narrower, as required. One of these instruments is adjusted in position behind each wire, so that the centre of the wire coincides exactly with the centre of the slit, the two instruments being placed at slightly different levels and screwed to plank brackets. When the instruments have been adjusted to position the wires are removed, and a lamp is placed behind each slit. The wires are thus replaced by two illuminated slits, which can be readily observed to.

Procedure when Vertical Wires Cannot be Used.—The vibration caused by pumping machinery, in or near the shaft, may prevent the use of vertical wires in transferring the line underground. In such a case, the method employed on the Severn Tunnel may be found useful. The underground heading was driven a short distance approximately in the right direction, and two frames were fixed in the heading, 100 yards apart. The frames were fitted with horizontal **V**-threaded screws and a wire, heavily weighted at the ends, was placed over the screws resting on the screw threads. About 14 feet of the wire was visible through the telescope of a large transit theodolite erected over the mouth of the shaft; by rotating the underground screws the extremities of the visible portions of the wire were brought exactly into the line given by the instrument, thus giving an underground base of 100 yards, which could be continued as required. During observation the wire was illuminated by electric light.

Apparatus for Suspending the Vertical Wires.—The suspended wires cannot be left in position, as their presence would interfere with the progress of the mining operations. Some handy arrangement must, therefore, be made use of whereby the wires may be quickly placed in position, or removed as required. A convenient apparatus for this purpose was used in the alignment of the Totley Tunnel, in which the wire was wound on a horizontal drum turned by a winch handle, the backwards rotation

of the drum being prevented by a pawl and ratchet wheel. The
drum was placed at a short distance from the shaft; two drums
and wires were, of course, required at each shaft. The wires
passed from the drums over timber baulks spanning the edges
of the shaft, the points of suspension being formed on the nuts
of horizontal screws which were supported in a frame fixed to
the inside vertical faces of the baulks; by rotating the screws,
the nuts, and with them the suspended wires, could be brought
quickly into the desired position.

A handy apparatus for supporting the wire is shown in Fig. 246.
It consists of a cast-iron frame F fitted with a screw S, which is
rotated by the winch handle H. The nut N is prolonged below
the screw and slides on the round stay A, below which is formed
the clamp C for holding the wire. The apparatus is fixed by the
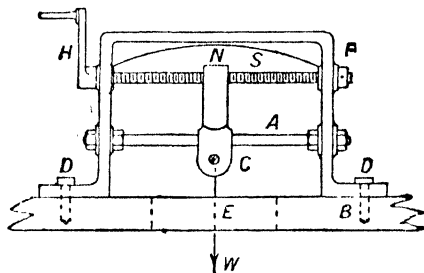
Fig. 246.

screws D to the baulk spanning the edge of the shaft, and through
the baulk B an oval slot E is cut to allow for the lateral move-
ment of the wire. The suspended wire is sighted with the theo-
dolite and the screw is rotated until the wire is accurately
bisected by the cross wires. The apparatus is easily removed
or placed in position by extracting or inserting the screws D.
If the wire is carried on a drum, a small grooved pulley may be
fixed in the clamp, the wire passing from the drum over the
pulley, and down the shaft.

When raising or lowering the wire in the shaft, a light weight
should be substituted for the heavy stretching weight perchance
the wire should break, and thereby cause an accident.

Underground Sights.—For sighting underground, various
forms of illuminated signal are made use of. For short sights,
a plumb line seen against the white background formed by a
sheet of oiled paper illuminated from behind by a lamp is the

best. For longer sights, ordinary carriage candles are commonly used ; and for still longer sights, argand oil lamps of from 40 to 50 candle-power have been used. Both candles and lamps are supported in suitable metal frames, which are adjusted in position until the axis of the flame is vertically under the point of suspension. A convenient form of target lamp is shown in Fig. 247. For sighting to stations marked on the floor of the tunnel, a plumb line illuminated from behind, the vertical illuminated slit of Mr. Watkins, or the plummet lamp shown in Fig. 248, may be used.
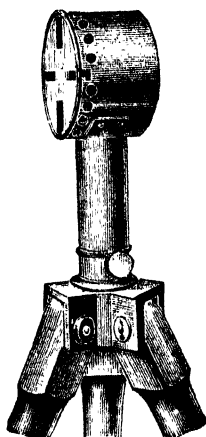
Fig. 247.

Fig. 248.

The Levels.—Wherever possible, the longitudinal section along the whole course of the surface alignment is obtained, and bench marks are established near each shaft and at the ends of the tunnel. Before proceeding to carry the levels underground, the levels of all the bench marks must be carefully checked, as an error in level may be quite as serious in its effects as an error of alignment.

Transferring the Levels Underground.—At the ends of the tunnel no difficulty is met with, the levels are transferred underground by levelling from the nearest bench mark, in the usual way.

Various apparatus, such as steel bands, chains, and specially constructed rods, have been used for transferring the levels down vertical shafts.

Where wooden guides are fitted in which the cage travels, the level of a mark near the top of one of the guides is obtained from the nearest bench mark. An assistant then holds the end of the steel tape to the mark, and the engineer is lowered down the shaft and marks the position of the lower end of the tape, on the guide. The assistant is then lowered down the shaft and places his end of the tape to the mark made by the engineer, who proceeds to make a new mark at a still lower level, these operations being repeated until the bottom of the shaft is reached. Temporary platforms, at distances of 66 or 100 feet apart, must be placed across the shaft for the use of the engineer and his assistant holding the tape, or they may be carried on seats fixed to the winding rope. If wire rope guides are used, the measurements may be made on the sides of the shaft, in which case care must be exercised to see that the tape hangs quite vertical at each stage of the process.

When a chain is used for the vertical measurement, it must be first carefully tested under a tension equal to its own weight. It is then lowered down the shaft and suspended by the upper handle from a nail, the level of the upper surface of which is known, or is subsequently determined. The chain being in free suspension from the nail, a second nail is driven within the lower handle and just touching it. The chain is then lowered down the shaft and suspended from the lower nail, the process being repeated as often as necessary. Round nails of uniform diameter should be employed as a distance equal to the diameter (d) of the nail will be lost at each chain length measured. The actual distance between the upper surfaces of the highest and lowest nails, N whole chains apart, will be given by the equation, $D = A - N (d + 2 t)$, where A is the apparent distance, and t the thickness of the handle of the chain.

Borcher's measuring rods furnish the most accurate means of measuring a vertical distance. These are round steel rods from 0·16 to 0·24 inch diameter, and from 1 to 4 yards long. The ends of the rods are screwed and may be connected by double brass nuts until a measuring rod of the desired length is obtained. The end surfaces of the rods must be planes at right angles to their longitudinal axes, the brass connecting nuts are cut away at the opposite ends of a diameter, so that the contact, between the ends of each pair of rods, may be seen. The upper rod is provided with a hook by which the whole may be suspended from a nail, and the measurement is counted from the inner surface of the hook.

For deep shafts, the vertical may be changed into a horizontal measurement by the arrangement shown in Fig. 249. A steel wire loaded with a weight of 10 to 30 lbs. is passed over a pulley, at the top of the shaft, from a windlass, and lowered down the shaft. At the top and bottom of the shaft horizontal threads C D, E F are stretched in contact with the wire; the points of contact are marked on the wire either by tying threads around it, or by chalking the wire and marking the exact point with a pencil. The wire—still loaded with the stretching weight —is wound up, and the distance between the marks on the
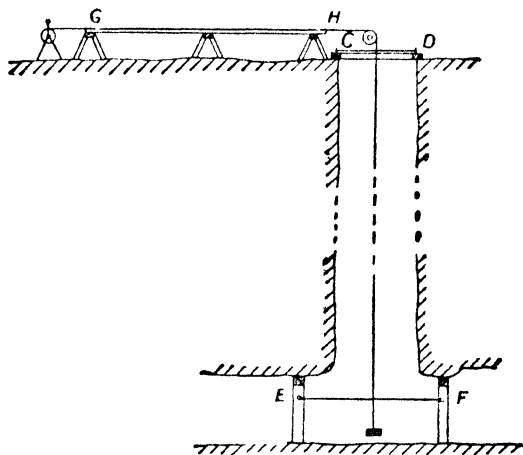
Fig. 249.

wire is obtained as it passes over the surface of a horizontal plank G H, which is suitably supported on trestles.

The elongation of the wire due to the attached weights does not interfere with the accuracy of the measurement, as the wire is under a nearly constant tension during the whole operation.

The method is convenient and rapid, and with ordinary care very accurate results are obtained.

Underground Bench Marks.—The level of an underground point having been obtained by the vertical measurements, the staff is set to the point, the spirit level is set-up near the bottom of the shaft, and the level is then set on a permanent bench mark.

In the early stages of the mining operations, the bench mark

may be a point on the top of a stake driven firmly into the ground near the bottom of the shaft. The position of the bench mark should be chosen so that it will not be disturbed during the progress of the work, and its level should be checked from time to time to guard against any accidental disturbance.

If the tunnel is in rock, a flat projecting portion of the rock face may be used as the bench mark, the exact point being denoted by a suitable mark cut into it.

When the lining of the tunnel has progressed for some distance, the bench marks may be established on iron spikes driven into the joints of the brickwork ; if the tunnel is iron-lined, the flanges of the segments of the lining may be used for the same purpose.

As the work proceeds new bench marks are put in from time to time, so that one or more bench marks are conveniently placed near the working face, the levels being finally checked, after the headings from the two ends of the tunnel meet, by levelling through from end to end. As the headings are kept well in front of the lining, any error that may exist in the levels is allowed for by putting in a " junction gradient."

Setting Profiles and Ribs.—The brickwork in brick-lined tunnels is set to profiles, and it is the duty of the engineer to place them accurately in position, both as regards level and alignment.

In setting the profiles to the correct level, the engineer works from the nearest bench mark, and directs his assistants to raise or lower the profiles as required. If the profiles are being set to continue the lining, a check is obtained by taking level readings on corresponding parts of the brickwork to be joined up to the new length. In a new " break-up "—*i.e.*, where the heading is being enlarged to the full section of the tunnel, greater care is necessary, as no check on existing brickwork is possible.

The ribs of the centering of the overhead arch are supported on wedges, by means of which they are raised or lowered as required, and in setting the ribs to the desired level, both springings should be levelled, and also the crown of the rib.

Each profile or rib is set to line by right- and left-hand measurements with a steel tape from a point in the centre line opposite each profile or rib, the points on the centre line are obtained either with the theodolite or from plumb lines suspended from a cord roved through staples which are exactly in the line. After each adjustment of a profile or a rib for line, it must be tested for level, and *vice versâ*.

In iron-lined tunnels the segments of each ring are bolted to

those of the ring, which immediately precedes it by bolts passing through the flanges of the segments ; any adjustment that may be necessary is made by driving wooden packings between the flanges of the segments, the packing being inserted at the top or bottom of the ring to allow for changes in level and at the sides for changes in direction.

To test the levels, readings are taken at the crown and invert of the last rings. The mean of the readings gives the level of the centre line at the selected cross-section, and if this is too high or too low the new ring is packed accordingly.

When dealing with curves the packing is inserted on the right or left to throw the forward end of the ring in the desired direction. If the curve is very sharp special rings, cast to allow for the curvature, are employed.

The last ring is tested for line by stretching a cord through two bolt holes at the same height and near the top of the ring ; from the cord a plumb-bob is suspended, and the point of suspension is adjusted until the plumb line is in the centre line of the tunnel. Right- and left-hand measurements from the plumb line on a horizontal diameter will then give the desired check on the alignment of the ring.

Errors in Alignment and Level of Constructed Tunnels.—The lengths of some of the principal tunnels, with their errors at junction both in alignment and level, are given in the following table :—*

Tunnel.	Length.	Error at Junction.	
		In Alignment.	In Level.
	Feet.	Inches.	Inches.
St. Gothard, ...	48,872	12·99	1·97
Mont Cenis, ...	40,081	Nil.	12·00
Hoosac, Massachusetts, .	25,031	0·03	0·23
Ernst-August adit, Hartz (Division 2), ...	23,760	1·20	0·09
Totley, ...	18,687	4·50	2 25
Cowburn, ...	11,106	1·00	··
Croton aqueduct, New York (Division 1), . . .	6,400	0·09	0·01
Nepean, New South Wales (Division 2), . . .	4,341	0·42	0·25

Meaning of the Terms " Outcrop," " Strike," and " Dip."—It is a great advantage to an engineer engaged in underground work to have a prior knowledge, not only of the geological nature of the strata the work will pass through, but of such data as will enable him to predict the point at which any particular stratum will be met with in his underground operations. This is particularly the case when the underground works must pass through strata heavily charged with water, since, by knowing where this will be met with, the engineer may take precautions beforehand for dealing with it when liberated.

In some cases the layers of rock overlying the site are exposed at the surface by the denudation of the overlying material ; a rock face exposed in this way is called an " outcrop " or basset." A study of the outcrops over the site will often enable the engineer to determine the nature of the strata his underground operations will penetrate. Where the overlying rocks are completely covered by alluvial deposits, bore holes must be made in order to obtain the requisite information. If the overlying strata is of tabular or plane formation, the data obtained from the outcrop or from bore holes may be relied on ; but if the strata is much crumpled, the data obtained in this way may be quite misleading. In the following investigations we assume that the strata is of tabular structure.

To fix the position of any plane, it is sufficient to fix the direction of two lines lying in the plane and having a minimum and maximum inclination respectively. As applied to a stratum, the former line is necessarily horizontal, and its direction is termed the " strike " of the stratum. The latter is the line of greatest slope, and its inclination to the horizontal is termed the " dip " of the stratum. Obviously the two lines are contained by vertical planes which are at right angles to each other.

If a sufficient layer of rock is exposed at the outcrop, the direction of the strike may be found by finding the direction in which a long, straight rod lies when resting on the layer in a horizontal position. A cord stretched parallel to the layer may be used for the same purpose. The dip is found by determining the direction and inclination of the line of greatest slope by means of a clinometer.

The directions of the dip and strike, and of all lines lying on a stratum, are fixed by taking their bearings.

.If the strike of a stratum, and the bearing and inclination of any line lying upon it, are known, the dip may be calculated. Thus, let A B (Fig. 250) represent the strike of the stratum

A B C D, and A G the direction of any line lying on the plane
A B C D. Also, let B F be a horizontal plane meeting the plane
A B C D in the line A B. From any point G in A G drop a
perpendicular meeting the plane B F in H, and from H drop
a perpendicular meeting A B in K. Join K G and H A. Then
G K is the line of dip, and the angle H K G is the angle of dip;
also, H K and H A are the projections of K G and G A on the
plane B F, and the inclination of G A
is equal to the angle H A G = b.
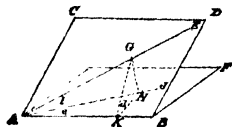
The angle H A K (or e) is the differ-
ence of the bearings of A B and A G,
and the bearing of K G will be equal to
the bearing of A B $\pm$ 90°. Let the angle
of dip be d, then from the triangle G A H
we have H G = A G . sin b; and H A

Fig. 250.

= A G cos b. From the triangle H A K, since the angle is
H L A a right angle,

$$\text{H K} = \text{H A . sin } e$$
$$= \text{A G . cos } b \text{ . sin } e.$$

Then in the triangle H K G,

$$\tan d = \frac{\text{H G}}{\text{H K}}$$

$$= \frac{\text{A G . sin } b}{\text{A G . cos } b \text{ . sin } e}$$

$$= \tan b \text{ . cosec } e.$$

If the angle of dip be known, then the direction of the strike
relative to A G is given by the above equation, thus :—

$$\sin e = \tan b \text{ . cot } d.$$

Example 2.—The bearing of the strike of a certain seam is
45° 30′, and that of a diagonal heading driven in the seam is
75° 46′. If the inclination of the heading is + 6° 20′, find the
dip of the seam.

Here, $e = 75° 46′ - 45° 30′$

                $= 30° 16′,$

and $b = 6° 20′.$

∴ $\tan d = \tan 6° 20′ \text{ . cosec } 30° 16′,$

and $d = 12° 25′.$

The bearing of the line of dip is equal to 45° 30′ + 90° or 135° 30′

Example 3.—A diagonal heading in a seam dips at an angle of 4° 35′, and its bearing is 330° 45′. If the dip of the seam is 20° 45′, find the bearing of the strike.

Here, $\sin e = \tan 4° 35′$. $\cot 20° 45′$, from which
                            $e = 12° 13′$.

Required bearing $= 330° 45′ - 12° 13′$
                          $= 318° 32′$.

Dip and Strike from Three Bore Holes.—The dip and strike of a stratum may be determined if any three points on it are given. The requisite data may be obtained from any three bore holes (not in the same straight line) by measuring the depths at which the stratum is met with, and fixing the positions of the bore holes by bearings and distances in the usual way. The levels of the three points on the stratum are reduced to the same datum from the known levels of the ground at the bore holes. It is convenient to take the datum plane coincident with that of the lowest point. From this data the problem may be solved either graphically or by calculation.

Graphical Method.—Let A, B, and C (Fig. 251) be the plans of the three bore holes, of which A and B are the highest and lowest points, and let the bearings of A B and A C be a and β respectively ; also let the levels of A and C with reference to the datum plane through B be p and q respectively. At A erect a perpendicular A D, p units long. Join D B. Then the angle A B D is the inclination of the line A B lying on the stratum. At A and C erect perpendiculars A E, C F, p and q units long respectively. Join E F and produce the line to meet A C produced in the point G. Then the angle E G A is the inclination of the line A G lying on the stratum ; also, the point G is in the datum plane and G B is the direction of the strike. A H, or any line, drawn at right angles to G B, is the direction of the line of dip. To determine the dip angle, set-out A I at right angles to A H, and make it p units long ; join I H, then the angle A H I is the required angle of dip (θ).

Computation of Dip and Strike.—To determine the distance A. G. (Fig. 251), we have from the similar triangles G E A and G F C,

$$G A : G C :: p : q,$$

or $$G A : G A - C A :: p : q.$$

from this $$G A = \frac{C A \cdot p}{p - q}$$

In the triangle G A B, the sides A G and A B, and the angle G A B are known. To determine the bearing of the strike, it is necessary to find either the angle A G B or the angle A B G. Let the angle A G B = G, and the angle A B G = B, then

$$\frac{A G - A B}{A G + A B} = \frac{\tan \frac{1}{2} (B - G)}{\tan \frac{1}{2} (B + G)},$$

hence $\tan \frac{1}{2} (B - G) = \dfrac{A G - A B}{A G + A B} \tan \frac{1}{2}(180 - \overline{\beta - \alpha})$
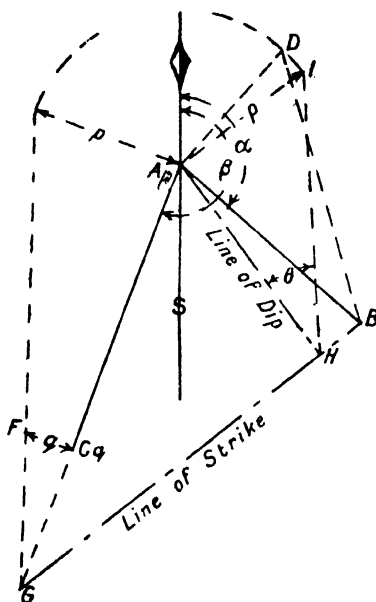
Fig. 251.

This gives $\frac{1}{2}$(B — G), and since $\frac{1}{2}$(B + G) = $\frac{1}{2}$(180 — $\overline{\beta - \alpha}$), both the angles B and G are known. The bearing of the strike = bearing of G A + G, or = bearing of B A — B.

The bearing of the dip line = bearing of strike ± 90°.

To determine the angle of dip, we have, from the triangle A H B,

$$A H = A B . \sin B.$$

Again
$$\tan \theta = \frac{A\,I}{A\,H}$$

$$= \frac{p}{A\,B \cdot \sin B}$$

The strike of the stratum can be set out on the surface of the ground, from the computed bearing, in the usual way.

Example 4.—From the following data determine the bearing of the strike and the bearing and inclination of the dip line :—Bearing of A B = 60° 30′ ; bearing of B C = 135° 15′ ; A B = 500 yards ; B C = 650 yards ; depths of A, B, and C below a common datum 30, 54, and 38 yards respectively.

Here, $p = 54 - 30 = 24$ yards,

   $q = 38 - 30 = 8$ yards,

and $B\,G = \dfrac{650 \times 24}{24 - 8}$

   $= 975$ yards.

Again, the angle A B G $= 60° 30′ + 180° - 135° 15′$

   $= 105° 15′,$

and $A + G = 180° - 105° 15′ = 74° 45′,$

∴ $\tfrac{1}{2}(A + G) = 37° 22′ 30″.$

Again, $\dfrac{\tan \frac{1}{2}(A - G)}{\tan 37° 22′ 30″} = \dfrac{975 - 500}{975 + 500},$

or $\tan \tfrac{1}{2}(A - G) = \dfrac{475}{1,475} \tan 37° 22′ 30″.$

From this $\tfrac{1}{2}(A - G) = 13° 49′$

and $(A - G) = 27° 38′$

   $A + G = 74° 45′$

   _____

∴ $2A = 102° 23′$

and $A = 51° 11′ 30″.$

Similarly, $G = 23° 33′ 30″.$

Bearing of strike from
   $A = 60° 30′ + 51° 11′ 30″$
   $= 111° 41′ 30″.$

Bearing of dip line at
$$A = 111° 41' 30'' - 90°$$
$$= 21° 41' 30''.$$

Let the line of dip through A cut the strike through B at H, then
$$A H = 500 . \sin 51° 11' 30''$$
$$= 389·5 \text{ yards.} \quad \text{If } \theta \text{ is the dip angle, then}$$
$$\tan \theta = \frac{24}{389·5}$$
and $$\theta = 3° 31' 30''.$$

Point of Intersection of Centre Line of a Tunnel and a Stratum.

—The problem of determining the point of intersection of the centre line of a tunnel and a stratum of rock fixed by three bore holes is easily solved graphically, but the solution by calculation is somewhat tedious.

Since the assumption that the stratum is a plane surface may or may not be true in a particular case, the graphical mode of solution will determine the desired result with a sufficient degree of precision in most cases occurring in practice.

The positions of the bore holes should be selected so that the point sought falls within the triangle formed by the lines joining them. This not only renders the construction more convenient, but tends to greater accuracy by limiting the extent of the stratum dealt with to that defined by the bore holes.

The method of solving this problem, both graphically and by computation, is illustrated by the following example :—

Example 5.—A, B, and C are three holes drilled to a stratum of rock, K L is the centre line of a tunnel constructed on a rising gradient of 1 in 100. The bearings, levels, and distances are as follows :—

Line.	Bearing.	Distance. Yards.	Reduced Levels. Yards.
K A	58°	730	K. 300·5
K L	79°	. .	A. 390·8
K B	92°	600	B. 298·3
A C	120°	500	C. 230·4

Determine the distance from K to the point at which the centre line meets the plane of the stratum.

Graphical Solution.—Prepare a plan of the points K, A, B, and C, from the given bearings and distances. The plotting of the plan may be done either from the latitudes and departures of the points, or by protracted bearings and distances. Lay down the direction of K L from the given bearing, let this line meet the lines A B and A C (Fig. 252) in the points E and F respectively. Scale off the distance K F. It is convenient to reduce the levels of the points K, A, B, and C to the datum plane passing through the point B. On doing this, the reduced levels of the points become 2·2, 92·5, 0·0, and — 67·9 yards.
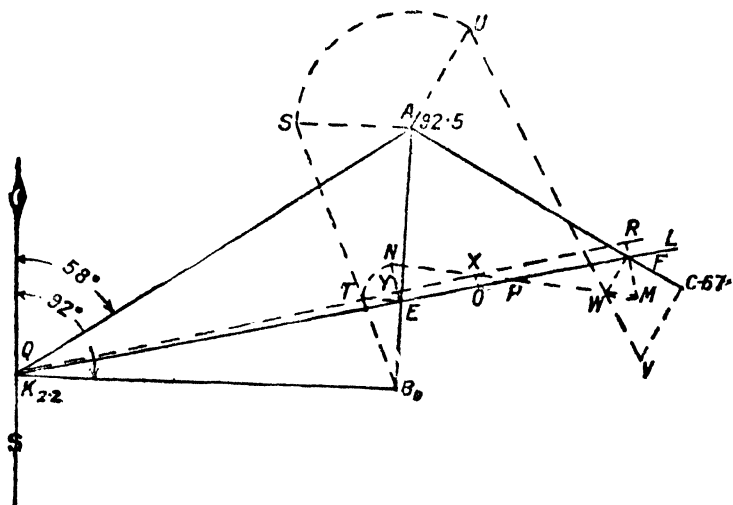
Fig. 252.

At K and F erect K Q and F R perpendicular to K F, make K Q = 2·2 yards, and F R = $\frac{K F}{100}$ + 2·2 yards. Join Q R. At A and E erect perpendiculars to the line A B; make the perpendicular A S = 92·5 yards. Join S B, and let this line cut the perpendicular at E in the point T. Then E T is the height of E above B. At A and C erect perpendiculars A U = 92·5 yards, and C V = — 67·9 yards. Join U V, and let this line meet the perpendicular at F in the point W. Then F W is the depth of F below B. Next, erect E N and F M perpendicular to E F, making E N = E T, and — F M = — F W. Join N M,

and let the line meet Q R at X, then X is the elevation of the
point sought; from X drop a perpendicular X O on the line
K F, and meeting it in the point O. The point O is the plan
of the required point of intersection, and K O is its horizontal
distance from K, measured on the centre line of the tunnel.

The student conversant with the methods of descriptive
geometry should have no difficulty in following, and under-
standing, the above construction.

By Calculation.—(*a*) To determine the angles B and A in the
triangle K A B.

$$\text{The angle A K B} = 92° - 58°$$
$$= 34°.$$

$$\text{K A} = 730 \text{ yards}; \quad \text{K B} = 600 \text{ yards.}$$

Hence, $\tan \frac{1}{2} (B - A) = \dfrac{730 - 600}{730 + 600} \cdot \cot \dfrac{34°}{2}$,

and $\qquad \frac{1}{2} (B - A) = 17° 44'$

∴ $\qquad (B - A) = 35° 28'$

$\qquad (B + A) = 146° 00' = 180° - 34°$,

By addition, $\qquad 2 B = 181° 28'$

and $\qquad B = 90° 44'.$ Similarly we get

$\qquad A = 55° 16'.$

(*b*) Determine A B from the triangle K A B.

$$A B = \frac{730 \cdot \sin 34°}{\sin 90° 44'}$$
$$= 408 \cdot 3 \text{ yards.}$$

(*c*) Determine A E from the triangle K A E.

$$\text{Angle A K E} = 79° - 58°$$
$$= 21°.$$

$$\text{The angle K E A} = 180° - (55° 16' + 21°)$$
$$= 103° 44'.$$

Also, $\qquad A E = \dfrac{730 \cdot \sin 21°}{\sin 103° 44'}$

$$= 269 \cdot 3 \text{ yards,}$$

and $\qquad B E = 408 \cdot 3 - 269 \cdot 3$

$$= 139 \text{ yards.}$$

(d) Find A F and K F from the triangle K A F.

First, find the angle K F A.

Reduced bearing of B A = $92° - (180° - 90° 44')$
$$= 2° 44'$$

Bearing of A B = 182° 44'
Bearing of A F = 120° 00'

Angle B A F = 62° 44'
Angle K A E = 55° 16'

Angle K A F = 118° 00'

Angle K F A = $180 - (118 + 21)$,
$$= 41°.$$

$$A F = \frac{730 \cdot \sin 21°}{\sin 41°}$$
$$= 398 \cdot 7 \text{ yards.}$$

$$K F = \frac{730 \cdot \sin 118°}{\sin 41°}$$
$$= 982 \cdot 4 \text{ yards.}$$

(e) Find K E from the triangle A K E.

$$K E = \frac{730 \cdot \sin 55° 16'}{\sin 76° 16'}$$
$$= 617 \cdot 5 \text{ yards.}$$

(f) Find level of E on A B with respect to B.

$$\frac{\text{Level of E}}{390 \cdot 8 - 298 \cdot 3} = \frac{B E}{B A}$$

$$\text{Level of E} = \frac{92 \cdot 5 \times 139}{408 \cdot 3}$$
$$= 31 \cdot 49 \text{ yards.}$$

Reduced level of E = $31 \cdot 49 + 298 \cdot 3$
$$= 329 \cdot 72 \text{ yards.}$$

Similarly, the level of F in A C, relative to C

$$= \frac{(390{\cdot}8 - 230{\cdot}4)\,(500 - 398{\cdot}7)}{500}$$

$$= 32{\cdot}48 \text{ yards.}$$

Reduced level of F $= 230{\cdot}4 + 32{\cdot}48,$

$$= 262{\cdot}88 \text{ yards.}$$

(*g*) Determine the level of E in K F relative to K.

Here, required level $= \dfrac{K\,E}{100},$

$$= \frac{617{\cdot}5}{100},$$

$$= 6{\cdot}175 \text{ yards.}$$

Reduced level of E in

$$K\,F = 300{\cdot}5 + 6{\cdot}175$$

$$= 306{\cdot}675 \text{ yards.}$$

The levels of the points K, E, and F reduced to the datum at 298·3 are :—K = 2·2 ; E on K F = 8·37 ; E on A B = 31·49 ; and F on A C = — 35·42 yards.

(*h*) Find the point P on E F at level 298·3.

Here, $\dfrac{E\,P}{E\,N} = \dfrac{E\,F}{E\,N + F\,M},$

and $E\,P = \dfrac{31{\cdot}49\,(982{\cdot}4 - 617{\cdot}5)}{31{\cdot}49 + 35{\cdot}42}$

$$= 171{\cdot}7 \text{ yards.}$$

(*i*) To find K O. From the similar triangles K E Y and K O X we have

$$\frac{K\,O}{K\,E} = \frac{X\,O}{E\,Y}$$

∴ $X\,O = \dfrac{K\,O \times 6{\cdot}175}{617{\cdot}5} = \dfrac{K\,O}{100}.$ . . (1)

Also, from the similar triangles P N E and P X O we get

$$X O = \frac{O P . E N}{E P} = \frac{(K P - K O) . E N}{E P}$$

$$= \frac{(K E + E P - K O) E N}{E P}$$

$$= \frac{(617 \cdot 5 + 171 \cdot 7 - K O) . 31 \cdot 49}{171 \cdot 7}. \quad (2)$$

Equating (1) and (2), thus

$$\frac{K O}{100} = \frac{(617 \cdot 5 + 171 \cdot 7 - K O) . 31 \cdot 49}{171 \cdot 7}.$$

Transposing and reducing, we get K O = 748·4 yards, the required distance of the point of intersection from K.

EXAMPLES.

1. What are the chief points to be taken into consideration when deciding whether to construct a tunnel, a cutting, or to make a detour, in order to connect two points on a proposed line of communication ?

2. Describe how you would carry out the surface alignment of a tunnel, (a) when the tunnel is straight, and (b) when partly straight and partly curved.

3. Explain why it is so important to determine accurately the length of a proposed tunnel. The actual distance between the extremities of a tunnel is 5,002 yards, and the distance as computed 5,000 yards. If the difference of level of the two ends of the tunnel is 200 yards, and it is driven from the two ends, find the error in level at the junction of the two headings.

(*Ans.* 2·88 inches.)

4. If in Question 3 the difference of level of the two ends of the tunnel be 50 yards, find the error in level at the junction.

(*Ans.* 0·72 inch.)

5. Describe by aid of sketches two methods of determining the distance between the two ends of a tunnel when the surface of the ground is such as to render direct measurement impossible.

6. How are the surface and underground lines connected in tunnelling ? How would you set out the centre line of a tunnel from a shaft which is (say) 100 yards from the centre line ?

7. If an error of 0·01 inch is made in bisecting the vertical

wires at the bottom of a shaft, and the wires are 3 yards apart, what would be the probable error at the junction 1,000 yards from the shaft, if the other heading is run true to line ?

(*Ans.* 3⅛ inches.)

8. A surface line A B is cut by a block of buildings, around which a traverse B C D E is placed, the point E being in A B produced. Given that B C = 840 feet, C D = 1,250 feet, $\angle$ E B C = 63° 30', $\angle$ B C D = 108° 25', and $\angle$ C D E = 126° 45', find the distance from D to E and the gap in the surface line. The point B not being visible from E, how would you continue the line A B with a theodolite ?   (*Ans.* 656·6 feet ; 1,927·7 feet.)

9. Sketch a suitable apparatus for suspending the wires down a shaft, and explain clearly how you would set the wires in true alignment with the surface line.

10. Describe two ways of transferring the levels underground, down a vertical shaft.

11. In transferring the levels down a vertical shaft, the point of suspension of the (100-feet) measuring chain was set on one of the cage guides at a reduced level of 540·32 feet. The apparent depth to a convenient point at the lower end of the guide was 375 feet, as measured with the chain suspended on nails ⅛ inch diameter. The levelling staff was set to this mark and the reading 2·41 was obtained, while the reading on the bench mark was 4·62. The thickness of the handle of the chain is ½ inch, and the chain was known to be ⅛ inch too short, find the reduced level of the underground bench mark.   (*Ans.* 163·39 feet.)

12. How would you test the centre line of the last ring in an iron-lined tunnel for both alignment and level ?

13. What is meant by " dip " and " strike " of a stratum? The strike of a certain seam of ore is observed to be N. 55° W., and the vein dips N. 35° E. at an angle of 14° with the vertical. From a point on the outcrop the distance of the mouth of a tunnel is 250 feet (measured on the horizontal), the bearing being S. 35° W. and the slope of the ground − 20°. Find the length of a horizontal tunnel from the mouth to the vein.

(*Ans.* 272·7 feet.)

14. The bearing of a diagonal heading driven on a seam is 120° 30', and its inclination 5° 15'. If the bearing of the strike is 40° 35', find the dip of the seam and the bearing of the dip line.

(*Ans.* 5° 20' ; 130° 35'.)

15. The dip of a certain seam is 20° 45'. A diagonal heading in the seam has a dip of 10° 15' and courses 224° 30'. Find the bearing of the strike.   (*Ans.* 196°.)

16. In locating a seam, three bore holes A, B, and C were driven from the surface of the ground, and they cut the seam at depths of 100, 150, and 175 yards, respectively, below the datum point. A B measured 400 yards, A C 500 yards, and the bearings of A B and A C were N. 30° 30′ E., and S. 40° 30′ E. Determine either graphically or by calculation the angle of dip of the seam, and the bearings of the dip and strike. (*Victoria University, B.Sc. Tech.*, 1912.)

(*Ans.* 13° 21′ ; N. 88° 43′ E ; N. 1° 17′ W.)

17. The centre line (D E) of a tunnel, on a rising gradient of 1 in 150, meets a stratum of rock in the point E, and three bore holes, A, B, and C, have been drilled from the surface of the ground to meet the stratum. Determine the position of E from the following data :—

Line.	Bearing.	Distance.	Reduced Levels.
		Feet.	Feet.
A B	N. 45° 34′ W.	895	A 154·4
A C	S. 75° 15′ W.	1,024	B. 263·8
D E	S. 76° 45′ W.	..	C. 294·5
B D	90° 00′ E.	1,850	D. 250·7

(*Ans.* D E = 1,986 feet.)

18. In a certain mine the lode has a true bearing of 314° 16′, and is worked from two levels 150 feet, vertically, apart. The levels are connected to a vertical shaft by cross-cuts, and both cross-cuts and levels rise from the shaft at a uniform rate of 6 inches in 100 feet. It is proposed to connect the lower (No. 2) level to the upper (No. 1) level by a rise in a direction at right angles to the course of the lode. The traverse from a point A in No. 2 level, the starting point on the rise, to any point E in No. 1 level, is as follows :—

Line.	True Bearing.	Distance.	Lat. N.	Lat. S.	Dep. E.	Dep. W.
		Feet.				
A B	147° 12′	353·6		297·2	191·6	
B C	226° 20′	896·4		618·9		648·4
C′ D	36° 45′	538·7	431·6		322·3	
D E	314° 26′	261·3	182·9			186·6

Station C′ is vertically over station C in the shaft. Determine (1) the point to start sinking in No. 1 level ; (2) the bearing of the rise ; (3) the angle of elevation of the rise ; and (4) the length of the rise from level to level.

(*Ans.* (1) 19·5 feet south of E. ; (2) A to E 224° 16′ ; (3) 18° 33′ ; (4) 464 feet.)

19. A straight footpath, 6 feet wide, is to be made on a piece of ground which rises at a gradient of 1 in 10 and which may be considered as plane. The footpath is to have a gradient of 1 in 20. Find (*a*) the gradient of the ground on perpendicular cross-sections and (*b*) the volume of earthwork to be handled per 100 feet, the formation on the centre line being in the undisturbed surface of the ground. Take side slopes 1½ to 1.

(*Victoria University, B.Sc.Tech.,* 1926).

(*Ans.* 1 in 11·53 ; 1·66 c. yds.)

20. A and B are two stations on a location traverse, their total co-ordinates being :—

Station.	Lat. (feet) N.	Dep. (feet) E.
A	43,850	4,850
B	38,465	3,430

A straight reach on a proposed railway runs from C roughly south of B to D (invisible at C) and roughly north of A, the offsets from A and B, perpendicular to C D, being 1,420 and 584 feet respectively. Calculate the bearing of C D and the distance from A of its point of intersection with A B.

(*Victoria University, B.Sc.Tech.,* 1924).

(*Ans.* N. 6° 18′ 25″ W. ; 3948·17 feet.)

CHAPTER XVI.

SPHERICAL TRIGONOMETRY—ASTRONOMICAL TERMS.

No attempt is here made to deal fully with the subject of spherical trigonometry, sufficient only is attempted to render the matter dealing with the geodetic and astronomical work required by surveyors more intelligible.

Plane trigonometry deals with the properties of figures situated wholly in one plane—the plane of the paper on which they are drawn—spherical trigonometry, on the other hand, deals with the properties of figures drawn on the surface of a sphere, the sides of the figures being situated in three or more planes which pass through one point, thus enclosing a pyramidal space. If the base of the pyramid is a polygon, the enclosed pyramidal space may be divided into several pyramids, each bounded by three planes containing the common point. A triangular pyramid of this kind is called a *spherical triangle*.

The *sides* of a spherical triangle are the arcs of intersection of the plane faces of the pyramid and the surface of the sphere, and are portions of great circles of the sphere since the planes contain as a common point, the centre of the sphere. The sides are not represented by their lengths, but by the angles they subtend at the centre of the sphere. If the length of a side is given, the angle it subtends is easily found from the proportion —Given length of side : πR :: subtended angle : 180°, R being the radius of the sphere.

The angle between the sides of a spherical triangle is the angle between the planes containing the sides; to determine this angle in any particular case, we assume the plane to be cut by a third plane perpendicular to both planes, and the required angle is given by the angle between the lines if intersection of the cutting plane and the planes under consideration. Such an angle is obtained when we measure the angle between two intersecting lines on the earth's surface with a theodolite, the direction

of the cutting plane is horizontal, and consequently is perpendicular to the direction of the plumb line, which represents the direction of the line of intersection of the planes containing the two lines.

Fig. 253 represents (in orthographic projection) the plan and elevation of a spherical triangle A B C, whose corresponding sides are *a b c*. The side *c* is placed in the horizontal plane of projection, and consequently the edges O A and O B of the triangle are in that plane. The elevation is drawn on a vertical plane parallel to the edge O C, thus showing the true length and inclination of this edge.

To construct the plan, with centre O, and any convenient radius, strike the arc K B A F. From K, set off the angles K O B, B O A, A O F respectively equal to the angles subtended by the sides *a*, *c*, and *b*. Now, imagine the sectors B O K and A O F to be turned about O B and O A as hinge lines ; during rotation the points K and F will trace out the lines K C and F C, meeting at the point C. The edge O C is formed by the junction of O F and O K after rotation, while points such as *p* and *q* on the arcs K B and F A, take up the positions shown by *p'* and *q'*, the curves passing through all such points as *p'* and *q'* are the plans of the sides *a* and *b*.

It is evident that H K and F G are the real lengths of the lines H C and G C respectively. To determine the angles A and B, between the sides *b* and *c*, *a* and *c*, consider H C and G C to be the horizontal traces of vertical planes cutting the inclined faces of the pyramid ; these cutting planes are perpendicular to the faces of the pyramid meeting in the edges O B and O A, since they are respectively perpendicular to these edges ; also, these planes meet in the vertical line dropped from C to the horizontal plane. Hence, the sections made by the cutting planes will be represented by the right-angled triangles C E H and G C D, of which the bases are H C and G C, the heights C E and C D are equal, since they represent the height of C above the horizontal plane ; also, the hypotenuse H E = H K, and the hypotenuse G D = G F. The required angles A and B are evidently equal to the angles D G C and E H C.

To determine the angle C ; from C' the elevation of the angular point C, draw C' J' perpendicular to O' C'. The line C' J' is the elevation of a plane perpendicular to the edge O C, and consequently perpendicular to the planes of the sides *a* and *b* of the spherical triangle. Let the horizontal trace of this plane meet the horizontal traces of the planes O C A and O C B in the points

P and Q, join C P, C Q, and P Q. Now, imagine the triangle P C Q to be rotated about P Q, as shown in the figure, until
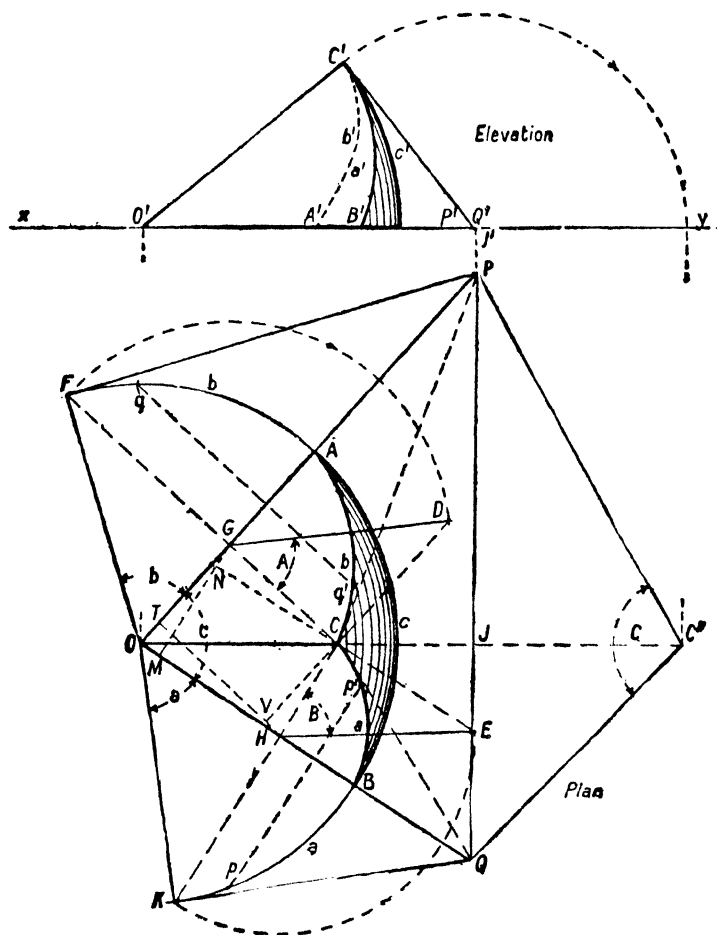
Fig. 253.

its plane is horizontal, then the required angle is the angle P C″ Q.

In the following investigations, we shall assume the truth of

the following :—In any spherical triangle having sides a, b, **and** c, and angles A, B, and C.

1. A + B + C is less than 3π and greater than π.
2. Any angle A is less than π, and any side a is less than π R.
3. $a + b + c$ is less than 2π R, and $a + b + c$ is less than 2π
4. $a + b$ is greater than c, and $a - b$ is less than c.
5. If $a + b = \pi$, then A + B = π.
6. If a is $>$ = or $<$ b, then will A be $>$ = or $<$ B.
7. " If a general equation be established between the sides and angles of a spherical triangle, a true result is obtained if in the equation the supplements of the sides and of the angles respectively be written for the angles and sides which enter into the equation."

Trigonometrical Relations between the Sides and Angles of a Spherical Triangle.

I. Given two angles and a side, or two sides and an angle, to find the other angle or side.

Let A and B be the given angles and a the given side, and let R be the radius of the sphere.

Referring to Fig. 253, we have

$$\text{Sin A} = \frac{C D}{G D} = \frac{C D}{G F} = \frac{C D}{R \cdot \sin b},$$

also,
$$C D = C E = H E \cdot \sin B$$
$$= H K \cdot \sin B$$
$$= R \cdot \sin a \cdot \sin B,$$

∴
$$\sin A = \frac{R \cdot \sin a \cdot \sin B}{R \cdot \sin b},$$

or
$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b}.$$

In the same way, by turning the pyramid over until the side a is in the horizontal plane, it may be shown that

$$\frac{\sin C}{\sin c} = \frac{\sin B}{\sin b};$$

therefore, we may write,

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}. \qquad \bullet \quad \bullet \quad \bullet \quad \bullet \quad (1.)$$

II. Given the three sides, to find the values of the **angles.**
Here we have to establish a formula of the form

$$\cos C = \frac{\cos c - \cos b \cdot \cos a}{\sin b \cdot \sin a}.$$

In triangles P C Q, P O Q (Fig. 253), we have

$$P Q^2 = C Q^2 + C P^2 - 2 C P \cdot C Q \cdot \cos C; \text{ also}$$

$$P Q^2 = O P^2 + O Q^2 - 2 \cdot O P \cdot O Q \cdot \cos c,$$

$$\therefore (O P^2 - C P^2) + (O Q^2 - C Q^2) - 2 \cdot O P \cdot O Q \cdot \cos c$$
$$= - 2 \cdot C P \cdot C Q \cdot \cos C,$$

or $2 \cdot O C^2 - 2 \cdot O P \cdot O Q \cos c = - 2 \cdot C P \cdot C Q \cdot \cos C;$

dividing out by $- 2 O P \cdot O Q$, we get :—

$$- \frac{O C}{O P} \cdot \frac{O C}{O Q} + \cos c = \frac{C P}{O P} \cdot \frac{C Q}{O Q} \cdot \cos C,$$

or $- \dfrac{O F}{O P} \cdot \dfrac{O K}{O Q} + \cos c = \dfrac{F P}{O P} \cdot \dfrac{Q K}{O Q} \cdot \cos C,$

that is, $- \cos b \cdot \cos a + \cos c = \sin b \cdot \sin a \cdot \cos C,$

or, $\text{Cos } C = \dfrac{\cos c - \cos a \cdot \cos b}{\sin a \cdot \sin b},$

Similarly, it may be shown that

$$\cos A = \frac{\cos a - \cos b \cdot \cos c}{\sin b \cdot \sin c},$$

and $\cos B = \dfrac{\cos b - \cos c \cdot \cos a}{\sin c \cdot \sin a}.$ $\Bigg\}$ **.** **.** **.** **.** **(II.)**

III. Given the three angles, to find the sides.
The formula to be established in this case is of the **form :—**

$$\cos c = \frac{\cos C + \cos A \cdot \cos B}{\sin A \cdot \sin B}.$$

From equation II., we have :—

$$\cos C = \frac{\cos c}{\sin a \cdot \sin b} - \cot a \cdot \cot b, \qquad \text{.} \quad \text{.} \quad \text{.} \quad \boldsymbol{(a)}$$

and
$$\frac{1}{\sin a} = \frac{R}{H\,K} = \frac{R}{H\,E} = \frac{R}{h}\,.\,\sin B,$$

where $h = $ the height of C above the plane of the face A O B (Fig. 253).

Similarly,
$$\frac{1}{\sin b} = \frac{R}{F\,G} = \frac{R}{h}\,.\,\sin A,$$

also,
$$\cot a = \frac{O\,H}{H\,K},\quad \cot b = \frac{O\,G}{G\,F};$$

substituting in equation (a), we get :—

$$\cos C = \cos c\,.\,\frac{R^2}{h^2}\sin A\,.\,\sin B - \frac{O\,H}{H\,K}\,.\,\frac{O\,G}{G\,F},$$

$$= \cos c\,.\,\left(\frac{R^2 - h^2}{h^2}\right)\sin A\,.\,\sin B + \cos c\,.\,\sin A\,.\,\sin B$$
$$- \cos A\,.\,\cos B\,.\,\frac{O\,H\,.\,O\,G}{C\,H\,.\,G\,C}$$

$$= \cos c\,.\,\sin A\,.\,\sin B - \cos A\,.\,\cos B$$
$$+ \cos c\left(\frac{R^2 - h^2}{h^2}\right)\sin A\,.\,\sin B$$
$$- \cos A\,.\,\cos B\left(\frac{O\,H}{C\,H}\,.\,\frac{O\,G}{G\,C} - 1\right).$$

Now $\cos c\left(\dfrac{R^2 - h^2}{h^2}\right)\sin A\,.\,\sin B - \cos A\,.\,\cos B\dfrac{O\,H}{C\,H}\,.\,\left(\dfrac{O\,G}{G\,C} - 1\right)$

$$= \cos c\left(\frac{G\,C^2 + O\,G^2}{h^2}\right)\frac{C\,D}{G\,D}\,.\,\frac{C\,E}{E\,H} - \frac{G\,C}{G\,D}\,.\,\frac{C\,H}{E\,H}\left(\frac{O\,H\,.\,O\,G - C\,H\,.\,G\,C}{C\,H\,.\,G\,C}\right),$$

$$= \frac{\cos c\,(G\,C^2 + O\,G^2) - O\,H\,.\,O\,G + C\,H\,.\,G\,C}{G\,D\,.\,E\,H}\,.\quad \cdot\quad \cdot\quad (\beta)$$

In Fig. 253, draw G M perpendicular to O H, C N perpendicular to G M ; then

$$\frac{O\,M}{O\,G} = \frac{G\,N}{G\,C} = \cos c,$$

$\therefore$ G C . $\cos c = $ G N, O G $\cos c = $ O M ; substituting in equation (β), we get :—

$$\frac{G\,C\,.\,G\,N + O\,G\,.\,O\,M - O\,H\,.\,O\,G + C\,H\,.\,G\,C}{G\,D\,.\,E\,H}$$

$$= \frac{O\,G\,(O\,M - O\,H) + G\,C\,(G\,N + C\,H)}{G\,D\,.\,E\,D}$$

$$= \frac{G\,C\,.\,G\,M - O\,G\,.\,M\,H}{G\,D\,.\,E\,D}$$

$$= \frac{G\,M\,.\,C\,N \cosec c - G\,M\,.\,M\,H\,.\, \cosec c}{G\,D\,.\,E\,D}$$

$$= \frac{G\,M\,.\, \cosec c\,(C\,N - M\,H)}{G\,D\,.\,E\,D}$$

$$= 0, \text{ since } C\,N = M\,H.$$

∴ $\cos C = \cos c \,.\, \sin A \,.\, \sin B - \cos A \,.\, \cos B,$

or, $\cos c = \dfrac{\cos C + \cos A \,.\, \cos B}{\sin A \,.\, \sin B},$

Similarly for the other angles, we have :—

$$\cos a = \frac{\cos A + \cos B \,.\, \cos C}{\sin B \,.\, \sin C}.$$

$$\cos b = \frac{\cos B + \cos C \,.\, \cos A}{\sin C \,.\, \sin A}.$$

.   .   (III.)

The above equations may be more easily derived from equation II. by substituting the values $(\pi - A)$ for a, $(\pi - B)$ for b, $(\pi - a)$ for A, etc., thus :—

$$\cos A = \frac{\cos a - \cos b \,.\, \cos c}{\sin b \,.\, \sin c}.$$

On making the above substitutions, we get :—

$$\cos (\pi - a) = \frac{\cos (\pi - A) - \cos (\pi - B) \,.\, \cos (\pi - C)}{\sin (\pi - B) \,.\, \sin (\pi - C)},$$

or, $\cos a = \dfrac{\cos A + \cos B \,.\, \cos C}{\sin B \,.\, \sin C},$

similarly for the other sides.

IV. Given two angles and the included side, or two sides and the included angle, to find the remaining functions.

In this case the formula required is of the form,

$$\cot A = \frac{\cot a \cdot \sin c - \cos c \cdot \cos B}{\sin B}.$$

In Fig. 253, draw T H at right angles to O G and C **V** at right angles to T H.

From the figure we observe that

$$\cot A \cdot \sin B = \frac{G C}{D C} \cdot \frac{C E}{E H}.$$

$$= \frac{G C}{E H}, \text{ since } C E = C D$$

$$= \frac{T H}{E H} - \frac{V H}{E H}$$

$$= \frac{T H}{O H} \cdot \frac{O H}{E H} - \frac{V H}{C H} \cdot \frac{C H}{E H}$$

$$= \sin c \cdot \cot a - \cos c \cdot \cos B,$$

or,
$$\cot A = \frac{\sin c \cdot \cot a - \cos c \cdot \cos B}{\sin B}.$$

Similarly,
$$\cot B = \frac{\sin c \cdot \cot b - \cos c \cdot \cos A}{\sin A} \quad \left. \vphantom{\begin{array}{c} \\ \\ \\ \\ \\ \end{array}} \right\} \quad \bullet \quad \bullet \quad (IV.)$$

$$\cot C = \frac{\sin a \cdot \cot c - \cos a \cdot \cos B}{\sin B}.$$

V. To adapt the formula,

$$\cos A = \frac{\cos a - \cos b \cdot \cos c}{\sin b \cdot \sin c}$$

to logarithmic computation.

$$1 - \cos A = 1 - \frac{\cos a + \cos b \cdot \cos c}{\sin b \cdot \sin c}$$

$$= \frac{\sin b \cdot \sin c + \cos b \cdot \cos c - \cos a}{\sin b \cdot \sin c}$$

$$= \frac{\cos (b - c) - \cos a}{\sin b \cdot \sin c}.$$

$$2 \cdot \sin^2 \frac{A}{2} = \frac{2 \sin \cdot \dfrac{a + b - c}{2} \sin \dfrac{a + c - b}{2}}{\sin b \cdot \sin c},$$

$$\therefore \qquad \sin^2 \frac{A}{2} = \frac{\sin (s - c) \cdot \sin (s - b)}{\sin b \cdot \sin c}, \qquad \bullet \qquad \bullet \qquad \bullet \quad \textbf{(V.)}$$

where $\qquad s = \dfrac{a + b + c}{2}.$

Similarly,

$$1 + \cos A = \frac{\cos a - \cos b \cdot \cos c + \sin b \cdot \sin c}{\sin b \cdot \sin c},$$

$$= \frac{\cos a - \cos (b + c)}{\sin b \cdot \sin c},$$

$$2 \cos^2 \frac{A}{2} = \frac{2 \cdot \sin \dfrac{a + b + c}{2} \cdot \sin \dfrac{b + c - a}{2}}{\sin b \cdot \sin c};$$

that is,

$$\cos^2 \frac{A}{2} = \frac{\sin s \cdot \sin (s - a)}{\sin b \cdot \sin c}. \qquad \bullet \qquad \bullet \qquad \bullet \qquad \bullet \quad \textbf{(VI.)}$$

Also, $\quad \tan^2 \dfrac{A}{2} = \dfrac{\sin^2 \dfrac{A}{2}}{\cos^2 \dfrac{A}{2}}$

$$= \frac{\sin (s - c) \cdot \sin (s - b)}{\sin s \cdot \sin (s - a)}. \qquad \bullet \qquad \bullet \qquad \bullet \quad \textbf{(VII.)}$$

Again, $\quad \sin^2 A = 4 \cdot \sin^2 \dfrac{A}{2} \cdot \cos^2 \dfrac{A}{2}$

$$= \frac{4}{\sin^2 b \cdot \sin^2 c} \sin s \cdot \sin (s - a) \cdot \sin (s - b) \cdot \sin (s - c),$$
$$\textbf{(VIII.)}$$

From formulæ V., VI., VII., and VIII., we may find any angle when the three sides are known.

VI. If we are given the three angles of the triangle, by starting with the formula, $\cos a = \dfrac{\cos A + \cos B \cdot \cos C}{\sin B \cdot \sin C}$, and treating it as in V., we arrive at the formulæ :—

$$\sin^2 \frac{a}{2} = - \frac{\cos S \cdot \cos (S - A)}{\sin B \cdot \sin C}. \qquad \bullet \qquad \bullet \qquad \bullet \quad \textbf{(IX.)}$$

$$\cos^2 \frac{a}{2} = \frac{\cos (S - C) \cdot \cos (S - B)}{\sin B \cdot \sin C}. \qquad . \qquad . \qquad . \qquad . \qquad \textbf{(X.)}$$

$$\tan^2 \frac{a}{2} = - \frac{\cos S \cdot \cos (S - A)}{\cos (S - C) \cdot \cos (S - B)}. \qquad . \qquad . \qquad . \qquad \textbf{(XI.)}$$

$$\sin^2 a = \frac{4}{\sin^2 B \cdot \sin^2 C} \cos S \cdot \cos (S - A) \cdot \cos (S - B) \cdot \cos (S - C),$$
$$\textbf{(XII.)}$$

where $S = \dfrac{A + B + C}{2}$.

VII. Given two sides and the included angle, to determine the remaining angles.

In this case the required formula is of the form :—

$$\tan \frac{A + B}{2} = \frac{\cos \dfrac{a - b}{2}}{\cos \dfrac{a + b}{2}} \cdot \cot \frac{C}{2}.$$

We have, from equation III.,

$$\cos A + \cos B \cdot \cos C = \cos a \cdot \sin B \cdot \sin C,$$

Similarly, $\cos B + \cos A \cdot \cos C = \cos b \cdot \sin A \cdot \sin C,$

by addition

$$(\cos A + \cos B) \cdot (1 + \cos C) = (\cos a \cdot \sin B$$
$$+ \cos b \cdot \sin A) \sin C,$$

or, $\quad (\cos A + \cos B) \cdot 2 \cdot \cos^2 \dfrac{C}{2} = (\cos a \cdot \dfrac{\sin A}{\sin a} \cdot \sin b$

$$+ \cos b . \sin A) 2 \sin \frac{C}{2} \cos \frac{C}{2}.$$

$$\cos A + \cos B = \frac{\sin A}{\sin a} (\cos a \cdot \sin b + \cos b \cdot \sin a) \tan \frac{C}{2}$$

$$= \frac{\sin A}{\sin a} \sin (a + b) \tan \frac{C}{2}. \qquad . \qquad . \qquad . \qquad \textbf{(a)}$$

Again,

$$\sin A + \sin B = \sin A \left(1 + \frac{\sin B}{\sin A} \right),$$

$$= \sin A \left(1 + \frac{\sin b}{\sin a} \right),$$

$$= \frac{\sin A}{\sin a} (\sin a + \sin b). \qquad . \qquad . \qquad . \qquad \textbf{(b)}$$

Dividing equation (*b*) by equation (*a*), we have :—

$$\frac{\sin A + \sin B}{\cos A + \cos B} = \frac{\sin a + \sin b}{\sin (a + b)} \cot \frac{C}{2}$$

that is,

$$\frac{2 . \sin \dfrac{A + B}{2} \cos \dfrac{A - B}{2}}{2 . \cos \dfrac{A + B}{2} \cos \dfrac{A - B}{2}} = \frac{2 . \sin \dfrac{a + b}{2} \cos \dfrac{a - b}{2}}{2 . \sin \dfrac{a + b}{2} \cos \dfrac{a + b}{2}} \cot \frac{C}{2}.$$

or, $$\tan \frac{A + B}{2} = \frac{\cos \dfrac{a - b}{2}}{\cos \dfrac{a + b}{2}} \cot \frac{C}{2}, \qquad \text{. (XIII.)}$$

In a similar way we obtain,

$$\frac{\sin A - \sin B}{\cos A + \cos B} = \frac{\sin a - \sin b}{\sin (a + b)} \cot \frac{C}{2},$$

therefore,

$$\tan \frac{A - B}{2} = \frac{\sin \dfrac{a - b}{2}}{\sin \dfrac{a + b}{2}} \cot \frac{C}{2}. \qquad \text{. (XIV.)}$$

By writing ($\pi - a$) for A, etc., formulæ XIII. and XIV. become

$$\tan \frac{a + b}{2} = \frac{\cos \dfrac{A - B}{2}}{\cos \dfrac{A + B}{2}} \tan \frac{c}{2}, \qquad \bullet \qquad \bullet \qquad \bullet \quad \text{(XV.)}$$

$$\tan \frac{a - b}{2} = \frac{\sin \dfrac{A - B}{2}}{\sin \dfrac{A + B}{2}} \tan \frac{c}{2}, \qquad \bullet \qquad \bullet \qquad \bullet \quad \text{(XVI.)}$$

Formulæ XV. and XVI. are useful when we are given two angles and the adjacent side.

Formulæ for Right-angled Triangles.—When one of the angles (say C) is a right angle, we have :—

In general case.	*In right-angled triangle.*	
Sin C . sin a = sin A sin c.	sin a = sin c . sin A,	(1)
sin C . sin b = sin c . sin B.	sin b = sin c . sin B,	(2)
cos c = cos a . cos b + sin a . sin b . cos C.	cos c = cos a . cos b,	(3)
cos c . sin A . sin B = cos A . cos B + cos C.	cos c = cot A . cot B,	(4)
cos A + cos B . cos C = cos a . sin B . sin C.	cos A = cos a . sin B,	(5)
cos B + cos A . cos C = cos b . sin A . sin C.	cos B = cos b . sin A,	(6)
cot A . sin C + cos b . cos C = cot a . sin b.	cot A = cot a . sin b,	(7)
	or, tan a = tan A . sin b,	(8)
cot B . sin C + cos a . cos C = cot b . sin a.	cot B = cot b . sin a,	(9)
	or, tan b = tan B . sin a,	(10)
cot c . sin a = sin B . cot C + cos a . cos B.	cot c . sin a = cos a . cos B,	(11)
	or, tan a = tan c . cos B,	(12)
sin b . cot c = sin A . cot C + cos b . cos A.	tan b = tan c . cos A,	(13)

All cases of the right-angled triangle can be solved by means of the above formulæ.

Area of a Lune.—*Definition.*—The portion of a spherical surface intercepted between two great semicircles is called a *lune.*

Let the lune be represented by A B C D (Fig. 254). To find its area.

It is clear that the area of the lune varies as the angle between the planes of the semicircles A C B and A D B,

∴ $\dfrac{\text{Area of a lune whose angle is A}}{\text{Area of the sphere}} = \dfrac{A}{360}$.

or,

$$\text{area of lune} = \frac{A}{360} . 4 \pi R^2$$

$$= \frac{A}{180} . 2 \pi R^2,$$

where R is the radius of the sphere.

To find the Area of a Spherical Triangle.—Let A B C be the given triangle drawn on the hemisphere B C F D (Fig. 255), and let the arcs C A, A B, be continued both ways until they meet at E. Then, since A D E = a semicircle = D A C, and the arc A D is common, the arc A C = the arc D E. Similarly, the arc A B = the arc F E ; also, the angle D E F = B A C, there-fore the triangles D E F and A B C are equal.
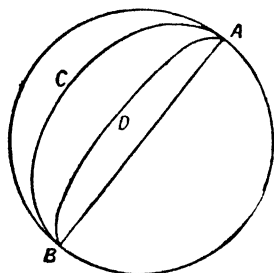
Fig. 254.
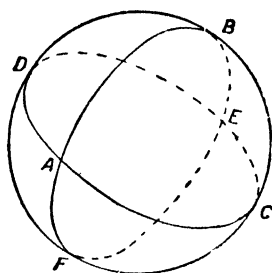
Fig. 255.

If S = the area of the triangle A B C, then S = surface of hemisphere D F C B — D A F — F A C — A D B

$$= 2\,\pi\,R^2 - (\text{lune A F D E} - S) - (\text{lune F A B C} - S)$$
$$- (\text{lune C A D B} - S)$$

$$= 2\,\pi\,R^2 \left(1 - \frac{A}{180} - \frac{B}{180} - \frac{C}{180} \right) + 3\,.\,S$$

$$\therefore \quad S = \pi\,R^2 \left(\frac{A + B + C - 180}{180} \right)$$

$$= \frac{E}{180}\,\pi\,R^2, \text{ if } E = A + B + C - 180.$$

The quantity E, by which the sum of A, B, and C exceeds 180°, is called the *spherical excess* of the triangle, and, as the area of a spherical triangle varies as its spherical excess, for any two triangles A B C and A′ B′ C′ on the same sphere, we have

Area of A B C : area of A′ B′ C′ :: E : E′.

To find the Area of a Spherical Polygon.—Suppose the area of the polygon to be divided into as many triangles as the polygon has sides (say N) by means of great circles passing through the

angular points and any point within the polygon.   Then the area of the polygon

$$= \text{sum of areas of the triangles}$$

$$= \frac{\pi \, \mathrm{R}^2}{180} \, (\text{sum of angles of all the triangles} - \mathrm{N} \cdot 180)$$

$$= \frac{\pi \, \mathrm{R}^2}{180} \{\text{sum of angles of the polygon} - (\mathrm{N} - 2) \, 180\}.$$

To determine the number of seconds in the spherical excess of a geodetic triangle.

Let $\mathrm{R} = $ the mean radius of the earth in feet,

$ \mathrm{A} = $ area of the given triangle, in square feet.

Then,
$$\mathrm{A} = \frac{\mathrm{E} \times 60 \times 60}{180 \times 60 \times 60} \, \pi \, \mathrm{R}^2,$$

and,
$$\mathrm{E} \times 60 \times 60 = \frac{180 \times 60 \times 60}{\pi \, \mathrm{R}^2} \times \mathrm{A}.$$

If we take $\mathrm{R} = 20{,}889{,}000$ feet, by reduction as on p. 274, we obtain

$$\log (\mathrm{E} \times 60 \times 60) = \log \mathrm{A} - 9 \cdot 3254098.$$

Taking Colonel Clarke's more recent determination of the value of R—viz., 20,890,172 feet—the equation for spherical excess becomes

$$\log (\mathrm{E} \times 60 \times 60) = \log \mathrm{A} - 9 \cdot 3254589.$$

Astronomical Terms.

The following terms are of such frequent occurrence in the descriptions of the methods of determining time, azimuth, latitude, and longitude, which follow, that it is necessary to have a clear conception of their meaning at the outset.

Horizon.—A plane tangent to the earth's surface is spoken of as the horizon. Obviously such a plane is swept out by the line of collimation of an accurately set-up spirit level when rotated about the vertical axis of the instrument.

The Sensible or Apparent Horizon is the circle of contact, with the earth, of the cone of visual rays passing through the

point of observation. The circle of contact is a small circle of the earth and its radius depends on the altitude of the point of observation.

The Rational or True Horizon is a great circle of the celestial sphere parallel to the horizon and passing through the earth's centre.

Zenith.—The upper pole of the rational horizon, it is the point on the celestial sphere immediately above the observer's station. The lower pole is called the **Nadir.**

Altitude.—The altitude of a heavenly body is its angular distance from the horizon measured on a great circle passing through the zenith. Thus, in Fig. 256, if the small circle represents the earth, the large circle the celestial sphere, Z the zenith, and b any fixed star, then the altitude of the star is H b (or the angle H O b).

Note.—In dealing with the altitudes of fixed stars, *horizon* and *rational horizon* are treated as synonymous terms, as the fixed stars are so distant that they have the same altitude both at the centre of the earth and on its surface. When dealing with the sun or moon, since these bodies are comparatively near the earth, a correction for altitude is required, as we shall see later.
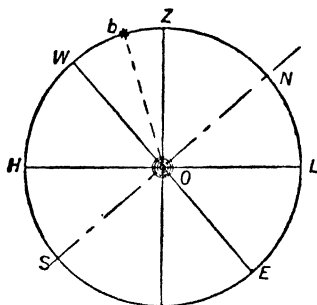
Elevation on Plane of Meridian.

Fig. 256.

Co-altitude or Zenith Distance.—The co-altitude of a heavenly body is the complement of its altitude, or is the distance of the heavenly body from the zenith. In Fig. 256 the co-altitude of the star b is the arc b Z (or the angle b O Z).

Poles.—The poles are the points of intersection of the earth's axis and the celestial sphere ; the upper pole (in the northern hemisphere) is called the North, and the lower pole the South pole (N and S in Fig. 256).

Equator is the great circle of the earth at right angles to its axis. The intersection of the plane of the equator and the celestial sphere is the *celestial equator.* It is represented in Fig. 256 by the line W O E.

Declination.—The declination of a heavenly body is its distance north or south of the equatorial plane, the angular distance being measured on the meridian—*i.e.,* the great circle

passing through the body and the poles. The declination of the star *b* (Fig. 256) is the arc W *b*.

Co-declination is the complement of the declination, or is the *polar distance* of any celestial body. The co-declination of *b* (Fig. 256) is the arc *b* N.

Angle of the Vertical.—Owing to the earth being a spheroid of revolution, and not a perfect sphere, with the exception of the poles and points on the equator, the normal to the earth's surface at any point does not pass through the point of intersection of the earth's axis and the equatorial plane. The angle between the normal at any point on the earth's surface and the line joining the point to the centre of the earth is called "the angle of the vertical." It may also be defined as the angle between the geographical and geocentric latitudes of any place. The angle is zero at the equator, increases to the 45th degree of latitude where it amounts to 11′ 30″, and thence diminishes to zero at the poles.

Latitude, Geographical, is the angular distance of any place on the earth's surface north or south of the equator, and is measured on the meridian of the place. It is the same as celestial declination. In Fig. 256, if Z is a point on the earth's surface, the latitude of Z is the arc W Z.

Latitude, Geocentric, is the angular distance of a place north or south of the equator corrected for the oblateness of the earth. It is the geographical latitude diminished by the angle of the vertical.

Co-latitude is the complement of the latitude and equals the distance from the zenith to the pole. For, in Fig. 256, since the latitude of Z is the arc W Z, and the arc W N = arc H Z = a right angle, therefore the arc H W = the arc Z N, or the co-latitude (H W) = the zenith distance of the pole (Z N).

Similarly, the arc W N = the arc Z L = a right angle, and the arc Z N is common to these two angles, hence the arc W Z = the arc N L, or *the elevation of the pole gives the latitude of the place*.

Prime Vertical.—The great circle passing through the zenith and nadir and the east and west points of the horizon is called the prime vertical. Its plane is, therefore, vertical and at right angles to the meridian of any place.

Circum-polar Stars.—These are stars whose apparent daily courses are wholly above the horizon, and consequently these stars never rise or set. All stars having declinations of the same sign as the place of observation but greater than its co-latitude
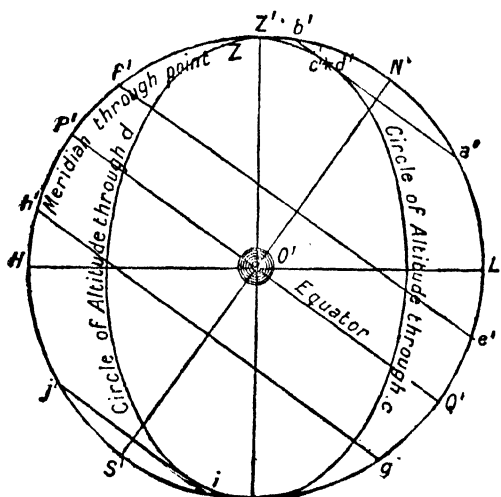
Fig. 257.—Elevation on Plane of Meridian through **Z**.
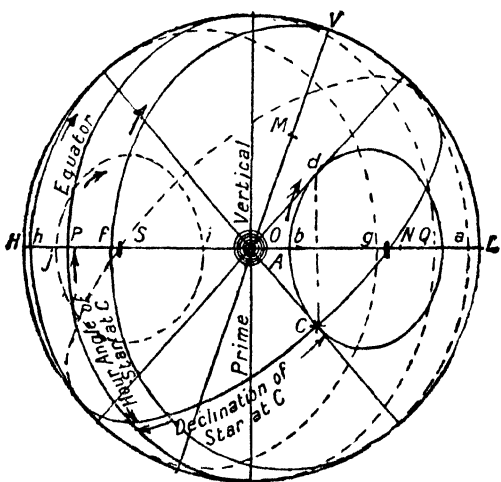
Fig. 258.—Plan on Plane of Horizon.

are circum-polar stars. Thus, in latitude 54° N., stars having a north declination greater than 90° — 54°, or 36°, are circum-polar stars. This is evident from Fig. 257, when Z' is the place of observation, H' L' the horizon, and P' Q' the equator, the co-latitude of Z' = the arc Z' N' = the arc Q' L', and any star having a N. declination greater than the arc Q' L' would not cut the plane of the horizon, whereas if the declination of the star is less than the arc Q' L' the path of the star will cut the plane of the horizon in two points, the points of rising and setting Similarly, stars which have a S. declination greater than the arc H' P'—i.e., greater than the co-latitude of Z', will never be visible at that place. In Fig. 257 and Fig. 258, a' b' is the path of a circum-polar star, e' f' and g' h' are the paths of rising and setting stars, and i' j' the path of a star never visible at Z'.

Azimuth and Amplitude.—The azimuth of a star is the angle between the plane of the meridian of the place of observation and the plane passing through the star, the zenith, and nadir —i.e., the angle between the vertical planes containing the poles, the star, and the place of observation. The amplitude of a star is its angular distance from the prime vertical and, consequently, is equal to 90° — azimuth.

Culmination or Transit.—Owing to the rotation of the earth, every celestial body appears to cross, or transit, the meridian twice each day, once above and once below the pole. These transits are called the upper and lower culminations of the star. Clearly, the star has its greatest and least altitudes at the instants of culmination. In Fig. 257, if a' b' represents the path of a star (circum-polar or otherwise), the points b' and a' are its upper and lower culminations respectively. From the figure we note that the arc N' L' = $\dfrac{\text{arc } b' \, L' + \text{arc } a' \, L'}{2}$ —i.e., the mean of the altitudes at the upper and lower culminations is the altitude of the elevated pole. This gives the most accurate method of determining latitude.

Elongation.—A celestial body is said to be at its east or west elongation when it is at its greatest distances east or west of the earth's axis. In Fig. 258, which is the plan of Fig. 257 projected on the plane of the horizon, the points of elongation of the upper circum-polar star are shown at c and d. If A (Fig. 258) is the point of observation, and A M a plane passing through a second station M, the azimuth of A M is the angle V A N, and it is evident from the figure that the angle V A N = $\dfrac{\text{V A } d + \text{V A } c}{2}$. This

gives the best method of determining the azimuth of a line, but as the points c and d are a considerable distance apart in time, one of the observations will, in general, have to be made in daylight; hence this method is inapplicable with an ordinary theodolite. The same remark applies to the method, described above, of determining latitude from the two points of culmination.

It should be noted that as the line joining the points c and d is horizontal, the points of elongation are points of equal altitude.

Ecliptic.—The great circle of the heavens which the sun appears to describe in the course of a year is called the ecliptic. The plane of the ecliptic is inclined to the plane of the equator at an angle (called the obliquity) of about 23° 27′, but is subject to a diminution of about 50″ in a century. The change in the declination of the sun is due to the obliquity of the ecliptic.

Solstices and Equinoxes.—The points at which the north and south declination of the sun is a maximum are called the solstices, the former being the summer and the latter the winter solstice, and *vice versâ* for the Southern Hemisphere.

At the points of intersection of the sun's apparent path and the plane of the equator, the declination of the sun is zero. These points are six months apart in time, one marks the commencement of spring, and is called the " vernal equinox," the other occurs at the beginning of autumn, and is called the " autumnal equinox." The interval of time between two successive vernal equinoxes is that adopted for the solar or equatorial year, and amounts to 365¼, or, more accurately, 365·2422 days.

Sidereal and Mean Solar Days.—In the period of time between two successive vernal equinoxes, which take place at or about the 21st of March, the earth makes 366·2422 revolutions on its axis, but, in the same period the sun makes one complete revolution in the stellar sphere, moving from west to east or opposite to that of the apparent motion of the stellar sphere; from this cause, the sun transits any meridian of the earth 365·2422 times in a solar year, thus producing 365·2422 days. Any star will, however, transit a meridian 366·2422 times in a year, hence the interval between two successive transits of a star (or the length of a sidereal day) is shorter than the mean solar day by about four minutes.

First Point of Aries, Right Ascension.—We have already seen that, in order to fix the position of a point in a plane, it is sufficient to know the two rectangular co-ordinates of the point. Similarly, to fix the position of a point on a sphere two rectangular co-ordinates are sufficient; thus, for example, the position of

a point on the earth's surface is fixed when its latitude and longitude are known. In both cases, however, we must have a fixed point of departure and a fixed line, or plane of reference. On the stellar sphere the fixed point of reference is called *the first point of Aries*, and is the point in the heavens opposite which the sun arrives at the vernal equinox. The fixed plane of reference is that of the equator, hence in Fig. 259, if P be any star, N P Q S and N ϒ S great circles of the stellar sphere passing through P, and the first point of Aries ϒ, and cutting the equatorial circle ϒ Q B at the points Q and ϒ respectively, the co-ordinates of the star are ϒ Q and Q P. The distance ϒ Q (expressed in
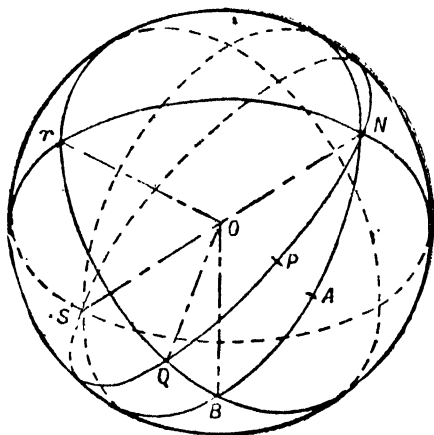
Fig. 259.

hours, minutes, and seconds) is called the *Right Ascension** of the star, and P Q is, of course, the star's declination.

Right ascensions and declinations of a large number of fixed stars are given in the *Nautical Almanac*, or, in an abridged form, in *Whitaker's Almanac*.

Mean Sun.—The increase in the right ascension of the sun in its apparent daily course is not uniform, being sometimes more than a quarter of an hour behind, and at others as much in advance of its mean position. If, therefore, common clocks were to register noon at the exact instant of the sun's transit, they would have to vary their rates continually in order to keep true sun time. To obviate this inconvenience, an imaginary

* Right Ascensions are *always* measured *eastwards* from Aries to the object.

(mean) sun is assumed to exist, which moves, in the equator, uniformly in right ascension throughout the year.

Local Mean Noon is the instant at which the mean sun crosses the meridian of any place, and at this instant a clock keeping local mean time would strike twelve, or we say it is noon.

In astronomical work, angles which are measured in the plane of the equator are usually expressed in hours, minutes, and seconds ; longitudes, for example, are frequently expressed in hours, etc., as well as in degrees. The conversion of degrees into hours, etc., or *vice versâ*, is very simple, for since the earth turns through 360° in 24 hours, any fraction of 360° is the same fraction of 24 hours ; thus an angle of 80° corresponds to $\dfrac{24 \times 80}{360}$, or 5⅓ hours. For any other angle (a) we have $360 : 24 :: a : x$, or $15 : 1 :: a : x$.

Example 1.—The longitudes of (a) Manchester and (b) New York are 2° 13′ 45″ and 73° 57′ 30″ west of Greenwich. Express these angles in hours, minutes, and seconds.

			H.	**M.**	**S.**
(a) Time corresponding to $2° = \dfrac{2}{15}$ hours		$=$	0	8	0
,, ,, $13′ = \dfrac{13}{15}$ mins.		$=$	0	0	52
,, ,, $45″ = \dfrac{45}{15}$ secs.		$=$	0	0	3
Time corresponding to long. 2° 13′ 45″		$=$	0	8	55

			H.	**M.**	**S.**
(b) Time corresponding to $73° = \dfrac{73}{15}$ hours		$=$	4	52	0
,, ,, $57′ = \dfrac{57}{15}$ mins.		$=$	0	3	48
,, ,, $30″ = \dfrac{30}{15}$ secs.		$=$	0	0	2
Time corresponding to long. 73° 57′ 30″,		$=$	4	55	50

Example 2.—The longitude of Tokyo is 9 hours 18 minutes 58 seconds east of Greenwich. Express this angle in degrees, minutes, and seconds.

Angle corresponding to 9 hours = 9 × 15 deg. = 135° 0′ 0″ *
,, ,, 18 mins. = 18 × 15 m. = 4° 30′ 0″
,, ,, 58 secs. = 58 × 15 sec. = 0° 14′ 30″

Longitude of Tokyo = 139° 44′ 30″ E.

Since the instant of noon at any place is decided by the transit of the mean sun at that place, it is evident that common clocks at different places record different times; thus, from the preceding examples, a common clock in New York will be 4 hours 55 mins. 50 secs. slow of the clock at Greenwich; similarly, a clock in Tokyo will be 9 hours 18 mins. 58 secs. fast of the clock at Greenwich. It is important to remember this, as all the data used in astronomical calculations are compiled for Greenwich Mean Time.

Since January 1st, 1925, the Astronomical Day as reckoned in the *Nautical Almanac* is considered to begin at 0 h. midnight instead of 0 h. noon, thus bringing astronomical and civil time and date into agreement. The dial of the astronomical clock is, however, divided into 24 hours, thus dispensing with the terms *a.m.* and *p.m.*, but these are still used in civil time reckoning.

To convert local mean (civil) time to Greenwich mean time, proceed as follows :—*If the given time be a.m. make no change ; if p.m., add 12 hours ; in both cases the date remains unchanged ; to this result, if the longitude be* $\left\{ \begin{array}{l} west \\ east \end{array} \right\}$ $\left\{ \begin{array}{l} add \\ subtract \end{array} \right\}$ *the longitude in hours, etc. If the sum exceeds 24 hours, deduct 24 hours and add one day to the date; if the sum is negative add 24 hours and deduct one day from the date.*

Example 3.—Find the Greenwich mean time corresponding to 6 hrs. 30 mins. 0 secs. a.m., April 6th, in Newfoundland. (Long. 52° 45′ W.)

	H.	M.	S.	
6 hrs. 30 mins. 0 secs. a.m., April 6th	6	30	00	April 6th, Newfoundland mean time.
Long. 52° 45′ 00″ W.	=+ 3	31	00	
Corresponding Greenwich mean time=	10	1	00	April 6th.

* The N.A. now contains tables for converting time to arc, and *vice versâ.*

Example 4.—Determine the Greenwich mean time corresponding to 5 p.m., June 15th, in Tokyo. (Long. 139° 44′ 30″ E.)

		H.	M.	S.	
5 hrs. 0 mins. 0 secs., June 15th	=	17	0	0	June 15th,
Long. 139° 44′ 30″ E. . .	=	−9	18	58	Tokyo
					mean time.
Required G.M.T. . . .	=	7	41	02	June 15th.

Hour Angle—Local Sidereal Time.—The hour angle of a star is the angular distance between the plane of the meridian of a place and the plane passing through the star and containing the earth's axis ; the angular distance being measured, east or west of the meridian, in the plane of the equator. Hour angles are stated in degrees, etc., on the whole circle system, with the upper transit of the star as the point of departure, round to 180° at the lower transit, and so on to 360°. They are also expressed in hours, minutes, and seconds on the four-quadrant system, the position of the star east or west of the meridian at the time of observation being noted. Thus, in Fig. 259, if N A B S is the meridian through the place of observation A, and cutting the equatorial circle at B, the westerly hour angle of the star P is equal to B Q.

We have seen that ♈ Q (Fig. 259) is the right ascension (R.A.) of the star ; similarly, the R.A. of the meridian of the place is equal to ♈ B ; from this it follows that, *the R.A. of the meridian of a place = R.A. of star + westerly hour angle of star.*

If this sum is greater than 24 hours, deduct 24 hours and add a like amount if the sum is negative.

The right ascension of the meridian of a place is called the *Local Sidereal Time* (or L.S.T.) of the place. It is the time interval which has elapsed since the transit of the first point of Aries over the meridian of the place.

Conversion of Mean into Sidereal Time Intervals.—As the mean sun, moving uniformly, makes a complete circuit of his apparent path in one mean solar year, the right ascension of the mean sun increases from zero to 360°, or 24 hours in the interval of time between two successive vernal equinoxes—that is, in 365·2422 days. Therefore,

366·2422 sidereal days $= 365\cdot2422$ mean solar days,

or 1 sidereal day $\qquad = 0\cdot99727$ mean solar days.

∴ 24 hours sidereal time $\quad = 24 \times 0\cdot99727$ hours M.T.

$\qquad\qquad\qquad\qquad = 23$ hrs. 56 mins. $4\cdot09$ secs. **M.T.**

∴ 1 sidereal hour $\qquad\quad = 1$ hr. $- 9\cdot8295$ secs. M.T.

Similarly, 1 mean solar day $= \dfrac{366\cdot2422}{365\cdot2422}$

$\qquad\qquad\qquad\qquad = 1\cdot00273$ sidereal days,

and 1 hour M.T. $\qquad\qquad = 1$ hr. $+ \dfrac{3600}{365\cdot2422}$ secs. S.T.

$\qquad\qquad\qquad\qquad = 1$ hr. $+ 9\cdot8564$ secs. S.T.

The quantity $9\cdot8564$ is called the *acceleration* of sidereal time on mean time, and the quantity $9\cdot8295$ is the *retardation* of mean time on sidereal time.

Tables for the conversion of M.T. into S.T., and *vice versâ*, are given in the *Nautical Almanac*, and in *Chambers' Mathematical Tables*.

The right ascension of the mean sun $+ 12$ h. at 0 h. G.M.T., for every day of the year, is given in the *Nautical Almanac* (N.A.), but is there called " Sidereal Time."

We have seen that the R.A. of any meridian $=$ L.S.T. $=$ R.A. of any star $+$ W.H.A. of the star $=$ R.A. of mean sun (R.A.M.S.) ± 12 h. $+$ mean time at the place. Another form of these equations is :—

L.S.T. $=$ S.T. at 0 h. L.M.T. $+$ L.M.T. (in S.T. units).

The following examples show the necessary steps in the computation.

Example 5.—Find the R.A. of the mean sun at 5.30 a.m., August 14th, 1932, in Montreal. Longitude $73^\circ\ 34'\ 45''$ W. $= 4$ hrs. 54 mins. 19 secs.

		H.	M.	S.	
5 h. 30 m. 00 s. a.m., Aug. 14th	$=$	5	30	00	Aug. 14th.
Add long. 4 h. 54 m. 19 s. W.		$+$ 4	54	19	
Corresponding G.M.T.	$=$	10	24	19	
	$=$	$10\cdot405$ h.			

		H.	M.	S.
From N.A., S.T. at 0 h. G.M.T., Aug. 14, 1932 =		21	28	48·41
Accel. for 10·405 h. = 10·405 × 9·856	= +		1	42·56
R.A.M.S. ± 12 h.	=	21	30	30·97
Deduct 12 h.		−12	00	00·00
R.A.M.S. at given instant	=	9	30	30·97

Example 6.—Find the R.A. of the meridian of Montreal (or the L.S.T.), from the data given in Example 5.

		H.	M.	S.	
The R.A. of the mean sun at given moment,	=	9	30	30·97	
Mean time at place	=	5	30	00	Aug. 14th.
Sum	=	15	00	30·97	
Subtract 12 hours	=	12	00	00	
Difference = R.A. of meridian	=	3	00	30·97 = L.S.T.	

When the problem is to determine L.S.T. the equation :

L.S.T. = S.T. at 0 h. L.M.T. + L.M.T. (in S.T. units) is to be preferred.   Applying this to Example 6, we have :

		H.	M.	S.
From N.A., S.T. at 0 h. G.M.T., Aug. 14, 1932	=	21	28	48·41
Corrn. for long. = 4·905 × 9·856	+W			48·35
S.T. at 0 h. L.M.T.	=	21	29	36·76
S.T. equiv. for 5 h. 30 m. = 5 h. 30 m. + 5·5 × 9·856	=	5	30	54·21
Required L.S.T.	=	3	00	30·97

Example 7.—Find the R.A. of the meridian of Bombay at 4.30 p.m., February 24th, 1932.   Longitude of Bombay, 72° 48′ 46·8″ E. = 4 h. 51 m. 15·15 s.

		H.	M.	S.	
4.30 p.m., Feb. 24th	=	16	30	00	Feb. 24th.
Longitude 4 h. 51 m. 15·15 s. E.	= −4	51	15·15		
Corresponding G.M.T.	=	11	38	44·85	Feb. 24th.
	=	11·646 h.			

		H.	M.	S.
From N.A., S.T. at 0 h. G.M.T., Feb. 24/32 =		10	10	40·73
Accel. for 11·646 h. = 11·646 × 9·856 =			1	54·78
R.A.M.S. ± 12 h. at given instant* =		10	12	35·51
Add		12		
R.A.M.S. at given instant =		22	12	35·51
Add W.H.A. of mean sun		4	30	00·00
Required L.S.T. =		2	42	35·51

The processes involved in the above computations are illustrated in Figs. 260 and 261, in which the R.A. of the mean sun at the given time and place is shown by the angle α, and the local mean time (or hour angle $\pm$ 12 h.) by the angle β. The required L.S.T. is represented by the angle θ. It is obvious from the figures that, $\theta = \alpha + \beta - 12$ hours in fig. 260 and $\alpha + \beta - 36$ h. in fig. 261.

Second Method.—Local mean time may be converted into local sidereal time from the known value of the Greenwich Mean Time at Sidereal Noon (G.M.T. at S.N.). This is given for each day in the *Nautical Almanac*, but is there called the " Transit of First Point of Aries." The method of Conversion is as follows :— Correct the G.M.T. at *preceding* S.N. for longitude at the rate of 9·8295 seconds per hour, subtracting the correction if the longitude be west, and adding the correction in the case of east longitude. Subtract the corrected angle from the L.M.T. at observation (adding 24 hours and deducting one day from the date, if necessary) ; the remainder is the mean time interval which has elapsed since the preceding transit of the first point of Aries, or Local Sidereal Noon. The sidereal time equivalent for this mean time interval is then obtained, the result being the required local sidereal time.

In Figs. 260 and 261, the L.M.T. at preceding S.N. is represented approximately by the angle φ, and the local mean time by the angle β ; in both cases it is obvious from the figures that $\theta = \beta \pm 12$ h. $- \varpi$.
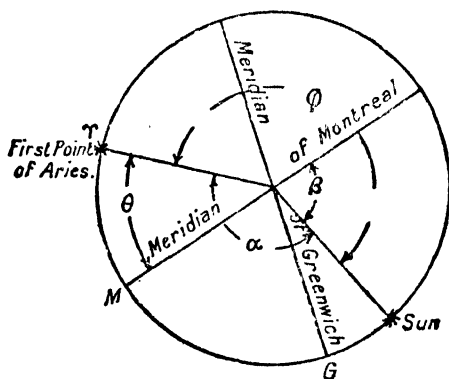
Fig. 260.
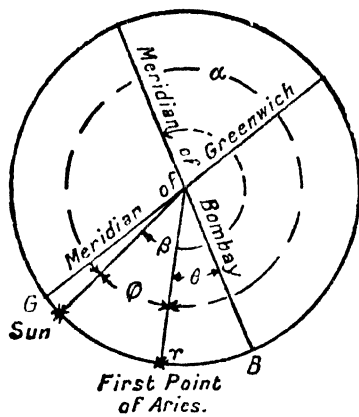
Fig. 261.

The necessary steps in the computation are shown in the following examples :—

Example 8.—From the data given in Example 5, find the R.A. of the meridian of Montreal.

	H.	M.	S.	
In this case, 5.30 a.m., Aug. 14 =	5	30	00	Aug. **14.**

	H.	M.	S.			
From N.A., G.M.T.						
at preceding G.S.N.						
(Aug. 13, 1932)	=	2	30	46·82		
Corrn. for long. 4·905						
= — 4·905×9·829	= (—W)			48·22		
L.M.T. at L.S.N.,						
Aug. 14, 1932	=	2	29	58·60	2 29	58·60

Interval from L.S.N. in M.T. units	=	3	00	01·40
To this add 3·0004 × 9·856 s.	=			29·57

Required L.S.T.	=	3	00	30·97

Example 9.—Determine the L.S.T. of the meridian of Bombay from the data given in Example 7.

	H.	M.	S.	
Here, 4.30 p.m., Feb. 24th =	16	30	00	Feb. **24.**
Longitude 4 h. 51 m. 15·15 s. E. =	4·854 h.			

	H.	M.	S.
L.M.T. =	16	30	00

		H.	M.	S.			
From N.A., G.M.T.							
at preceding G.S.N.							
(Feb. 24, 1932)	=	13	47	03·41			
Corrn. for long.							
= 4·854 × 9·829	= (+E)			47·71			
L.M.T. at L.S.N.	=	13	47	51·12	13	47	51·12

S.T. interval from L.S.N. in M.T. units	} =	2	42	08·88
To this add 2·702 h. × 9·856	=			26·64

Required L.S.T.	=	2	42	35·52

If preferred, the given mean time interval may be converted into the equivalent sidereal time by aid of the tables given in the *Nautical Almanac;* for example, in the preceding case, we have :—

Interval of mean time.			From N.A.—Corresponding sidereal time.		
H.	M.	S.	H.	M.	S.
2	0	0	2	0	19·713
0	42	0	0	42	06·900
0	0	8	0	0	08·022
0	0	0·88	0	0	0·882

Required L.S.T. = 2　42　35·517

Conversion of Sidereal into Mean Time.—The conversion of sidereal into mean time intervals is the converse of the preceding operation. It is effected by aid of the fundamental equation,

L.S.T. = R.A. of mean sun + 12 h. + mean time at place.

∴ Mean time at place = L.S.T. — (R.A. of mean sun. + 12 h.)

or L.M.T. (in S.T. units) = L.S.T.— S.T. at 0 h., L.M.T.

The method of calculation is illustrated in the following example :—

Example 10.—Find the local mean time corresponding to 15 h. 21 m. 18·52 s. sidereal time in Sydney on June 3rd, 1932. Longitude of Sydney, 10 h. 4 m. 49·54 s. E.

	H.	M.	S.
Given L.S.T.	15	21	18·52

From N.A. S.T. at
0 h. G.M.T.,
June 3rd, 1932　= 16　44　56·25
Corrn. for long.
= 10·0804 h. ×9·856= (—E) 1　39·36

S.T. at 0 h. L.M.T.　= 16　43　16·89,—16　43　16·89

S.T. interval from 0 h. L.M.T.　= 22　38　01·63
From this subtract
22·635 h. × 9·829 s.　=　3　42·48

Required L.M.T.　= 22　34　19·15 June 3rd.

The conversion of the S.T. interval to its equivalent M.T. is shown below :—

	Interval of sidereal time.			From N.A.—Corresponding mean time.	
H.	M.	S.	H.	M.	S.
22	0	0	21	56	23·750
0	38	0	0	37	53·775
0	0	01	0	0	0·997
0	0	0·63	0	0	0·628

Required mean time interval from L.M.M. = L.M.T. $\Big\}$ = 22   34   19·150 June 3rd

Local Mean Time is the westerly hour angle of the *mean* sun ± 12 h. expressed in hours, minutes, and seconds.

If the given L.S.T. is less than the R.A. of the mean sun + 12 h. at G.M.M., 24 hours must be added to maintain a positive angle. The G.S.T. of G.M.M. is taken from the *Nautical Almanac* for the same nominal date as the local date.

Example 11.—The star δ Ursæ Majoris was observed at 5.30 a.m. in Montreal on Aug. 14th, 1932. The right ascension of the star was 12 h. 12 m. 5·38 s., find its hour angle.

In this case we use the equation :—

The westerly hour angle of star = L.S.T. — R.A. of star.

	H.	M.	S.
From Ex. 6, L.S.T. at given time and place =	3	00	30·97
Add 24 hours (to avoid a negative angle) =	24	00	00
Sum =	27	00	30·97
R. A. of star =	— 12	12	5·38
Required hour angle =	14	48	25·59

Example 12.—In longitude 60° W. on May 4th, 1932, an observation was made on β Tauri, whose R.A. was 5 h. 21 m. 59·48 s.

If the hour angle of the star was 9 h. 15 m. 8 s., find the local mean time of the observation.

Here, we use the equation :—

S.T. at L.M.M., May 4th + mean time at place
$\qquad$ = R.A. of star + westerly hour angle of star.

From this we have :—

Mean time at place
$\qquad$ = R.A. of star + westerly hour angle of star
$\qquad\qquad$ — S.T. at L.M.M., May 4th.

		H.	M.	S.
R.A. of star	=	5	21	59·48
W.H.A. of star	=	9	15	08·00
L.S.T.	=	14	37	07·48

	H.	M.	S.			
From N.A., S.T. at 0 h.						
G.M.T., May 4th, 1932, =	14	46	39·53			
Corrn. for long.						
4 h. × 9·856 (+W) =			39·42			
L.S.T. at 0 h. L.M.T. =	14	47	18·95	14	47	18·95
S.T. interval from 0 h. L.M.T.			=	23	49	48·53
M.T. equivalent for 23 h. 49 m. 48·53 s.						
= required L.M.T.			=	23	45	54·29

Second Method.

Obtain the Transit of the first point of Aries from 0 h. L.M.T. To this add the M.T. equivalent for the given L.S.T. The result is the required L.M.T.

		H.	M.	S.
From N.A. Transit of first point of Aries from 0 h. G.M.T., May 4th, 1932.	=	9	11	49·82
Corrn. long. 4 h. × 9·83 (—W + E)	=—			39·32
Transit of first point of Aries from 0 h. L.M.T.,	=	9	11	10·50
M.T. equivalent for L.S.T. 14 h. 37 m. 07·48 s.,	=	14	34	43·79
Required L.M.T.	=	23	45	54·29

Interpolation.—In the more recent Nautical Almanacs the variations per hour of the rapidly varying quantities in connection with the true sun are not given. In their place the differences per day in arc and time are inserted. The computer must decide what method of interpolation he will use. This will depend on the accuracy required ; by assuming that the variation between two consecutive differences is linear, the required value may be found by simple interpolation ; but, if a higher degree of precision is required, the method due to the Prussian astronomer Bessel (for which the tabulation is designed) should be used.

Let f_{-1}, f_0, f_1, f_2 etc. be the successive values of the function we desire to interpolate, and let the first differences between these values be $\Delta'_{-\frac{1}{2}}, \Delta'_{\frac{1}{2}}, \Delta'_{\frac{3}{2}}$, i.e., $\Delta'_{-\frac{1}{2}} = f_0 - f_{-1}$, $\Delta'_{\frac{1}{2}} = f_1 - f_0$, etc., and the second differences, i.e., $\Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}}$ etc., be Δ''_0, Δ''_1.

The operations are best shown thus :

$$
\begin{array}{ccc}
f_{-1} & & \\
 & \Delta'_{-\frac{1}{2}} & \\
f_0 & & \Delta''_0 \\
 & \Delta'_{\frac{1}{2}} & \\
f_1 & & \Delta''_1 \\
 & \Delta'_{\frac{3}{2}} & \\
f_2 & &
\end{array}
$$

Third and higher orders of differences are not required in surveying. The quantity (f_n) we wish to find lies somewhere between f_0 and f_1, i.e., it is greater than f_0 and less than f_1. To determine f_n Bessel's equation

$$ f_n = f_0 + n \, \Delta'_{\frac{1}{2}} + \frac{n \, (n-1)}{4} \, (\Delta''_0 + \Delta''_1) + \text{etc.} $$

is most suitable ; n is the fraction of the interval between two tabulated values.

In the above equation it should be noted (1) that by neglecting the 3rd term the result will be equivalent to simple linear interpolation, which will produce a maximum error (when $n = 0.5$) of one-eighth of the average second difference ; (2) that $\Delta''_0 + \Delta''_1 = \Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}}$, hence it is necessary to have the tabulated differences on four dates, selected so that the given instant lies between the two middle dates. The coefficient $\frac{n \, (n-1)}{4}$ is called B″ in the Nautical Almanac, and its value is given to four places of decimals in Table XVII. and three places in Table XVIII.

Three decimal places are usually enough for surveying purposes.

For general use we may write the above formula :

$$f_n = f_0 + n \, \Delta'_{\frac{1}{2}} + B'' \, (\Delta'_{\frac{3}{2}} - \Delta'_{-\frac{1}{2}})$$

The coefficient B'' is always negative.

In determining n we use Table IX. in the N.A. for converting hours, minutes and seconds into decimals of a day.

The use of the formula is shown in the examples which follow.

The **apparent solar day** is now considered to begin and end at *apparent midnight*, hence apparent time at any instant = W.H.A. of true sun + 12 h. Apparent time and mean time differ by a variable quantity (E) called the equation of time, and its daily value is given in the N.A. at 0 h. G.M.T. in the form :—

Apparent time — mean time = ± E.

The value of E at any given instant must be interpolated. The Almanac also gives the instant of Transit of the true sun over the meridian of Greenwich in G.M.T.

To Convert L.M.T. to its Equivalent L.A.T.—*Rule.*—Interpolate the value of E at the given instant and add it algebraically, to the given L.M.T.

Example 13.—In longitude 45° 30' E. at 9 h. 45 m. 15 s. a.m., mean time, June 10th, 1936, find the corresponding local apparent time.

		D.	H.	M.	S.
Here, given	L.M.T. =	10	9	45	15
	East long. =—		3	2	0
	Corresponding G.M.T. =	10	6	43	15
		= June 10·28003 d.			

From N.A. 1936.	DATE.	E.		VAR.
		M.	S.	S.
	June 9th			− 11·59
	„ 10th	+ 0	51·49	− 11·85
	„ 11th	+ 0	39·64	− 12·08
	„ 12th			

$$n \, \Delta'_{\frac{1}{2}} = 0{\cdot}28003 \times - 11{\cdot}85 \qquad = - 3{\cdot}32$$

$$B'' \, (\Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}}) = - 0{\cdot}0504 \times - 0{\cdot}49 \qquad = + 0{\cdot}02$$

Correction to E = − 3·30

		M.	S.
± E. = (App. − mean) at 0 h. G.M.T.	= + 0	51·49	
	Correction = − 0	3·30	

		H.	M.	S.
E. at given instant	= +	0	48·19
Given L.M.T.	= June 10 d.	9	45	15·00
Required L.A.T.	= June 10 d.	9	46	03·19

To Convert L.A.T. into its Equivalent L.M.T.—Here we may work from the equation, L.A.T. + E = L.M.T., or since the N.A. now gives the value of the G.M.T. at apparent noon we can solve the problem by aid of the equation

L.M.T. = L.A.T. + G.M.T. of transit at Greenwich, corrected for G.M.T. instant ± 12 h.

Example 14.—Find the L.M.T. corresponding to 4 h. 15 m. 30 s. p.m. L.A.T. in longitude 75° W. on Nov. 18th, 1936.

First Method.		H.	M.	S.
Given	L.A.T. = Nov. 18 d.	16	15	30
	Long. W.= +	5	0	0
	G.A.T. = Nov. 18 d.	21	15	30
	E at 0 h. G.M.T. = —		14	52·26
M.T. interval from 0 h., G.M.T., Nov. 18		21	00	37·74
	= Nov. 18·87544 d.			

From N.A., 1936.

DATE	E.		VAR.
	M.	S.	S.
Nov. 17			
,, 18	+ 14	52·26	− 12·02
,, 19	+ 14	39·41	− 12·85
,, 20			− 13·65

$n\,\Delta'_{\frac{1}{2}} = 0.87544 \times -12.85 \qquad = -\qquad 11.25$

$B''\,(\Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}}) = -0.0273 \times -1.63 = +\qquad 0.04$

Correction to E = −   11·21

	M.	S.
E at Nov. 18 d. 0 h. = +	14	52·26
Correction = −		11·21
E at Nov. 18·87544 d. = +	14	41·05

		H.	M.	S.
Given	L.A.T. = Nov. 18 d.	16	15	30
	E = —		14	41·05
	Required L.M.T. =	16	00	48·95

Second Method.

Here, L.M.T. = L.A.T. + corrected G.M.T. at G.A.N. — 12 h.

From above,   G.A.T. = Nov. 18 d. 21 h. 15 m. 30 s.

           = ,,   18 d. 12 h. + 9 h. 15·5 m.

           = ,,   18 d. 12 h. + 0·38577 d.

From N.A., 1936.

DATE.	G.M.T. AT G.A.N.			VAR.
	H.	M.	S.	S.
Nov. 17				+ 12·43
,, 18	11	45	13·93	+ 13·25
,, 19	11	45	27·18	+ 14·05
,, 20				

$n \Delta'_{\frac{1}{2}} = 0.38577 \times 13.25$ = + 5·11

$B'' (\Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}}) = -0.059 \times +1.62$ = — 0·09

                    Correction = +5·02

		H.	M.	S.
G.M.T. at G.A.N., Nov. 18 d. 12 h.	=	11	45	13·93
Correction	=+			5·02
G.M.T. at G.A.T., 18 d. 21 h. 15 m. 30 s.,	=	11	45	18·95
Given L.A.T.	=	16	15	30·00
Sum — 12 h. = required L.M.T.,	=	16	00	48·95

To Determine the Sun's Right Ascension and Declination at any Instant of Local Time.—First obtain the corresponding G.M.T. or G.A.T., as the case may be. If the former, interpolate the required value from those given for 0 h. G.M.T.; if the latter, interpolate from the values given for transit at Greenwich, which are for G.A.T. + 12 h.

Example 15.—From the following data determine the R.A. of the true sun: W.H.A. 5 h. 30 m. 15 s.; date Sept. 1st, 1936; longitude 75° 30′ E.

		H.	M.	S.
Given	W.H.A. =	5	30	15
	Add 12 h.	12	0	0
	L.A.T. = Sept. 1 d.	17	30	15
	Long. E. =	− 5	2	0
	G.A.T. = Sept. 1 d.	12	28	15

$$= \text{Sept. 1 d. 12 h.} + 0.01961 \text{ d.}$$
$$B'' = - 0.0048.$$

From N.A., 1936.

R.A.

DATE.	H.	M.	S.	VAR.
				S.
Aug. 31 d. 12 h.				
Sept. 1 „ „	10	41	49·40	217·67
„ 2 „ „	10	45	26·77	217·37
„ 3 „ „				217·08

		H.	M.	S.
R.A. at Sept. 1 d. 12 h.	=	10	41	49·40
$n \Delta'_{\frac{1}{2}} = 0.01961 \times 217.37$	= +			4·263
$B'' (\Delta'_{\frac{3}{2}} - \Delta'_{-\frac{1}{2}}) = - 0.0048 \times - 0.59$	= +			0·003
R.A. at G.A.T. Sept. 1 d. 12 h. 28 m. 15 s.	=	10	41	53·67

Example 16.—Find the R.A. of the true sun at 10.30 a.m. on Aug. 15th, 1936, in longitude 45° 30′ W.

		D.	H.	M.
Given	L.M.T. = Aug.	15	10	30
	Long. W. =	+	3	2
	Corresponding G.M.T. = Aug.	15	13	32

$$= \text{Aug. } 15.56389 \text{ d.}$$
$$B'' = - 0.0615.$$

From N.A., 1936.

R.A. AT 0 H.

DATE.	H.	M.	S.	VAR.
				S.
Aug. 14				225·45
„ 15	9	37	21·24	224·92
„ 16	9	41	06·16	224·39
„ 17				

		H.	M.	S.
R.A. Aug. 15 d. 0 h.	=	9	37	21·24
$n \Delta'_{\frac{1}{2}} = 0.56389 \times 224.92$	= +		2	6·83
$B'' (\Delta'_{\frac{3}{2}} - \Delta'_{-\frac{1}{2}}) = - 0.0615 \times - 1.06$	= +			0·06
R.A. at G.M.T. Aug. 15 d. 13 h. 32 m.	=	9	39	28·13

Example 17.—Find the declination of the sun at 10.30 a.m., Aug. 16th, 1936, in longitude 25° 30′ E.

		D.	H.	M.
Given	L.M.T. = 1936, Aug.	16	10	30
	Long. E =−		1	42
	Corresponding G.M.T. =	16	8	48
	=	16·36667 d.		

From N.A., at 0 h. G.M.T.

DATE.	DEC.	VAR.
Aug. 15		− 1125·6″
„ 16	+ 13° 52′ 39·5″	− 1138·9
„ 17	+ 13° 33′ 40·6″	− 1151·8
„ 18		

$n \, \Delta'_{\frac{1}{2}} = 0 \cdot 36667 \times - 1138 \cdot 9$ $=- \ 417 \cdot 60''$
$B'' \, (\Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{4}}) =- \ 0 \cdot 058 \times -26 \cdot 2$ $=+ \quad 1 \cdot 52$

$$\text{Corrn.} \ =- \ 416 \cdot 08''$$
$$=-6' \ 56 \cdot 08''$$

Declination at 0 h. G.M.T., Aug. 16	=	13° 52′ 39·5″
Corrn. =	−	6 56·1
Required declination	=+	13 45 43·4

Example 18.—If the given time in Example 17 be apparent time, find the declination.

		D.	H.	M.
Here, given L.A.T. = 1936, Aug.		16	10	30
	Long. E. =−		1	42
	Corresponding G.A.T. =	16	8	48
	= Aug. 15 d. 12 h. + 20 h. 48 m.			
	= Aug. 15 d. 12 h. + 0·86667 d.			

From N.A., Aug. 15 d. 12 h.

DATE.	DEC.	VAR.
Aug. 14		− 1118·8″
„ 15	+ 14° 02′ 00·5″	− 1132·1
„ 16	+ 13° 43′ 08·4″	− 1145·2
„ 17		

$$n \, \Delta'_{\frac{1}{2}} = 0.86667 \times - 1132.1 \qquad = - \qquad 981.16''$$
$$B'' (\Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}}) = - 0.0289 \times - 26.4 \qquad = + \qquad 0.76''$$

$$\underline{\qquad - \qquad 980.40''}$$
$$= - \qquad 16' \quad 20.4''$$

Dec. Aug. 15 d. 12 h. $\qquad = +14° \quad 02' \quad 00.5''$

$\qquad$ Corrn. $= - \qquad 16 \quad 20.4$

$\qquad$ Required dec. $\quad +13 \quad 45 \quad 40.1$

Example 19.—Find the declination of the sun at apparent noon on Oct. 23rd, 1936, in longitude 60° 30′ W.

$\qquad$ Given long. W. $= + 4$ h. 2 m.
$\qquad \qquad = 0.16806$ d.

$\qquad \qquad$ 1936, Oct. : at transit at Greenwich.

DATE.	DEC.	VAR.
Oct. 22		
,, 23	− 11° 26′ 17.4″	− 1267.3″
,, 24	− 11° 47′ 14.3″	− 1256.9
,, 25		− 1246.0

Dec. Oct. 23 at G.A.N. $\qquad = -11° \quad 26' \quad 17.4''$
$n \, \Delta'_{\frac{1}{2}} = 0.16806 \times - 1256.9 \qquad = - \qquad 3 \quad 31.2$
$B'' (\Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}}) = - 0.035 \times + 21.3 \qquad = - \qquad 0.7$

$\qquad$ Required dec. $= -11° \quad 29' \quad 49.3''$

Semi-diameter—Sidereal Time of Semi-diameter passing Meridian.—In taking observations on stars, the centre of the star is observed to, as all stars only appear as points of light ; but, in taking observations to the sun, readings to its centre cannot be made with sufficient accuracy, as the sun's diameter is considerable, subtending nearly half a degree. Consequently, it is usual in reading altitudes to observe either the upper or lower edge of the sun's disc. These edges are called the upper and lower limbs respectively. If one limb only is observed, in order to obtain the true altitude, the angle subtended by the semi-diameter of the sun must be added to, or deducted from, the observed altitude, according as the lower or upper limb is observed.

It should be noted that the upper limb appears at the bottom when the sun is viewed in an inverting telescope.

When using a sextant, the diameter of the sun's disc may be obtained with the instrument, one-half of the result being applied to the observed altitude.

The amount of the semi-diameter is given for every day of the year in the early pages of the *Nautical Almanac.*

When observing the instant of the sun's transit over the meridian, the mean of the observed times of contact of the leading and following limbs with the vertical wire of the telescope is taken as the instant of transit of the sun's centre ; but, if one or other of the observations cannot be made, owing to the sun being obscured by driving clouds, or other cause, a knowledge of the time of transit of the semi-diameter will enable the observer to compute the instant of transit. The " sidereal time of the semi-diameter passing the meridian " is given daily for G.A.N. in the *Nautical Almanac.*

Parallax.—As we have observed (*vide* p. 538), the altitude of the sun or moon when measured on the earth's surface is not the same as it would be if measured from the earth's centre. This error, which is caused by the comparative proximity of the sun or moon, is called the sun's or moon's parallax in altitude. It may be defined as the angle subtended at the centre of the sun or moon by the line joining the place of observation to the earth's centre.

In Fig. 262, let A be the point of observation, O the earth's centre, C and D the position of the sun when on the horizon and in an elevated position respectively. To an observer at A, the elevation of the sun when at C would be zero, but at the earth's centre its elevation would be equal to the angle C O B, which is equal to the angle O C A. This angle is called the sun's (or moon's) *horizontal parallax,* and for the sun its value is practically constant, since the variation in the distance O C is comparatively small ; but in the case of the moon the variation is large, and the value of its horizontal parallax varies considerably. The moon's horizontal parallax is given in the *Nautical Almanac* on the pages headed " MOON."

It is evident from Fig. 262 that the sun's horizontal parallax (α) is given by the equation,

$$\text{Sin } \alpha = \frac{\text{A O}}{\text{O C}} = \frac{3,958 \text{ miles}}{93,000,000 \text{ miles}},$$

and from this $\alpha = 8 \cdot 8''.$

Again, the true altitude of the sun when at D

$$= \angle\, D\, O\, B$$
$$= \angle\, D\, E\, C = \angle\, D\, A\, E + \angle\, A\, D\, E$$
$$= \text{observed altitude} + \text{parallax at D.}$$

From Fig. 262 we see that

$$\frac{\operatorname{Sin} \angle\, O\, D\, A}{\operatorname{Sin} (90° + \beta)} = \frac{O\, A}{O\, D} = \frac{O\, A}{O\, C},$$

hence

$$\operatorname{Sin} \angle\, O\, D\, A = \frac{O\, A}{O\, C} \cos \beta.$$

Since $\angle\, O\, D\, A$ is very small, we have

$$\text{parallax at D} = \text{horizontal parallax} \times \cos \boldsymbol{\beta}$$
$$= 8 \cdot 8'' \cos \boldsymbol{\beta}.$$
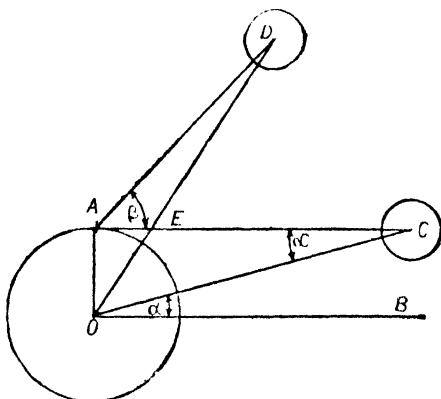
Fig. 262.

This applies to the sun only. In the case of the moon the value of sin α (*i.e.*, the moon's horizontal parallax) must be obtained for the given date and interpolated for the time of observation from the values given in the N.A.

The correction for parallax is always positive.

Refraction is the error in altitude due to the bending of the rays of light caused by their passage through layers of air of varying density. Its effect is to make objects appear higher than they really are, hence the correction for refraction to be applied to observed altitudes is always negative.

Under ordinary conditions of barometric pressure and temperature, refraction may be taken equal to $57''$ cot β where β

is the observed altitude. For more accurate work, or if the observations are made in extremes of pressure and temperature, the amount of the refraction may be obtained from the refraction tables given in " Chambers' Mathematical Tables," or from the tables given in Appendix A of this book.

Example 20.—In an observation of the sun, the altitude of the lower limb at a certain instant was found to be 30° 34′ 15″, barometer 29·15 inches, thermometer 40° F. If the date of the observation was April 18th, 1916, find the true altitude.

	Observed altitude =	30° 34′ 15″
From N.A.,	semi-diameter, April 18th =	0° 15′ 56·9″
	Sum =	30° 50′ 11·9″
	Parallax, 8·8″ cos 30° 34′ =+	0° 0′ 7·58″
		30° 50′ 19·48″

By Chambers' Tables—

Correction for alt. 30° 34′, temp. 40° F. =+ 3″
Correction for alt. 30° 34′, bar. 29·15 =− 1·5″

Correction for both =+	1·5″	
Mean refraction =1′	36·5″	
True refraction =1′ 38″	=−0°	1′ 38·00″
True altitude =	30° 48′ 41·48″	

Dip.—When taking observations of the sun with the sextant at sea, it is customary to read the sun's altitude from the sensible horizon. This introduces an error, which is termed " the dip," being the angle of depression from the observer's eye to the sensible horizon. Its amount depends on the height of the observer's eye above mean sea level, and is usually obtained from tables such as that given in " Chambers' Mathematical Tables." The correction for dip is always negative.

EXAMPLES.

1. The longitude of Adelaide is 9 h. 14 m. 20 s. E., and of Quebec 4 h. 44 m. 49 s. W. Express these angles in degrees, minutes, and seconds.   (*Ans.* 138° 35′ E. ; 71° 12′ 15″ W.)

2. The following table gives the approximate mean time to be added to, or subtracted from, Greenwich mean time to give the mean time at the places named. Determine their approximate longitudes east or west of Greenwich.

Place.	A.M. (Subtract).		P.M. (Add).	
	H.	M.	H.	M.
(a) Dublin, ...	0	25	..	..
(b) Newfoundland, ...	3	31	..	..
(c) Vancouver's Isle, ...	8	14	..	..
(d) Tokyo, ...	..	..	9	19
(e) Bombay, ...	..	..	4	51
(f) Petrograd, ...	..	..	2	1

(*Ans.* (a) 6° 15′ W. ; (b) 52° 45′ W. ; (c) 123° 30′ W. ; (d) 139° 45′ E. ; (e) 72° 45′ E. ; and (f) 30° 15′ E.)

3. Find the Greenwich mean time corresponding to 10 h. 25 m. 8 s. a.m., June 16th, in Ottawa (longitude 75° 43′ W.).

(*Ans.* 15 h. 28 m., June 16th.)

4. Find the R.A. of the mean sun at 8·40 a.m., June 4th, 1933, in Melbourne. Longitude of Melbourne, 9 h. 39 m. 54 s. E. Sidereal time at G.M.M., June 3rd, 1933, 16 h. 43 m. 54·277 s.

(*Ans.* 4 h. 47 m. 46 s.)

5. From the data given in Example 4, find the local sidereal time at the given instant. (*Ans.* 1 h. 27 m. 46 s.)

6. Find the R.A. of the meridian of Moscow at 10 h. 40 m. 15 s. a.m. on Oct. 12th, 1933. Longitude of Moscow, 2 h. 30 m. 17 s. E. Sidereal time at G.M.M., Oct. 12th, 1933, 01 h. 20 m. 28·141 s. (*Ans.* 12 h. 02 m. 3·6 s.)

7. Using the method given on p. 549, find the L.S.T. at 5 h. 24 m. 18 s. p.m. on Nov. 20th, 1933, in Naples. Longitude of Naples, 0 h. 57 m. 2 s. E. G.M.T. at sidereal noon, Nov. 19th, 1933, 20 h. 06 m. 24·57 s. (*Ans.* 21 h. 21 m. 13·9 s.)

8. Find the local mean time corresponding to 13 h. 5 m. 18·5 s. sidereal time at Liverpool on Dec. 21st, 1933. Longitude of Liverpool, 0 h. 12 m. 17·3 s. W. Sidereal time of G.M.M., Dec. 21st, 1933, 05 h. 56 m. 27·129 s. (*Ans.* 7 h. 07 m. 39·1 s., Dec. 21st.)

9. Determine the hour angle of the star β Argus at 3.30 a.m., Jan. 10th, 1933, in Wellington, N.Z., longitude 11 h. 39 m. 4·27 s. E. Right ascension of β Argus, 9 h. 12 m. 30·75 s. S.T. at G.M.M., Jan. 9th, 1933, 07 h. 12 m. 18·712 s. (*Ans.* 1 h. 32 m. 24·2 s.)

10. Find the local mean time from the given data :—Longitude of place, 45° 30′ 15″ E. ; star observed γ Geminorum, R.A. of star, 6 h. 33 m. 52·13 s. ; date, March 3rd, 1933 ; westerly hour angle of star at given instant, 4 h. 3 m. 18 s. ; S.T. at G.M.M., 10 h. 41 m. 16·19 s.   (*Ans.* 23 h. 52 m. 28·5 s.)

11. Find the local apparent time corresponding to 8 h. 3 m. 15 s. p.m., June 16, 1936, in longitude 75° 30′ 45″ E.

Data from N.A.

E. AT 0 H.

DATE.	G.M.T.		VARIATION.	
1936.	M.	S.	S.	$n = 0.62583$ d.
June 15	− 0	09·90		B″= −0·0585
,, 16	− 0	22·71	− 12·81	
,, 17	− 0	35·64	− 12·93	
,, 18	− 0	48·66	− 13·02	

(*Ans.* 20 h. 2 m. 44·2 s.)

12. Find the local mean time corresponding to 6 h. 30 m. 18 s. apparent time in longitude 60° 40′ E., on Aug. 10, 1936.

Data from N.A.

E. AT 0 H.

DATE.	G.M.T.		VARIATION.	
1936.	M.	S.	S.	$n = 0.10252$ d.
Aug. 9	− 5	26·59		B″= −0·023.
,, 10	− 5	18·20	+ 8·39	
,, 11	− 5	09·26	+ 8·94	
,, 12	− 4	59·76	+ 9·50	

(*Ans.* 6 h. 35 m. 35·3 s.)

13. Find the declination of the sun at 3.40 p.m., Sept. 8th, 1936, in longitude 40° 15′ W.

Data from N.A.

DEC. AT 0 H.

DATE.	G.M.T.			VARIATION.	
1936.	°	′	″	″	$n = 0.76458$ d.
Sept. 7	+ 6	13	43·9		B″=−0·045
,, 8	+ 5	51	15·0	− 1348·9	
,, 9	+ 5	28	40·1	− 1354·9	
,, 10	+ 5	05	59·6	− 1360·5	

(*Ans.* + 5° 33′ 59·59″).

CHAPTER XVII.

AZIMUTH, LATITUDE, TIME, AND LONGITUDE. PROJECTION OF MAPS.

IN surveys of an extensive character, in route surveys, and in setting out boundaries between contiguous states, astronomical observations for azimuth, latitude, and time are necessary, may be for the purpose of checking the work already done, for fixing the positions of important stations, or for setting out a cardinal direction.

As the instruments used by the surveyor or traveller are essentially of a portable character, the results arrived at, from astronomical observations, cannot be of that high order of accuracy which is attained in permanent observatories where every refinement tending to accuracy is at the disposal of the observer, who is also specially trained for the work. The astronomical work that a surveyor is called upon to do may be a mere (but by no means unimportant) incident in his many duties, and consequently under these conditions extreme accuracy is not to be expected.

For many purposes, the determination of azimuth, latitude, and time may be effected with a sufficient degree of accuracy by an observer whose knowledge of astronomy is very meagre ; a fair but correct knowledge of the subject is, however, necessary for an intelligent understanding of the processes involved.

Corrections of Observations.—As astronomical observations are in the main based on the measurement of space in arc and time, which involves the determination of the altitudes of heavenly bodies and their times of transit over the meridian of a place, or when arriving at some special position in the stellar sphere favourable for observation, it is necessary that the observer should possess considerable practical skill in the manipulation of his instrument, and be able to correct, or clear, his observations from all sources of error. The following are the corrections

to be applied to observed altitudes before they can be used in computation :—

(1) Instrumental and personal errors.

(2) Index error when using a sextant ; collimation and level error, when using a theodolite.

(3) Reduction of the double altitude, when using a sextant and artificial horizon.

(4) Correction for " dip," when sighting to the sensible horizon.

(5) Refraction, to be applied in all cases.

(6) Semi-diameter,   ⎰ when observing the sun or moon
    Parallax in altitude, ⎱   in altitude.

Correction (1) depends on imperfections in the construction of the instrument, and also on its manipulation by the observer. This correction can only be arrived at by the observer after many trials with the same instrument.

Correction (2) has been dealt with on pp. 245 and 257, the remaining corrections and the methods of applying them are explained in the preceding chapter.

The necessary equipment of a surveyor engaged in astronomical work consists of (1) a good theodolite ; (2) a sextant and an artificial horizon ; (3) a good chronometer of known time rate ; (4) a barometer (either mercurial or aneroid) ; (5) one or more thermometers ; (6) the Nautical Almanac for the year ; (7) seven-figure mathematical tables ; (8) refraction and dip tables ; (9) set of star charts ; and (10) maps or charts of the locality.

Azimuth and Bearing—Convergence.—The azimuth of a line A B may be defined as the angle between two great circles of the earth, one of the great circles being the meridian through A and the other containing the line A B. If the meridians at A and B were parallel (as they would be if A and B were on the equator), the azimuth of B from A would be equal or supplementary to the azimuth of A from B ; but as the meridians are not parallel, since they all converge on the earth's poles, the azimuth of B from A is not equal to, or supplementary to, that of A from B. In taking bearings (magnetic or otherwise), we assume that the plane of reference at each station is parallel to some standard plane—preferably near the middle of the area surveyed—and consequently the fore- and back-bearings of each line are supplementary angles. If a surveyor was engaged to set-out the course of a railway 10 miles long in a mean latitude

of (say) 53°, the direction being approximately east and west, he would first determine the azimuth of the first course astronomically, thereafter taking this angle as a bearing for fixing the direction of his other lines by measured angles. If from these angles he computes the bearing of the last line, and, to check his work, obtains the azimuth of this line, he would find the computed bearing and azimuth did not agree, the difference being about 11·5 minutes, in this case. This apparent error (it is not a real error) is due to the convergence of the meridians, and is termed the " convergence."

To determine a general equation for convergence, we proceed as follows :—Let P Q (Fig. 263) be the meridian passing through
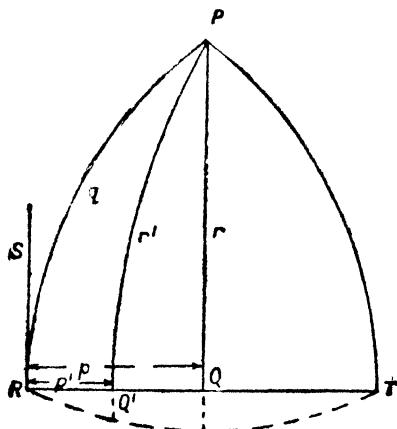
Fig. 263.

the central station Q in the parallel R T ; P the earth's pole ; Q R a line at right angles to the meridian P Q ; R P and P T the meridians through R and T ; and R S a line at right angles to R Q. Then the convergence for the departure R T is the difference between the angle Q R S and the spherical angle Q R P, and = 90° — R. In the right-angled spherical triangle R Q P, we have

$$\tan R = \frac{\tan r}{\sin p} \text{ (Eq. 8, p. 535)},$$

but $\qquad r = 90° —$ latitude of Q,

∴ $\qquad \tan r = \cot$ latitude of Q,

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and   the convergence $= 90° - $ R,

.˙.   tan R $=$ cot convergence,

consequently cot convergence $= \dfrac{\text{cot latitude}}{\sin p}$.

This may be written :—

          tan convergence $=$ tan latitude $\times \sin p$.   (1)

The calculation of p involves the conversion of length into angle, but if R Q be not more than 60 miles, the value of the convergence and likewise that of p will be small, and for small angles we may take the tangents, sines, and circular measure as equal. Hence, when p is small,

$$\sin p = \frac{\text{departure}}{\text{radius of the earth}},$$

also, tan convergence $=$ circular measure of convergence
                          $=$ convergence in minutes $\times$ tan 1′.

Making these substitutions in equation (1), we get :—

Convergence in minutes $\times$ tan 1′ $= \dfrac{\text{tan latitude} \times \text{departure}}{\text{radius of earth}}$,

or,   convergence in minutes $= \dfrac{\text{tan latitude} \times \text{departure}}{\text{radius of earth} \times \text{tan } 1'}$.

On adapting this equation to logarithmic computation, we obtain the equation,

Log convergence in minutes $=$ log K $+$ log tan latitude
                                                    $+$ log departure,

where K $= \dfrac{1}{\text{radius of earth} \times \text{tan } 1'}$.

If the departure is in nautical miles, and ρ be the radius of the earth, then since

$$1 \text{ nautical mile} = \frac{\pi \times \rho}{180 \times 60},$$

and   tan 1′ $= \dfrac{\pi}{180 \times 60}$, log K $= 0$.

If the departure is in statute miles,

$$\log K = \frac{5,280}{20,889,000 \times \tan 1'}$$

$$= \bar{1}\cdot 93899.$$

Similarly, if the departure is in Gunter chains,

$$\log K = \bar{2}\cdot 03589,$$

and if the departure is in feet,

$$\log K = \bar{4}\cdot 21635.$$

Example 1.—Find the convergence for a departure of 25 statute miles in a mean latitude of 62° 30′.

Here,   constant log = $\bar{1}\cdot 93899$
        Log tan 62° 30′ = 10·28352
            Log 25 = 1·39794
        ──────────
    Log convergence in minutes = 1·62045
∴   convergence = 41′ 43·8″, or about 1·67′ per
                    mile.

Example 2.—Determine the convergence in a traverse having a departure of 58,000 feet in a mean latitude of 15°.

In this case,   constant log = $\bar{4}\cdot 21635$
        Log tan 15° = 9·42805
        Log 58,000 = 4·76343
        ──────────
    Log convergence in minutes = 0·40783
and   convergence = 2′ 33·48″, or 0·233′ per mile.

North and South Lines not Parallel.—The equation for convergence shows that its amount varies both with the departure and the latitude in which the departure is situated. Examples (1) and (2) show that the convergence per mile in latitude 62° 30′ is 1·67′, and, in latitude 15°, 0·233′ per mile. Now, since perpendiculars to the same line are parallel, and convergence is the angle between a perpendicular to the parallel of latitude at any place and the meridian at the place, it follows that north and south lines at different places are not parallel.

Setting-out a Parallel of Latitude.—Suppose a surveyor desires to set-out a parallel of latitude between two places R and T (Fig. 263), whose distance apart is 364,602 feet in latitude 50°. (The length selected subtends an angle of 1° at the earth's centre, assuming the earth to be a perfect sphere of radius 20,890,172 feet.)

In setting-out the parallel of latitude, the surveyor may proceed to set-out very short lines, each having an azimuth of 90°, and continue in this way until he has chained off the given distance. The line thus set out would appear curved both on the plan and the ground. As this would be a very laborious process, involving the determination of the meridian at the end of each short length of his course, the surveyor may proceed to compute the azimuth of R T at R and set-out the line as straight. If he does this, and if after chaining off 364,062 feet he again determines the azimuth of the line, the result would agree with that obtained at its commencement, thus showing he is in latitude 50°; but, if he stopped at any other part of the line and determined the azimuth, his result would be different to that previously obtained. He would, in fact, be in a higher latitude as the line set-out would be an arc of a great circle, intersecting latitude 50°, at its beginning and end. To set-out the parallel of latitude, the pegs marking out the line would have to be moved south by variable amounts, which may be calculated as follows :—Let R T (Fig. 263) be the line set-out. Bisect R T at Q, and through Q draw the meridian Q P. Then the triangle R P T is an isosceles spherical triangle, since the arc R P = the arc P T = colatitude of 50° = 40°. Also, the triangle P R Q is right-angled at Q, and of this triangle we also know the side R P, and that R Q = ½ R T = 30'.

Now, $\cos R = \cot q \cdot \tan p$ (Eq. 13, p. 535)

$$= \cot 40° \cdot \tan 30'$$

$$\log \cot 40° = 10\cdot0761865$$

$$\log \tan 30' = 7\cdot9408584$$

$$\log \cos R = 8\cdot0170449$$

and $R = 89° \ 24' \ 15''$

          = the azimuth of R T at both R and T.

The latitude of Q may also be found from the same right-angled triangle. Thus :—

$$\cos r = \frac{\cos q}{\cos p} \text{ (Eq. 3, p. 535)}$$

$$= \frac{\cos 40°}{\cos 30'}$$

$$\log \cos 40° = 9 \cdot 8842540$$
$$\log \cos 30' = 9 \cdot 9999835$$

$$\log \cos r = 9 \cdot 8842705$$

and $\qquad r = 39° \, 59' \, 50 \cdot 6''$,

∴ $\qquad$ latitude of Q $= 50° \, 00' \, 09 \cdot 4''$.

The error in latitude at Q is thus $9 \cdot 4''$, and it remains to find the length of the arc on the meridian which subtends $9 \cdot 4''$. For this we use the proportion

Required arc : $\pi \times 20,889,000$ :: $9 \cdot 4$: $180 \times 60 \times 60$,

which gives the offset at Q equal to 952 feet, and this offset must be set off at Q south of the line to give the middle point of the parallel of latitude through R and T.

The offset at any other point, such as Q' (Fig. 263) is obtained by calculating the latitude of the point from the triangle R P Q', and from this the error in latitude is known. This error, in arc, converted into feet, gives the required offset. In the triangle R P Q', the angle P R Q' and the two sides q and p' are known. Hence we use the formula,

$\qquad \cos c = \cos a \, . \, \cos b + \sin a \, . \, \sin b \, . \, \cos C$ (Eq. 3, p. 535),

or, $\qquad \cos r' = \cos q \, . \, \cos p' + \sin q \, . \, \sin p' \, . \, \cos R$,

which gives r', and $90° - r'$ is the latitude of Q'.

Determination of Azimuth.—It is obvious from what we have already seen that the problem of finding the azimuth of a line may be at once solved by direct measurement when the direction of the meridian through a station on the line is known. The mode of procedure in determining the direction of the meridian at any place will depend on the accuracy required, the instruments available, and the method adopted. In the case of a chain survey covering a comparatively small area, and where the sole object is to show the approximate direction of the cardinal points on the plan, the two approximate methods given below will generally be sufficient.

Approximate Methods.

(a) **Method of Shadows.**—This method is based on the fact that (neglecting the change of declination in the interval), when the sun is at equal distances east and west of the meridian, it is at the same altitude, and consequently the shadows cast by an object, at these times, are of the same length. In applying this method, a rod (Fig. 264) is planted vertically in the ground, and the ring of the tape is attached to its foot by a piece of string. At some convenient time before noon, by allowing the shadow of the rod to fall on the tape, which is stretched horizontally, the length of the shadow is measured and the position of its extremity is marked with a peg (a') driven into the ground. In the same way other pegs (b', c', and d', Fig. 264) are placed in
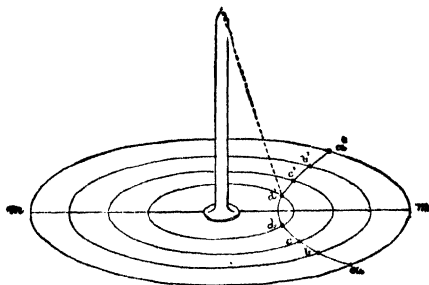
Fig. 264.

position when the sun is nearer the meridian. After the sun has passed the meridian by following the shadow as it lengthens, with the tape, the positions of the corresponding pegs d, c, b, and a are obtained. The line bisecting the angle formed by joining any corresponding pair of pegs to the foot of the rod is in the meridian through that point. Although one pair of pegs is sufficient to give the direction of the meridian, several pairs should be used, otherwise after passing the meridian the sun may be obscured at the critical moment and the observation be lost.

If the bisectors of the chords joining corresponding pegs in pairs are not in the same straight line, the mean direction must be taken as that of the meridian.

This method is correct only at the time of the solstices (June 21st, Dec. 22nd) ; at other times it gives a fair approximation.

(b) By Observation on the Pole Star and the Star ε in Ursæ Majoris.—In the northern hemisphere the direction of the meridian may be found by observing the instant at which the stars α (Polaris) Ursæ Minoris and Alioth (ε Ursæ Majoris) culminate. These stars culminate on opposite sides of the earth's axis at the same moment, and the vertical line passing through both stars at this instant is in the plane of the meridian (Fig. 265). The observation may be made by aid of a plumb line, the motion of the stars being followed by the eye until the plumb line covers both stars. At this instant the line joining the observer's eye and the plumb line is in the meridian.

The observation may also be made with a theodolite, by following the stars with the telescope, which will be pointing due north when, on rotating the telescope on its horizontal axis, the line of collimation bisects both stars. The pole star is exactly in the meridian about 29 minutes after being in the same vertical plane as Alioth.
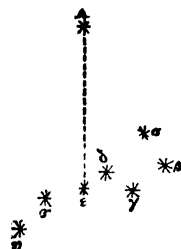
Fig. 265.

Exact Methods.—For the exact determination of azimuth the theodolite is the most satisfactory instrument, as its construction permits of both horizontal and vertical movements which are under precise control. In the following descriptions of determining azimuth, latitude, and time, we shall assume (unless otherwise stated) that this instrument is made use of.

As observations on stars must perforce be made at night, some form of illuminated referring object must be set up at the distant station; some provision must also be made for illuminating the cross wires, as described on p. 237.

A useful referring object is obtained by causing the light from a lamp to shine through a circular hole cut through a board, which is supported with the centre of the hole vertically over the distant station.

Determination of Azimuth.

(a) By the Method of Equal Altitudes of a Circum-polar Star.—1. **Assuming the Instrument is in Perfect Adjustment.**—The instrument is set up over the station at one end of the line, and the horizontal angle between the referring object at the other end and any circum-polar star is observed. The star selected should not be near either of its points of culmination.

The instrument is left undisturbed with the vertical circle clamped until the star is approaching the same altitude on the other side of the meridian; then, without altering the reading on the vertical circle, the telescope is directed towards the star, and, when it is in the field of view, the horizontal circle is clamped and the star is followed by aid of the tangent screw to the horizontal circle until its image is again bisected by the cross wires; the horizontal angle between the new direction of the star and the referring object is then noted. As the direction of the meridian is midway between the two positions of the star, the azimuth of the line is half the sum of the observed horizontal angles, if both positions of the star are on the same side of the line, but is half the difference if the two positions of the star are on opposite sides of the line. In Figs. 266 and 267, let A B be the line whose
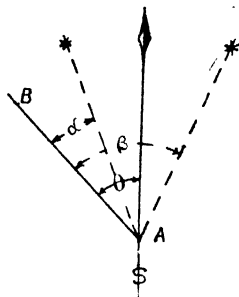
Fig. 266.   Fig. 267.
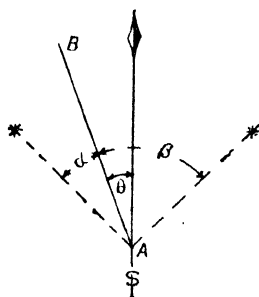
azimuth θ is required, and α and β the observed horizontal angles. When both positions of the star are on the same side of the line, as in Fig. 266,

$$\theta = \frac{\beta - \alpha}{2} + \alpha = \frac{\alpha + \beta}{2};$$

but in Fig. 267, where the two positions of the star are shown on opposite sides of the line,

$$\theta = \frac{\beta + \alpha}{2} - \alpha = \frac{\beta - \alpha}{2}.$$

This remark with reference to taking out the mean angle applies also to methods 2 and 3.

2. Assuming the Instrument is in Imperfect Adjustment.— In this case the instrument is set up as before, the referring object and the star are bisected in turn, the vertical circle being on (say) the left, and the horizontal and vertical circles are read as quickly as possible. The star is again immediately bisected with the vertical circle on the right, and the mean of the two horizontal angles is recorded as the angle between the line and the star at observation. The two vertical angles are also recorded and reserved for later use. These operations are repeated as many times as is thought desirable with the star on the same side of the meridian. After taking the last observation, the instrument is left undisturbed (with circle right and clamped at the last vertical reading) until the star approaches its corresponding position on the other side of the meridian, the horizontal circle is then unclamped and when the star is in the field of view of the telescope the horizontal circle is clamped, and the motion of the star is followed by turning the horizontal tangent screw, until the image of the star is again bisected by the cross wires. The horizontal angle is now observed, and the telescope is rapidly set to the corresponding vertical angle with circle left, and the star is followed and its image bisected as before. The instrument is then set with the telescope clamped at the vertical angle previously recorded for the next altitude of the star, and the operations are repeated (with circle right and circle left) for each of the remaining observations of the star in pairs. The azimuth of the line is obtained as in method 1, by taking the mean of the horizontal angles obtained from any pair of readings, but it is better to take the mean of the results given by the whole series of observations as the required azimuth.

The method of equal altitudes may be applied in observing any circumpolar star when at its points of elongation. In this case the star is followed with the telescope, both in azimuth and altitude, until it ceases to move further east (or west); when this occurs, several circle-left and circle-right observations of the star are made, the mean of the angles thus obtained being taken as the angle between the line and the star when at elongation. These observations are repeated when the star attains its corresponding position on the other side of the meridian. The mean of the position angles, as before, gives the required azimuth.

Although this last method gives the most accurate result, since the azimuth of the star changes very slowly at the points of elongation, thus giving time for several readings of the position angle, it is, however, seldom applicable with an ordinary theo-

dolite, as, in general, one of the observations must be made in daylight.

3. **By Equal Altitudes of the Sun.**—If the approximate latitude be known, the method of equal altitudes may be applied to the sun, face-right and face-left observations of the right and left-hand limbs of the sun being made in both the morning and afternoon observations. The readings should be made in adjoining (upper or lower) quadrants of the sighting webs. The time of each observation must be noted and the time elapsed between each pair of morning and afternoon observations obtained by difference. Several pairs of readings should be taken ; the result obtained from each pair of readings will, however, require correction, owing to the sun's change in declination. This correction is obtained by the following rule :—When the sun's declination is changing to the $\left\{ \begin{matrix} \text{south} \\ \text{north} \end{matrix} \right\}$ the direction of the meridian given by this method is too far to the $\left\{ \begin{matrix} \text{right} \\ \text{left} \end{matrix} \right\}$ by an amount given (in seconds) by the formula :—

$$\frac{\text{Change of sun's declination}}{2} \times \sec \text{ latitude} \times \operatorname{cosec} \tfrac{1}{2} \text{ angular}$$

motion of the sun between the observations.

The angular motion of the sun is obtained by multiplying the time interval in hours between a pair of observations by 15°. The rate and direction of the sun's change in declination is given daily in the early pages of the *Nautical Almanac,* and the rate multiplied by the time interval between the observations (in pairs) gives the total change to be used in the formula. The rule given above may be demonstrated as follows :—

Let $2\,\theta =$ mean change in sun's azimuth between observations at equal altitude,

$\Delta\,\theta =$ correction to this mean to reduce to true meridian,

$2\,\text{T} =$ time interval between observations,

$\delta =$ declination of sun at local apparent noon,

$\Delta\,\delta =$ increase in declination from the meridian to the west, and decrease to the east, observation,

$\alpha =$ altitude of sun at each observation,

$l =$ latitude of place.

Now, of the triangle having the pole (P), the zenith (Z), and the sun (S) at its angular points, we know the three sides and the angle P Z S. Hence in the formula

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A \text{ (Eq. 3, p.535)}$$

we have $a = 90 - (\delta \mp \Delta \delta)$; $b = 90 - a$; $c = 90 - l$;

$$A = (\theta + \Delta \theta)$$

at the first observation, and $(\theta - \Delta \theta)$ at the second. On substituting these values in the above equation, we get for the first observation :

$$\sin (\delta - \Delta \delta) = \sin a \cdot \sin l + \cos a \cdot \cos l \cdot \cos (\theta + \Delta \theta), \quad \textbf{(1)}$$

and for the second observation,

$$\sin (\delta + \Delta \delta) = \sin a \cdot \sin l + \cos a \cdot \cos l \cdot \cos (\theta - \Delta \theta). \quad \textbf{(2)}$$

Subtracting (1) from (2), we get,

$$\sin (\delta + \Delta \delta) - \sin (\delta - \Delta \delta)$$
$$= \cos a \cdot \cos l \cdot \{\cos (\theta - \Delta \theta) - \cos (\theta + \Delta \theta)\}.$$

On expanding the compound functions, and reducing, we obtain the equation,

$$2 \cos \delta \cdot \sin \Delta \delta = - 2 \cos a \cdot \cos l \cdot \sin \theta \cdot \sin \Delta \theta.$$

Consequently,

$$\sin \Delta \theta = - \frac{\cos \delta \cdot \sin \Delta \delta}{\cos a \cdot \cos l \sin \theta},$$

Since $\Delta \delta$ is very small, we may write this equation,

$$\Delta \theta = - \Delta \delta \cdot \frac{\cos \delta}{\cos a \cdot \cos l \cdot \sin \theta}$$

$$= - \Delta \delta \cdot \sec l \cdot \frac{\sin (90 - \delta)}{\sin (90 - a) \cdot \sin \theta},$$

$$= - \Delta \delta \cdot \sec l \cdot \frac{1}{\sin Z P S}, \text{ very nearly.}$$

But the angle Z P S is equal to half the angle turned through by the sun between observations,

$$\therefore \qquad \Delta\,\theta = -\,\Delta\,\delta \sec l \,.\, \operatorname{cosec} T \text{ (in arc)},$$

or, the required correction,

$$\Delta\,\theta = \mp \frac{\text{change in declination}}{2} \times \sec \text{ latitude} \times \operatorname{cosec} \text{ half the}$$

angle turned through by the sun between the observations.

When the sun's declination is decreasing the sign of the correction changes, and the positive sign must be used.

Example 3.—In latitude 53° 27′ 32″ N. on May 24th, 1936, the horizontal angles from the referring object to the sun when at equal altitudes were 232° 18′ 25″ and 322° 19′ 20″; time interval between the observations 4 h. 20 m. ; daily variation in sun's declination, May 23-24, 676·2″ ; May 24-25, 654·8″ ; find the azimuth of the line.

Note.—The given position angles being whole circle readings show that the line lies to the right of the observer.

Here,
$$\alpha = 232° \ 18′ \ 25″$$
$$\beta = 322° \ 19′ \ 20″$$
$$\overline{\alpha + \beta = 554° \ 37′ \ 45″}$$
$$\frac{\alpha + \beta}{2} = 277° \ 18′ \ 52·5″,$$

$\therefore$ approximate azimuth = S. 82° 41′ 7·5″ W.

To compute the required correction,

DATE.	VAR. AT G.A.N.	VAR. PER HOUR.
May 23	+ 676·2	
,, 24	+ 654·8	$= \dfrac{1331 \cdot 0}{48}$
,, 25	Sum = 1331·0	
		= 27·73″.

$$\tfrac{1}{2} \text{ change in declination} \quad = \tfrac{1}{2}\,(4\tfrac{1}{3} \times 27\cdot73)$$
$$= 60\cdot1″$$

$$\begin{aligned}
\log. \ 60\cdot1 &= \ 1\cdot77887 \\
\log \sec 53° \ 27′ &= 10\cdot22519 \\
\log \operatorname{cosec} \tfrac{1}{2}\,(4\tfrac{1}{3} \times 15°) &= \overline{10\cdot26978} \\
\log 187\cdot8″ &= \ 2\cdot27384 \\
\text{Corrn.} &= 3′ \ 07\cdot8″
\end{aligned}$$

too far to the right: but the line lies on the right of the observer:
therefore the correction is positive. Hence adding 3′ 07·8″ to
82° 41′ 7·5″ gives S. 82° 44′ 15·3″ W. as the required azimuth.

4. **By Observation of a Circum-polar Star when at a Point of
Elongation.**—In this case the declination of the star selected
and also the latitude of the place must be known.

The angle (β) between the line and the star when at a point
of elongation, east or west of the meridian, is obtained by circle-
right and circle-left observations in the usual way. The azimuth
(θ) of the star is then computed, and the azimuth (φ) of the line
is found from these angles as follows :—

(a) When the star lies between the line and the meridian
$$\varphi = \theta + \beta \;;$$

(b) when the line lies between the star and the meridian,
$$\varphi = \theta - \beta \;; \text{ and}$$

(c) when the star and line are on opposite sides of the meridian,
$$\varphi = \beta - \theta.$$

The relative positions of the star, line, and meridian should
be noted at the time of observation. A sketch will then show
at once whether the angles are to be added or subtracted.

By reference to Fig. 258, it will be observed that the spherical
triangle A N C, having its angular points at the zenith, the pole,
and the star, is a right-angled triangle, hence $\sin A = \dfrac{\sin a}{\sin c}$
(Eq. 1, p. 535).

Here, A = required azimuth of the star = θ,

$$a = \text{codeclination of star} = \frac{\pi}{2} - \text{declination,}$$

$$c = \text{colatitude of place} = \frac{\pi}{2} - \text{latitude.}$$

On substituting these values in the above equation, we get,

$$\sin \theta = \frac{\cos \text{declination}}{\cos \text{latitude}}.$$

To assist in finding the selected star, its altitude and hour
angle at the instant of elongation should be computed. The
instrument may then be set approximately to the former angle,

and from the latter the local mean time of the observation may be found, as shown in Example 5 below. To compute the altitude of the star. Let l = the latitude of the place; δ = the declination of the star selected; and a = the required altitude. In the right-angled triangle A N C (Fig. 258), A C is the side required, and $\cos b = \dfrac{\cos c}{\cos a}$ (Eq. 3, p. 535), but

$$b = \text{A C} = \left(\frac{\pi}{2} - a\right); \quad c = \text{A N} = \left(\frac{\pi}{2} - l\right);$$

$$a = \text{C N} = \left(\frac{\pi}{2} - \delta\right);$$

making these substitutions, we have

$$\cos\left(\frac{\pi}{2} - a\right) = \frac{\cos\left(\frac{\pi}{2} - l\right)}{\cos\left(\frac{\pi}{2} - \delta\right)},$$

or $\sin a = \sin l \cdot \operatorname{cosec} \delta.$ (Refraction will make a rather greater than this.)

To determine the hour angle of the star, we have to find the angle A N C in the same triangle. In this case we use the formula

$$\cos \text{B} = \frac{\tan a}{\tan c} \text{ (Eq. 12, p. 535),}$$

that is, $\cos \text{H} = \dfrac{\tan\left(\dfrac{\pi}{2} - \delta\right)}{\tan\left(\dfrac{\pi}{2} - l\right)} = \tan l \cdot \cot \delta,$

where H is the required hour angle.

The declination and R.A. of the star selected are obtained from the *Nautical Almanac*. If the longitude of the place is unknown, the approximate local mean time only can be determined. This is, however, sufficient for the present purpose, as this method of determining the azimuth of a line is independent of the time of elongation.

Example 4.—In latitude 54° 30′ N. on June 1st, 1936, the star *a* Ursæ Majoris was observed at its western elongation; the angle between the line and star 65° 18′ 42″, the star lying between the meridian and the line; declination of *a* Ursæ Majoris at the

given date 62° 05′ 51·3″ N. ; find the altitude of the star at elongation and the azimuth of the line.

Here, $\sin \alpha = \sin 54° 30′ 00″ \times \operatorname{cosec} 62° 05′ 51·4″$,
which gives $\alpha = 67° 06′ 13·0″$ = required altitude.

Also, $\sin \theta = \dfrac{\cos 62° 05′ 51″}{\cos 54° 30′ 00″}$,

and $\theta = 53° 41′ 37·1″$ = azimuth of star at elongation.

As the star lies between the meridian and the line, the azimuth of the line

$$\varphi = 53° 41′ 37·1″ + 65° 18′ 42″$$
$$= \text{S. } 60° 59′ 40·9″ \text{ W.}$$

Example 5.—Find the hour angle in Ex. 4, and the approximate local mean time at elongation.   R.A. of star at given date 10 h. 59 m. 51 s., S.T. at G.M.M., June 1st, 1936, 16 h. 37 m. 11 s.

Here, $\cos H = \tan 54° 30′ 00″ \times \cot 62° 05′ 51·3″$
and the required hour angle

		H.	M.	S.
$H = 42° 03′ 58·9″ =$		2	48	16
R.A. of star =		10	59	51
R.A. of meridian =		13	48	07
S.T. at G.M.M., June 1st, 1936 =	−16	37	11
Approximate L.M.T. at elongation in S.T. units,	=	21	10	56 June 1st.
Required L.M.T. =		21	07	27·8

5. By a Single Altitude of a Star, or the Sun.—For this purpose the star selected should be moving slowly in azimuth, hence the observation should be made when the star is on or near the prime vertical (*vide* p. 609)—*i.e.* when it is rising or setting.

The star at observation should not be too low, or the refraction will be great.   Where great accuracy is required, observations should be made on two stars, one being a rising and the other a setting star.

In making the observation, the angle between the star and the referring object is obtained by circle-right and circle-left observations, as already described, the mean of the results being used in computing the azimuth of the star and line.

If the sun be observed, the local mean time at observation must also be noted, and from this and the longitude of the place the declination of the sun at observation is computed. Four observations of the sun should be made, the image of the sun occupying successively the positions S^2, S^1, S, and S^3 (Fig. 268 or 269) between the cross wires, the angles for positions S^2 and S^1 being read with (say) circle-left, and for positions S and S^3 with circle-right. The time at each observation must be recorded. The mean of the four angles is taken as the altitude and position angle of the sun's centre, and the mean of the times as that at observation.

To find the azimuth of the sun or star at observation, it is necessary to determine the angle N A C (Fig. 258); the triangle
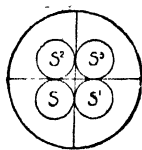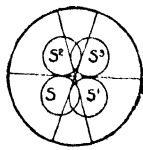
Fig. 268.   Fig. 269.

is, however, no longer right-angled at C. As we know the three sides of the triangle, we use the formula

$$\tan^2 \tfrac{1}{2} A = \frac{\sin (s - c) \cdot \sin (s - b)}{\sin s \cdot \sin (s - a)} \quad \text{(Eq. VII., p. 532).}$$

Here, $s = \dfrac{90 - \delta + 90 - a + 90 - l}{2}$

$= 90 - \dfrac{(a + l) + 90 - \delta}{2}$

$= 90 - \dfrac{(a + l - p)}{2},$

where p is the polar distance.

Let $\dfrac{a + l + p}{2} = z$, then $s = 90 - (z - p)$; $(s - a) = 90 - z$; $(s - b) = (z - l)$; and $(s - c) = (z - a)$.

Substituting these values in the above equation we get,

$$\tan \tfrac{1}{2} A = \sqrt{\sec z \cdot \sec (z - p) \cdot \sin (z - a) \cdot \sin (z - l)}.$$

It must be noted that A is measured from the elevated pole—*i.e.*, the pole which is above the horizon.

The following examples show the method of computation :—

Example 6.—In latitude 50° 18′ 30″ N., the mean altitude of the star α Leonis was found to be 38° 15′ 30″, on March 6th, 1936 ; bar. 30·5″; therm. 56° F. ; mean horizontal angle between the referring object and the star 50° 14′ 18″, the line lying to the west and between the star and the elevated pole ; declination of α Leonis March 6th, 1936, 12° 16′ 35·8″ N. Find the azimuth of the line.

By refraction tables.

Mean altitude $=$	38° 15′ 30″

Mean refraction due to alt. $+1′\ 13\cdot6″$
Correction for therm.,   . $-0′\ \ 0\cdot9″$
Correction for bar.,   . $+0′\ \ 1\cdot2″$

Refraction $=-1′\ 13\cdot9″$ Refraction $=-0°\ \ 1′\ 13\cdot9″$

True alt. (α) $=$ 38° 14′ 16·1″

Declination $=$ 12° 16′ 35·8″
90° 00′ 00″

Polar distance (p) $=$ 77° 43′ 24·2″
True alt. (α) $=$ 38° 14′ 16·1″
Latitude (l) $=$ 50° 18′ 30″

2) 166° 16′ 10·3″

83° 8′ 5·2″

$z = 83° \ 8′ \ 5\cdot2″$	log sec $=$	10·9225076
$z-p = \ 5° \ 24′ \ 41\cdot0″$	log sec $=$	10·0019399
$z-\alpha = 44° \ 53′ \ 49\cdot1″$	log sin $=$	9·8487027
$z-l \ = 32° \ 49′ \ 35\cdot2″$	log sin $=$	9·7340762

2) 20·5072264

log tan ½ A $=$ 10·2536132
½ A $=$ 60° 51′ 8·5″
A $=$ 121° 42′ 17·0″ from N. pole.

Mean angle between line and star $=$ 50° 14′ 18·0″

Azimuth of line from N. pole $=$ 71° 27′ 59·0″
and   true bearing of line $=$ N. 71° 27′ 59·0″ W.

Example 7.—On July 24th, 1936, an observation of the sun was made in latitude 41° 19′ 22″ N., longitude 4 h. 51 m. 41 s. W., and the following readings were obtained :—

L.M.T. at Observation, by Watch.	Angle between Sun and Referring Object.	Observed Altitude.	Remarks.
H. M. S.			
5 31 10 p.m.	40° 18′ 16″	37° 45′ 5″	Sun in position S² [see Fig. 268].
5 33 05 p.m.	41° 28′ 14″	37° 00′ 32″	Sun in position S¹. Changed face of instrument.
5 37 20 p.m.	42° 18′ 30″	35° 43′ 35″	Sun in position S.
5 40 32 p.m.	42° 44′ 17″	34° 34′ 10″	Sun in position S³.

Bar. 29·5″, therm. 80° F. Sun between the elevated pole and the line. Watch 4·5 seconds fast at noon, gaining 1·2 seconds per day.

Find the azimuth and true bearing of the line.

$$
\begin{array}{lrrr}
& \text{H.} & \text{M.} & \text{S.} \\
\text{Mean time of observation} = & 17 & 35 & 31 \cdot 75
\end{array}
$$

$$
\text{Corr. for watch} - \left(4 \cdot 5 + \frac{1 \cdot 2 \times 5 \cdot 59}{24}\right) = \quad 0 \quad\ 0 \quad\ 4 \cdot 78
$$

$$
\begin{array}{lrrr}
\text{Mean L.M.T. of observation} = & 17 & 35 & 26 \cdot 97 \ \text{July 24th} \\
\text{Add longitude W.} = & 4 & 51 & 41
\end{array}
$$

$$
\begin{array}{lrrr}
\text{Corresponding G.M.T.} = & 22 & 27 & 7 \cdot 97 \ \text{July 24th} \\
= & \text{July} & 24 \cdot 9355 \ \text{d.}
\end{array}
$$

From N.A., 1936.

DATE.	VAR. AT 0 H. G.M.T.	DEC.
July 23		
,, 24	− 737·7″	+ 19° 57′ 47·8″
,, 25	− 757·6″	
,, 26	− 777·2″	

$$
n \Delta'_{\frac{1}{2}} = 0 \cdot 9355 \times - 757 \cdot 6 = - \qquad 11' \ 48 \cdot 6''
$$

$$
\text{B}'' \left(\Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}}\right) = - 0 \cdot 0151 \times - 39 \cdot 5 = + \qquad 0 \cdot 6''
$$

$$
\begin{array}{lr}
\text{Dec. at given instant} = & 19° \ 45' \ 59 \cdot 8'' \\
& 90° \ 00' \ 00 \cdot 0''
\end{array}
$$

$$
\text{Polar distance } (p) = \quad 70° \ 14' \ 00 \cdot 2''
$$

Mean alt. by observation = 36° 15′ 50·5″.

By Appendix A.—

Refraction due to alt. = +1′ 19·2″

Corr. therm. = −0′ 4·8″

Corr. bar. = −0′ 1·3″

Refraction = −1′ 13·1″

Corr. for sun's parallax in
alt. = 8·8″ × cos 36° 16′ = +0′ 7·1″

Combined corr. = −1′ 6·0″ = − 0° 1′ 6·0″

True altitude (α) = 36° 14′ 44·5″

Polar distance (p) = 70° 14′ 00·2″

Latitude (l) = 41° 19′ 22″

2) 147° 48′ 06·7″

73° 54′ 03·4″

z = 73° 54′ 03·4″, log sec = 10·5570520

$z-p$ = 3° 40′ 03·2″, log sec = 10·0008903

$z-\alpha$ = 37° 39′ 18·9″, log sin = 9·7859765

$z-l$ = 32° 34′ 41·4″, log sin = 9·7311451

2) 20·0750639

log tan ½ A = 10·0375319

½ A = 47° 28′ 21·7″

and A = 94° 56′ 43·4″, measured from
the elevated pole.

Mean observed position angle = 41° 42′ 19·2″

Azimuth of line = 136° 39′ 02·6″

and reduced bearing of line = S. 43° 20′ 57·4″ W.

Graphical Construction for Azimuth.—From the data obtained by observation on the sun or star, a fair approximation of the required azimuth may easily be obtained by a graphical construction. The result obtained in this way gives us a means of detecting any gross clerical errors which may occur in the calculation.

The construction is as follows :—Draw a circle, centre A (Fig. 270), of fairly large radius. Through A draw the horizontal and vertical diameters H L and Z N' to represent the rational horizon and the direction of the plumb line at the place of observation Z. From L set off the angle L A N equal to the given latitude, draw the diameters N A S, E A Q at right angles to each other to represent the axis and equator of the celestial sphere, N and S being the elevated and depressed poles respectively. Set off

Fig. 270.

the angle C A Q equal to the declination (north or south) of the star, and through C, draw C D parallel to E Q. The line C D is the elevation of the circular path of the star. Next, set off the angle of altitude H A J, and from J draw J G parallel to H L. The line J G represents the circle of constant altitude, and we observe that the altitude and declination circles cut each other at K. So far the diagram represents the given conditions as seen in an elevation projected on the plane of the meridian through Z. The angles we require will, however, be seen on the

plan of the figure when projected on the plane of the rational horizon. To obtain this plan, on the same figure, imagine the semicircle H Z L to be rotated about the line H L until Z occupies the position A. H L now becomes the plan of the meridian through A, also J after rabatment occupies the position F, J F being at right angles to H L. With A as centre, A F as radius, draw the plan of the circle of altitude R F P. The plan of the star's position at the time of observation is situated somewhere on this circle. To determine the required position, through K draw the line P K R parallel to Z A N,' cutting the circle in the points P and R, then if at the time of observation the star is east of the meridian, the required point is at R, and if west it is at P ; also, the required azimuth is the angle P A L (or its equivalent R A L) as measured from the elevated pole.

To complete the figure, lay off the angle P A B equal to the angle between the star and the line at observation, the azimuth of the line is then the angle B A L as measured from the elevated pole, or B A H as measured from the depressed pole.

Fig. 270 has been drawn to illustrate the conditions given in Example 7.

Determination of Latitude.

As we have seen (*vide* p. 541), the latitude of a place may be obtained by observing the altitudes of a circumpolar star when at its points of culmination. The mean of the angles of altitude at upper and lower culmination gives the required latitude. Unfortunately, the points of culmination are 12 hours apart in time, hence, in general, one of the observations would have to be made in daylight. As this could not be done with the small telescope fitted on a theodolite, this method is seldom made use of by surveyors.

(1) **By One Meridian Altitude of a Star.**—This is the method generally used when the greatest accuracy is not required.

In determining the meridian altitude of the star selected, the star is sighted a little before it reaches the meridian, and its image is kept in contact with the cross wires by turning the tangent screws to the horizontal and vertical circles until the star ceases to rise (or fall), thereafter its motion is followed in azimuth only, by turning the tangent screw to the horizontal circle, until the image moves definitely downwards (or upwards). The reading on the vertical circle, when cleared of level error and refraction, gives the altitude at transit.

If the star observed is very near the pole (Polaris, for example), its motion in altitude is slow when passing the meridian, thus giving time for face-right and face-left observations; but, if the star is distant from the pole, an observation with one face of the instrument only can be made, and the index error of the vertical circle must be obtained and allowed for, as explained on p. 256.
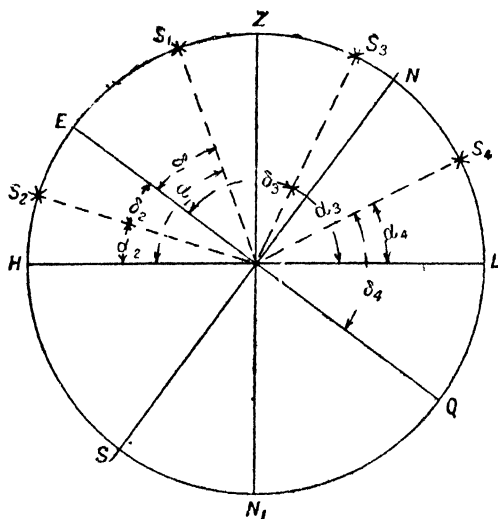
Fig. 271

The required latitude is obtained from the altitude (a) of the star and its declination (δ), as follows:—

(1) When the star lies between the equator and the zenith,

the latitude $L = \delta_1 + 90 - a_1 = \delta_1 + z_1$,

where z_1 is the zenith distance of the star.

(2) When the star lies between the equator and the horizon,

$$L = 90 - a_2 - \delta_2 = z_2 - \delta_2.$$

(3) When the star is between the elevated pole and the zenith,

$$L = \delta_3 - z_3, \text{ and}$$

(4) When the star lies between the pole and the horizon,

$$L = 180 - (\delta_4 + z_4).$$

The four cases are illustrated in Fig. 271, which represents an elevation of the stellar sphere projected on the plane of the meridian through the place of observation Z. The several positions of the star are indicated by the letters S_1, S_2, S_3, and S_4. In each case the angle to be determined is the angle E O Z, and an inspection of the figure will show the truth of the above precepts.

Example 8.—The meridian altitude of β Cassiopeiæ was observed to be 75° 18′ 25″ on Oct. 15th, 1916, the observation being made with face-left, the star lying between the zenith and the pole; index correction — 5″; declination of β Cassiopeiæ on given date 58° 41′ 43″ N. Find the latitude of the place.

$$\text{Observed altitude} = 75° 18′ 25″$$

$$\text{Corr. for refraction} = 57″. \cot 75° 18′ = -0′ 15″$$

$$\text{Index correction} = -0′ 5″$$

$$\text{Combined correction} = -0′ 20″ = -0° 0′ 20″$$

$$\text{True altitude} = 75° 18′ 5″$$
$$90° 00′ 00″$$

$$\text{Zenith distance } (z) = 14° 41′ 55″$$
$$\text{Declination } (\delta) = 58° 41′ 43″$$

$$\text{Required latitude} = \delta - z = 43° 59′ 48″ \text{ N.}$$

(2) Latitude by Meridian Altitude of the Sun.—Method (1) may be applied to the sun by obtaining the meridian altitude of either the upper or lower limb. To obtain the declination of the sun at the instant of observation, the longitude of the place must be known. The following example shows the method of computation :—

Example 9.—On the 1st June, 1936, in longitude 67° 30′ W., the meridian altitude of the sun's lower limb was 64° 30′ 18″; index error + 11″; sun south of zenith. Find the latitude of the place.

$$
\begin{aligned}
\text{Observed altitude} &= 64° \ 30' \ 18'' \\
\text{Index corr.} &= + \ 0° \ \ 0' \ 11'' \\
\hline
&\quad 64° \ 30' \ 29''
\end{aligned}
$$

$$
\begin{aligned}
\text{Ref.} = 57''. \cot 64° \ 46' &= - \ 0° \ \ 0' \ 27'' \\
\hline
&\quad 64° \ 30' \ 02''
\end{aligned}
$$

$$
\begin{aligned}
\text{From N.A.,} \qquad \text{semi-diam.} &= + \ 0° \ 15' \ 47 \cdot 7'' \\
\hline
&\quad 64° \ 45' \ 49 \cdot 7''
\end{aligned}
$$

$$
\begin{aligned}
\text{Parallax} = 8 \cdot 8''. \cos 64° \ 46' &= + \ 0° \ \ 0' \ \ 4'' \\
\hline
\text{True altitude} &= \ 64° \ 45' \ 53 \cdot 7'' \\
&\quad 90° \ 00' \ 00''
\end{aligned}
$$

$$
\begin{aligned}
\text{Zenith distance, } z &= \ 25° \ 14' \ 06 \cdot 3'' \\
&\qquad\qquad\qquad\qquad 25° \ 14' \ 06 \cdot 3''
\end{aligned}
$$

$$
\begin{array}{lrrr}
 & \text{H.} & \text{M.} & \text{S.} \\
\text{L.A.T. of obs.} = & 12 & 0 & 0 \\
\text{Long. W.} = & + \ 4 & 30 & 0 \\
\hline
\text{G.A.T. at obs.} = & 16 & 30 & 0
\end{array}
$$

= June 1 d. 12 h. + 4 h. 30 m. = June 1 d. 12 h. + 0·1875 d.

From N.A., 1936.

DATE.	VAR. AT G.A.N.	DEC.
May 31	+ 498·7″	
June 1	+ 475·6″	+ 22° 04′ 15·4″
,, 2	+ 452·4″	
,, 3		

$$
\begin{aligned}
n \, \Delta'_{\frac{1}{2}} &= + \ 0 \cdot 1875 \times + \ 475 \cdot 6 = + \qquad 1' \ 29 \cdot 2'' \\
B''(\Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}}) &= - \ 0 \cdot 038 \\
&\qquad\qquad\qquad \times - \ 46 \cdot 3 = + \qquad\qquad 1 \cdot 8''
\end{aligned}
$$

$$
\text{Dec. at L.A.N.} = \ 22° \ 05' \ 46 \cdot 4'' \ 22° \ 05' \ 46 \cdot 4''
$$

$$
\text{Required lat.} = \delta + z = \overline{47° \ 19' \ 52 \cdot 7'' \ \text{N.}}
$$

(3) **By the Meridian Altitudes of Two Stars.**—A still more accurate method of determining latitude than is given in the foregoing is to observe the altitudes of two stars, one culminating to the north and the other to the south of the zenith. The stars selected for this purpose should preferably be at, or about, the same altitude, and should culminate shortly after each other. When the meridian altitude of one star has been observed, without changing the face of the instrument, the telescope is rotated in azimuth through 180°, and the meridian altitude of

the second star is obtained. The mean value of the latitude obtained from the two observations depends on the *difference* of the zenith distances of the two stars ; hence, if owing to instrumental error both readings are a little too small or a little too great, by the same amount, both zenith distances will be equally affected, and their difference will be clear of instrumental error. If, however, the face of the instrument be changed between the observations, the sign of the instrumental error will change also, and the error will appear in the difference of the zenith distances.

We may demonstrate the truth of the statement that the mean latitude " depends on the difference of the zenith distance of the two stars " as follows :—

(1) Let the stars be in the positions S_1 and S_3 (Fig. 271), their zenith distances and declinations be z_1, z_3, and δ_1, δ_3, respectively, then for star S_1

$$L_1 = \delta_1 + z_1, \text{ for } S_3$$
$$L_3 = \delta_3 - z_3,$$

and the mean latitude

$$L_0 = \tfrac{1}{2}(L_1+L_3) = \tfrac{1}{2}(\delta_1+\delta_3)+\tfrac{1}{2}(z_1-z_3).$$

(2) When the stars occupy the positions S_2 and S_3.

$$L_2 = z_2 - \delta_2,$$
$$L_3 = \delta_3 - z_3,$$

and $$L_0 = \tfrac{1}{2}(\delta_3 - \delta_2) + \tfrac{1}{2}(z_2 - z_3).$$

(3) When the stars are in the positions S_1 and S_4,

$$L_1 = \delta_1 + z_1,$$
$$L_4 = 180° - (\delta_4 + z_4),$$
∴ $$L_0 = 90 + \tfrac{1}{2}(\delta_1 - \delta_4) + \tfrac{1}{2}(z_1 - z_4).$$

(4) Finally, when the stars are in the positions S_2 and S_4,

$$L_2 = z_2 - \delta_2,$$
$$L_4 = 180 - (\delta_4 + z_4),$$
and $$L_0 = 90 - \tfrac{1}{2}(\delta_4 + \delta_2) + \tfrac{1}{2}(z_2 - z_4).$$

Thus, in all cases, the accuracy of the result depends on the difference of the zenith distances, and not on the zenith distances themselves. The mean result obtained by observing several pairs of stars which culminate on opposite sides of the zenith

at, or about, the same altitude gives a very close approximation to the required latitude.

The latitude is computed from the observation of each star, as shown in Ex. 8, due regard being paid to the star's position. The mean result obtained from a pair (or series of pairs) of observations being taken as the latitude of the place.

(4) **By Observation on the Pole Star when out of the Meridian.** —In this method, circle-left and circle-right observations of the pole star are made as quickly as possible when the star is out of the meridian, and the correct local mean time at each observation is noted. The mean of the results is taken as the altitude and time respectively, at observation. To compute the L.S.T. at observation the longitude of the place must be known.

The computation of the required latitude by the following method is based on the fact that the difference of the observed altitude (a) and the latitude (l) cannot be greater than $1° 15'$, the polar distance (p) of the star. Hence, if x be the polar distance and the star be at upper culmination, $l = a - x$, and

$$\sin l = \sin (a - x)$$
$$= \sin a - x \cos a - \tfrac{1}{2} x^2 \sin a.$$

Similarly, $\cos l = \cos a + x \sin a - \tfrac{1}{2} x^2 \cos a.$

Also, $\sin p = p - \dfrac{p^3}{6} + \text{etc.},$

$$\cos p = 1 - \dfrac{p^2}{2} + \text{etc.}$$

In the spherical triangle having its angular points at the pole, the star and the zenith respectively, we know the hour angle (H) from the local mean time of observation, and the sides formed by the zenith distance and the polar distance of the star. We have to find the third side of the triangle.

From the formula,

$$\cos c = \cos a . \cos b + \sin a . \sin b . \cos C \text{ (Eq. 3, p. 535),}$$

or $\cos (90 - a) = \cos (90 - \delta) . \cos (90 - l)$
$$+ \sin (90 - \delta) . \sin (90 - l) . \cos H.$$

∴ $\sin a = \cos p . \sin l + \sin p . \cos l . \cos H.$

Substituting the above values in this equation, we get,

$$\sin \alpha = (\sin \alpha - x \cos \alpha - \text{etc.})(1 - \tfrac{1}{2}p^2 + \text{etc.}) + (\cos \alpha + x \sin \alpha - \text{etc.})$$
$$\left(p - \frac{p^3}{6} + \text{etc.} \right) \cos \text{H}.$$

Neglecting higher powers of p than the second this becomes,

$$\sin \alpha = \sin \alpha - x \cos \alpha + p \cos \alpha . \cos \text{H} - \tfrac{1}{2}(p^2 - x . p . \cos \text{H}) \sin \alpha.$$

$$\therefore \quad x = p . \cos \text{H} - \tfrac{1}{2}(p^2 - x . p . \cos \text{H}) \tan \alpha + \text{etc.},$$

x and p being in circular measure.

As a first approximation we take $x = p . \cos \text{H}$, and substituting this value in the preceding equation we get a second approximation, $x = p . \cos \text{H} - \tfrac{1}{2}p^2 . \sin^2 \text{H} . \tan \alpha$, and this may be shown to give a result within $1''$ of the true value; consequently our formula for latitude becomes,

$$l = \alpha - p . \cos \text{H} + \tfrac{1}{2}p^2 \sin 1'' . \sin^2 \text{H} . \tan \alpha,$$

the factor $\sin 1''$ being introduced to change the third term to seconds of arc. In using this formula, it must be remembered that $\cos \text{H}$ is negative in the second and third quadrants.

The *Nautical Almanac* * contains three tables for facilitating the computation of latitude by this method. Table I. gives the values of $p . \cos \text{H}$, the argument being the sidereal time of observation, which differs from H by a constant quantity. The first correction applied to α gives an approximate latitude $l = \alpha - p . \cos \text{H}$. Table II. gives the value of the term $\tfrac{1}{2}p^2 \sin 1'' . \sin^2 \text{H} . \tan \alpha$, the arguments being the sidereal time and true altitude. Table III. allows for the difference between the actual value of p and that value which is employed in the construction of Table I. The arguments are the date and the sidereal time.

The following example will show the method of computation in both cases :—

Example 10.—In longitude 5 h. 20 m. W., mean altitude of Polaris (when out of the meridian) at observation 40° 27′ 41″; L.M.T. of observation 10 h. 40 m. 15 s. p.m., on Oct. 11th, 1932. Bar. 30·4″, therm. 54·5° F. Watch 4 seconds slow at noon, gaining at the rate of 0·5 second per day. Find the latitude of the place.

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* Omitted in recent years, but given in the abridged edition.

By Chambers' Tables—

						H.	M.	S.

L.M.T. of observa-

			H.	M.	S.

Mean ref. due to alt., .   +0° 1′ 07″   L.M.T. of observa-

Corr. for bar., .   .   +0° 0′ 02″   tion,   .   .   22   40   15

Corr. for therm.,   .   −0° 0′ 01″   Chronometer corr.

    slow,   +4·0″

Corrected refraction, .   −0° 1′ 08″   Gained

Observed altitude,   .   40° 27′ 41″ $\frac{-0·5 \times 10·67}{24}$,   −0·22″

* True mean altitude,   40° 26′ 33″   +3·8″   0   0   3·8

† True L.M.T. at obs.,   22   40   18·8

Add long. [W.], .   5   20   00·0

Corresponding G.M.T.,

Oct. 12th,   .   4   00   18·8

From Abridged N.A., 1932.

			H.	M.	S.

Declination (δ)   = 88° 56′ 37·2″   S.T. at G.M.M., Oct.

      90° 00′ 00″   12th,   .   .   01   21   25·1

Polar distance *p*,   .   1° 03′ 22·8″   Acceleration for

    =   3802·8″   4·005 h.,   .   .   0   0   39·4

R.A.M.S. $\mp$ 12 h.

at obs.,   .   .   01   22   04·5

† Add L.M.T.,   .   22   40   18·8

L.S.T. at obs. =   24   02   23·3

R.A. star, = − 1   39   1·0

Hour angle in S.T. =   22   23   22·3

H. in arc = 335° 50′ 34·5″

or   24° 09′ 25·5″

<table>
<tr><td colspan="2">

Logarithmic Computation.

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Computation by Pole Star Tables in N.A.

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</table>

Log p in secs. = log 3802·8	= 3·5801035	True altitude,	. 40° 26·55′
Log cos H = log cos 24° 09′ 25·5″	= 9·9601981	From Table I.	
		with argument	
Log 1st correction	= 3·5403016	L.S.T. (2m. 23s.)	
1st correction in secs.	= 3,469·8″	1st corr., .	. −58·54′
Log p in secs.	= 3·58010		39° 28·01′
Log sin H = log sin 24° 09′ 25·5″	= 9·61198	From Table II.,	
		with arguments	
Log p + log sin H	= 3·19208	L.S.T. and alt.	
	2	40° 26′ 33″,	
		2nd corr.,	+0° 0·10′
Log p^2 . sin^2 H	= 6·38416		39° 28·11′
Log tan a = log tan 40° 26′ 33″	= 9·93061		
Log sin 1″	= $\bar{6}$·68557	From Table III.,	
Arith. comp. of log 2	= $\bar{1}$·69897	with arguments	
		L.S.T. and date,	
Log 2nd correction	= 0·69931	Oct. 10th,	+0° 0·70′
2nd correction in secs.	= 5″		
* True altitude, .	. 40° 26′ 33″	True latitude, 39° 28·81″ N.	
1st correction (hour angle in fourth quadrant),	− 57′ 49·8″		
	39° 28′ 43·2″		
2nd correction (always added), . .	. 0° 0′ 5″		
True latitude,	39° 28′ 48·2″ N.		

(5) **By the Altitude of a Star or the Sun when very near the Meridian.**—When a star is observed very near the meridian, the observation is spoken of as a circum-meridian observation, and the altitude as a circum-meridian altitude. The star selected for observation should have (1) a zenith distance of not less than 10°; and (2) an hour angle of not more than 20 minutes of time. The closer the star is to the meridian the better will be the result. This method is not applicable when the declination of the star is nearly equal to the latitude of the place.

In making the observation, the altitude of the star is obtained

not more than 15 minutes before, or after, the time of transit. After the first observation, the face of the instrument is changed before taking the second observation, thereafter it should be changed for each pair of readings. About six or eight readings should be taken, three or four on each side of the meridian, and the time of each observation must be noted. In observing the sun the upper and lower limbs should be read alternately.

It may be shown that the change in altitude of a heavenly body when near the meridian is proportional to the *square* of the time, and not proportional to the time simply. Hence, we cannot reduce a number of altitudes near the meridian by taking their mean to correspond with the mean of the observed times, as is done in observations taken on a star remote from the meridian. Each altitude must be separately reduced, and the mean of the results used in the final computation.

In addition to the altitudes and observed times, the longitude of the place must be known.

The reasoning is as follows :—In the spherical triangle having its angular points at the pole, the star, and the zenith respectively, we know the two sides meeting at the star, and also the (hour) angle at the pole, hence we use the formula

$$\cos c = \cos a \,.\, \cos b + \sin a \,.\, \sin b \,.\, \cos C \text{ (Eq. 3, p. 535).}$$

$$= \cos (a - b) - 2 \sin a \,.\, \sin b \,.\, \sin^2 \tfrac{1}{2} \, C, \qquad . \qquad . \quad (1)$$

Now, $a = 90 - \delta$; $b = 90 - l$, hence, $a - b = l - \delta.$

Also, $c = z = $ observed zenith distance,

$z - x = $ meridian zenith distance
$= l - \delta,$

and $C = H = $ hour angle of star.

Substituting these values in equation (1), we get,

$$\cos z = \cos (z - x) - 2 \cos l \,.\, \cos \delta \,.\, \sin^2 \tfrac{1}{2} \, H,$$

or $\cos (z - x) - \cos z = 2 \cos l \,.\, \cos \delta \,.\, \sin^2 \tfrac{1}{2} \, H,$

that is,

$$2 \sin (z - \tfrac{1}{2} x) \,.\, \sin \tfrac{1}{2} x = 2 \,.\, \cos l \,.\, \cos \delta \,.\, \sin^2 \tfrac{1}{2} \, H,$$

∴ $$\sin \tfrac{1}{2} x = \frac{\cos l \,.\, \cos \delta \,.\, \sin^2 \tfrac{1}{2} \, H}{\sin (z - \tfrac{1}{2} x)},$$

and approximately, since x is a small angle, we have,

$$\sin \tfrac{1}{2} x = \frac{\cos l \cdot \cos \delta \cdot \sin^2 \tfrac{1}{2} H}{\sin z}, \quad . \quad . \quad . \quad (2)$$

and the latitude $\qquad l = z - x + \delta. \quad . \quad . \quad . \quad . \quad . \quad (3)$

The value of x in equation (2) is given in terms of the unknown latitude, l, but if the star is very near the meridian we may replace l by its approximate value $z + \delta$. The value of the latitude obtained from equation (3) is accurate enough for many purposes, and, as will be noticed, it is obtained from one altitude of the star. For a closer approximation, several altitudes on both sides of the meridian should be observed, and the reduction of these altitudes to the meridian may be effected as follows :—
Since x is small, we may write equation (2) thus

$$\tfrac{1}{2} x = \frac{\cos l \cdot \cos \delta \cdot \sin^2 \tfrac{1}{2} H}{\sin z},$$

or in seconds, $\qquad x = \dfrac{2 \cdot \cos l \cdot \cos \delta \cdot \sin^2 \tfrac{1}{2} H}{\sin z \cdot \sin 1''}.$

If $z_0 =$ meridian zenith distance,

$$z_0 = z - x$$
$$= z - \frac{\cos l \cdot \cos \delta}{\sin z} \times \frac{2 \sin^2 \tfrac{1}{2} H}{\sin 1''}. \quad . \quad . \quad (\alpha)$$

Now, the nearest approximation to the true value of l, and consequently, the nearest approach we can get to the true value of z_0, is given us by the greatest altitude or the least zenith distance in the series of observations; hence, calling this value of the zenith distance z', and the corresponding value of l, l', we have

$$z_0 = z' - \frac{\cos l' \cdot \cos \delta}{\sin z'} \times \frac{2 \sin^2 \tfrac{1}{2} H'}{\sin 1''}.$$

Putting $C = \dfrac{\cos l' \cdot \cos \delta}{\sin z'}$, and h' for $\dfrac{2 \sin^2 \tfrac{1}{2} H'}{\sin 1''}$, we get

$$z_0 = z' - C \, h'.$$

For other values of the zenith distance, z'', z''', etc., and hour angles H'', H''', etc., obtained from the series of n observations, we have $\qquad z_0 = z'' - C \, h'',$
$$z_0 = z''' - C \, h''', \text{ etc.,}$$

hence the mean of the series is

$$\frac{n \cdot z_0}{n} = \frac{z' + z'' + z''' + \text{ etc.}}{n} - \frac{C \, (h' + h'' + h''' + \text{ etc.})}{n}$$

and, if for the mean zenith distance we write z_2, and for the mean value h we write h_2, then

$$z_0 = z_2 - C \cdot h_2.$$

If the star is observed at its upper culmination the required latitude, $l, = z_0 + \delta$, but if it is observed at its lower culmination, then $l = 180 - (z_0 + \delta)$.

It should be observed that H is obtained by taking the difference of the computed time of transit and the time of observation; the angle thus obtained will require to be multiplied by 15 to convert it to arc.

In the above demonstration we have assumed that the declination of the star remains constant during the time of observation. For a demonstration of the formula for determining latitude from circum-meridian altitudes of the sun, Chauvenet's " Spherical and Practical Astronomy " should be consulted.

Example 11.—In longitude 0 h. 9 m. 2 s. W. the star α Pegasi was observed on Oct. 18th, 1936, at its upper culmination, the altitudes and times at observation being as given below. Bar. 29·8″, therm. 25° F. Find the latitude of the place.

Observed Altitude (corrected for level).	Times of Observation Corrected for Watch. T.	$T^1 = 21$h. 12m. 55·4s. Hour Angles in Time, $T^1 - T = H$ in M.T. Units.	H. in S.T. Units.	* Corresponding Values of $h_2 = \dfrac{2 \sin^2 (\frac{1}{2} H \times 15)}{\sin 1''}$
	H. M. S.	M. S.	M. S.	
51° 14′ 12″	21 2 22	10 33·4	10 35·1	219·9
51° 14′ 28″	21 4 50	8 5·4	8 6·7	129·1
51° 14′ 40″	21 6 18	6 37·4	6 38·5	86·6
51° 14′ 48″	21 10 6	2 49·4	2 49·9	15·8
51° 14′ 35″	21 16 50	3 54·6	3 55·2	30·1
51° 14′ 15″	21 25 9	12 13·6	12 15·6	295·1
51° 13′ 55″	21 28 0	15 4·6	15 7·1	448·6

7) 51 (100) 53″   Sum.   Sum,   7) 1225·2

    51° 14′ 24·7″   Mean altitude.   Mean h_2 = 175·0

− 0° 0′ 48·7″   Refraction.

                                    Refraction due to alt.,   0′ 46·7″

    51° 13′ 36·0″   Corr. altitude.   Corr. for bar., −0′ 0·3″

    90° 0′ 0″   Corr. for therm., +0′ 2·3″

    38° 46′ 24·0″   Mean zenith dist. z_2.   Corr. refraction,   0′ 48·7″

* From Appendix C.

To compute the time (T') of transit :—

From N.A., 1936.

α Pegasi.

DATE.	R.A.	VAR.	DATE.	DECLINATION.	VAR.
	H. M. S.				S.
Oct. 6·9		61	Oct. 6·9		78
,, 16·9	23 01 38·554	86	,, 16·9	+ 14° 51′ 70·02″	53
,, 26·9	23 01 38·468	103	,, 26·9	+ 14° 51′ 70·55″	27
Nov. 5·8			Nov. 5·8		

$$\text{H. M. S.}$$
Mean time of obs., 18 d. 21 13 22·1
$$= 18\cdot 884 \text{ d.}$$
$$16\cdot 9$$

$$n = \frac{1\cdot 984}{10}$$
$$= 0\cdot 1984$$

$$\frac{n(n-1)}{4} = \frac{0\cdot 1984\,(0\cdot 1984 - 1)}{4} \qquad\qquad n = 0\cdot 1984$$
$$B'' = -0\cdot 0397$$
$$B'' = -0\cdot 0397 \qquad\qquad n\,\Delta'_{\frac{1}{2}} = 0\cdot 1984 \times + 53 = +10\cdot 52$$
$$n\,\Delta'_{\frac{1}{2}} = 0\cdot 1984 \times -86 = -17 \qquad B''(\Delta'_{\frac{3}{2}} - \Delta'_{-\frac{1}{2}})$$
$$B''(\Delta'_{\frac{3}{2}} - \Delta'_{\frac{1}{2}}) \qquad\qquad = -0\cdot 0397 \times -51 = -\ 2\cdot 01$$
$$= -0\cdot 0397 \times +42 = -\ 1\cdot 7$$
$$\qquad\qquad\qquad\qquad\qquad\qquad \text{Correction} = +12\cdot 53$$

$$\text{Correction} = -\ 18\cdot 7$$

	H. M. S.	Declination at	
R.A. Oct. 16·9 d. =	23 01 38·554	Oct. 16·9 d. =	14° 51′ 70·02″
Correction −	0·019	Correction = +	0·13″
R.A. Oct. 18·884 d. =	23 01 38·535	Dec. at 18·884 d. =	14° 52′ 10·15″

	H.	M.	S.
R.A. α Pegasi Oct. 18·884 d. =	23	01	38·54

	H.	M.	S.			
S.T. at G.M.M., Oct. 18 d.	01	45	12·59			
Accel. for 0 h. 9 m. 2 s. W.			1·48			
S.T. of L.M.M. =	01	45	14·07	01	45	14·07
S.T. interval from L.M.M.				21	16	24·47

From N.A.—

				H.	M.	S.
M.T. equivalent for 21 h.,	.	.	.	20	56	33·58
,,	,,	16 m.,	. . .	0	15	57·38
,,	,,	24 s.,	. . .	0	0	23·93
,,	,,	0·47 s.,	. .	0	0	0·47
L.M.T. at transit (T'),	.	.		21	12	55·36

Greatest obs. alt.,	.	.	.	51° 14′ 48″
Refraction,	.	.	.	−0° 0′ 48·7″

$$51° 13′ 59·3″$$
$$90° 0′ 0″$$

Approx. zenith dist. z',	.	.	.	38° 46′ 00·7″
Declination δ,	.	.	.	14° 52′ 10·1″

Approx. lat. l',	.	.	.	53° 38′ 10·8″

Computation of $C\,h_2$.

$$\log \cos l' = \log \cos 53° 38′ 10·8″ = \quad 9·77299$$
$$\log \cos \delta = \log \cos 14° 52′ 10·1″ = \quad 9·98521$$
$$\log \operatorname{cosec} z' = \log \operatorname{cosec} 38° 46′ 00·7″ = \quad 10·20332$$

$$\log C = \quad 29·96152$$
$$\log h_2 = \log 175·0 = \quad 2·24304$$

$$\log C \cdot h_2 = \quad 2·20456$$
$$C \cdot h_2 = 160·2$$
$$= 2′ 40·2″$$

and
$$z_2 = 38° 46′ 24·0″$$
$$C \cdot h_2 = -0° 2′ 40·2″$$

$$z_0 = z_2 - C \cdot h_2 = 38° 43′ 43·8″$$
$$\text{Declination } \delta = 14° 52′ 10·1″$$

$$\text{True latitude} = 53° 35′ 53·9″ \text{ N.}$$

Example 12.—At Y in South Africa, longitude 1 h. 23 m. 56 s. E., approximate latitude 27° S., the circum-meridian altitudes of the sun's upper limb given below were obtained on Oct. 25th, 1936. Angles read with circle-left, for which index correction = − 1′ 4″. Bar. 29·8 inches ; therm. 85° F. Watch 2 m. 5 s. fast. Find the latitude of the place.

Observed Altitudes.			Observed Times. T.			$T^1 = 11$ h.46m. 12·9s. Hour Angles in Time, $T^1 - T = H.$		* Corresponding Values of $h_2 = \dfrac{2\sin^2(\frac12 H \times 15)}{\sin 1''}$
°	′	″	H.	M.	S.	M.	S.	
76	10	4	11	38	8	8	4·9	128·3
76	11	7	11	40	12	6	0·9	71·1
76	11	48	11	42	24	3	48·9	28·6
76	12	15	11	44	48	1	24·9	3·9
76	12	21	11	46	48	0	35·1	0·7
76	12	01	11	51	06	4	53·1	46·8
76	11	18	11	53	31	7	18·1	104·7
76	10	30	11	55	28	9	15·1	168·1

Sums, 8) 76 (91) 24	8) 11 (372) 25	Sum = 8) 552·2
Means, 76 11 25·5	11 46 33·1	Mean h_2 = 69

* From Appendix C.

To Compute Chronometer Time at L.A.N.

Data from N.A., 1936.

DATE.	E.	VAR.
	M. S.	S.
Oct. 24	+ 15 41·47	+ 7·52
„ 25	+ 15 48·99	+ 6·82
„ 26	+ 15 55·81	+ 6·11
„ 27	+ 16 01·92	

$n\,\Delta'_{\frac12} = 0.44172 \times 6.82 = +3.012$

$B''(\Delta'_{\frac32} - \Delta'_{-\frac12})$

$= -0.0617 \times -1.41 = +0.087$

$\underline{\hspace{3cm}}$

$+3.099$

	M. S.
E. at 0 h. G.M.T. =	+ 15 48·99
Correction = +	3·099

$\underline{\hspace{3cm}}$

E. at L.A.N. = + 15 52·1

	H. M. S.
L.A.T. at L.A.N. =	12 0 0
Long. E. =	1 23 56

$\underline{\hspace{3cm}}$

G.A.T. Oct. 25 d. =	10 36 4
=	25·44172 d.
n =	0·44172
B'' =	− 0·0617

	H. M. S.
L.A.T. at L.A.N. =	12 0 0
E. = −	15 52·1

$\underline{\hspace{3cm}}$

L.M.T. at L.A.N. =	11 44 7·9
Chron. fast. +	2 5·0

$\underline{\hspace{3cm}}$

Chron. time   11 46 12·9

(T′) at L.A.N.

Mean of Hour Angles.	Mean Time Interval from G.M.M.

	H. M. S.		H. M. S.
(a) Mean of times,	11 46 33·1	L.M.T. of L.A.N.,	11 44 07·9
(b) Chron. time T′,	11 46 12·9	Correction for	
		long. E.,	−1 23 56·00

Mean of T′−T = +0 0 20·2

G.M.T. of L.A.N., 10 20 11·9

T′ − T is ± according as (a) is   Mean of T′−T,   +0 0 20·2
later or earlier than (b).

Mean time of obs. from G.M.M., Oct. 25/36 = 10 20 32·1
= Oct. 25·43093 d.

Declination.

From N.A., 1936.

DATE.	VAR.	DEC.
Oct. 24		
	−1251·5″	
,, 25	−1240·5″	−11° 57′ 52·3″
,, 26	−1229·0″	
,, 27		

0·43093 × − 1240·5 = −   8′ 54·6″
−0·0613 × +22·5 = −   1·4″

Declination (δ) = −12° 06′ 48·3″

Refraction.

Refr. due to alt.,	.	.	.	.	+14·2″
Corr. therm.,	.	.	.	.	− 1·0″
Corr. bar.,	.	.	.	.	− 0·1″

Corrected refraction, . ·. . 13·1″

Parallax.

Parallax in alt. = 8·8″ × cos 76° 11′ = 2·10″.

To Compute Mean Zenith Distance z_2.

Mean altitude, ...	76° 11′ 25·5″
Index corr., ...	—0° 1′ 4·0″

Diff., ...	76° 10′ 21·5″
Refraction, . . .—	0′ 13·1″
Semi-diameter, . .—	16′ 7·0″
Parallax in alt., . .+	0′ 2·1″

	—16′ 18·0″	—0° 16′ 18·0″

True mean alt., ...	75° 54′ 03·5″
	90° 0′ 0″

$$z_2 = 14° 05′ 56·5″$$

To Compute Approximate Latitude l'.

Greatest observed altitude, . .	76° 12′ 21″
Index corr., ...	—0° 1′ 4″

Diff., ...	76° 11′ 17·0″
Parallax. Semi-diameter. Refraction.—	0° 16′ 18·0″

Diff., ...	75° 54′ 59″
	90° 0′ 0·0″

Approx. zenith dist. z', . .	14° 5′ 01·0″
Declination δ, ...	12° 06′ 48·3″

Approx. latitude, l', . . .	26° 11′ 49·3″ S.

To Compute $C h_2$.

$$\log \cos l' = \log \cos \quad 26° 11′ 49·3″ = 9·95293$$
$$\log \cos \delta = \log \cos \quad 12° 06′ 48·3″ = 9·99022$$
$$\log \operatorname{cosec} z' = \log \operatorname{cosec} 14° 5′ 01·0″ = 10·61379$$

$$\log C = 0·55694$$
$$\log h_2 = \log 69·0 = 1·83885$$

$$\log C h_2 = 2·39579$$
$$C h_2 = 248·8″$$

$$\text{Mean zenith distance, } z_2 = 14° \ 5' \ 56{\cdot}5''$$
$$C \ h_2 = -0° \ 4' \ 8{\cdot}8''$$

$$z_0 = z_2 - C \ . \ h_2 = 14° \ 1' \ 47{\cdot}7''$$
$$\text{Declination } \delta = 12° \ 6' \ 48{\cdot}3''$$

$$\text{True latitude} = z_0 + \delta = 26° \ 8' \ 36{\cdot}0'' \text{ S.}$$

Finding the Time by Observation.

The determination of time by observation is necessary for the purpose of checking and correcting the chronometer, or other timekeeper the observer may be using, whenever an accurate knowledge of time is required in astronomical observations for determining azimuth or latitude.

When the error of the chronometer has been found, instead of altering the chronometer, it is usual to record the error (or correction), and to deduct it from, or add it to, a given time, according as the chronometer is fast or slow.

The rate at which the chronometer gains or loses should be determined by astronomical observations made at convenient intervals of time, such as one, two, three, or more days, and from the results of these observations the daily rate of the chronometer is obtained. Since no chronometer has a constant rate, it will be necessary to redetermine the rate from time to time, and this should always be done immediately before making astronomical observations in which time is an important factor.

A convenient epoch from which to reckon chronometer error and correction is that of mean noon on a given day.

The correction of the chronometer is that which must be added to, or subtracted from, the observed time of observation, to give the correct time at the given instant.

In a fixed observatory fitted with a transit instrument set accurately in the meridian, the most accurate way of finding time is by noting the time of transit of a known star over the vertical wires of the instrument. The mean time of transit, corrected for the instrument, gives the clock time of transit, and by comparison of the clock time with the R.A. of the star, reduced to mean time if necessary, the error of the clock is known.

In the field, the direction of the meridian cannot be defined with a sufficient degree of precision to enable the observer to

find the time in the above manner, hence the surveyor is compelled to make use of some method whereby he can compute the hour angle of the star, east or west of the meridian, and from this and the known R.A. of the star, the time of its transit may be computed. Only those methods in general use will be considered here.

To find the Time by the Method of Equal Altitudes of a Heavenly Body.—In this method, advantage is taken of the fact that (since the declination of a star remains constant during the interval of time necessary for the observation) the hour angles of a star at equal altitudes, east and west of the meridian, are the same.

In making the observation, the telescope is clamped at an inclination slightly greater (or less) than that of the star, and its motion is followed in azimuth by turning the tangent screw to the horizontal circle. The instant (T) as given by the chronometer at which the image of the star is bisected by the cross wires is recorded. The theodolite is then left undisturbed until the star approaches the same altitude on the other side of the meridian, and, when in the field of view of the telescope, its motion is again followed in azimuth, and the instant of bisection (T') is again recorded. The time of transit is now known, since it is equal to $\dfrac{T + T'}{2}$, and a comparison of this time with that obtained from the known R.A. of the star determines the chronometer error.

This method, if carefully carried out, gives a very accurate result ; it is independent of the errors of collimation and graduation of the instrument, and a knowledge of the latitude of the place or the declination of the star is not necessary. It requires, however, a clear sky over a considerable interval of time, and is liable to errors caused by the change in the amount of refraction, due to changes in temperature and barometric pressure.

If desired, a series of observations, in pairs, may be made on the same star, the mean of the results thus obtained will give a very close approximation to the time of transit.

The method of determining the S.T. and L.M.T. of transit is shown in Example 11, p. 600.

When the sun is used, the time of transit obtained as above will require correction, owing to the change in the sun's declination. This correction is obtained as follows :—

Let, l = the latitude of the place,
            δ = the sun's declination at local apparent noon,

$\Delta\,\delta$ = increase in declination to the west, or decrease to the east, observation,

a = sun's true altitude at each observation,

T = mean of times of observation,

$\Delta\,T$ = correction of this mean to reduce clock time to apparent noon,

$2\,t$ = time elapsed between observations,

then, $t + \Delta\,T$ = hour angle to first observation,

$t - \Delta\,T$ = hour angle to second observation,

$\delta - \Delta\,\delta$ = declination at first observation,

$\delta + \Delta\,\delta$ = declination at second observation,

and in the spherical triangle having the pole, the zenith, and the sun at its angular points, we know the three sides; therefore, from the formula,

$$\cos c = \cos a \,.\, \cos b + \sin a \,.\, \sin b \,.\, \cos C \text{ (Eq. 3, p. 535)},$$

we have

$$\sin a = \sin l \,.\, \sin (\delta - \Delta\,\delta) + \cos l \,.\, \cos (\delta - \Delta\,\delta) \,.\, \cos (t + \Delta T), \quad (1)$$

and

$$\sin a = \sin l \,.\, \sin (\delta + \Delta\,\delta) + \cos l \,.\, \cos (\delta + \Delta\,\delta) \,.\, \cos (t - \Delta T), \quad (2)$$

Expanding the compound functions and subtracting (1) from (2), we get

$$0 = 2 \sin l \,.\, \cos \delta \,.\, \sin \Delta\,\delta + 2 \cos l \,.\, \cos \delta \,.\, \cos \Delta\,\delta \,.\, \sin t \,.\, \sin \Delta T$$
$$- 2 \cos l \,.\, \sin \delta \,.\, \sin \Delta\,\delta \,.\, \cos t \,.\, \cos \Delta\,T.$$

Transposing and dividing by the coefficient of $\sin \Delta\,T$, we get

$$\sin \Delta\,T = \frac{\cos l \,.\, \sin \delta \,.\, \sin \Delta\,\delta \,.\, \cos t \,.\, \cos \Delta\,T - \sin l \,.\, \cos \delta \,.\, \sin \Delta\,\delta}{\cos l \,.\, \cos \delta \,.\, \cos \Delta\,\delta \,.\, \sin t}$$

$$= \tan \Delta\,\delta \,(\tan \delta \,.\, \cot t \,.\, \cos \Delta\,T - \tan l \,.\, \operatorname{cosec} t),$$

and since $\Delta\,\delta$ is a very small angle, this may be written

$$\Delta\,T = \Delta\,\delta \,(\tan \delta \,.\, \cot t - \tan l \,.\, \operatorname{cosec} t).$$

Now, $\Delta\,\delta = \Delta'\,\delta \times t$, where $\Delta'\,\delta$ is the hourly change in declination as given in the Nautical Almanac, and, converting this to seconds of time, we get:—

$$\Delta\,T = \pm \frac{\Delta'\,\delta \times t}{15} \,(\tan l \,.\, \operatorname{cosec} t - \tan \delta \,.\, \cot t),$$

and this quantity must be added to or subtracted from the mean of the times of observation according as the sun is *leaving or approaching* the " elevated pole."

The method of computing L.M.T. at L.A.N. is shown in Example 12, p. 602.

(2) **By a Single Altitude of a Heavenly Body.**— In this method the altitude of a known star, or the sun, is determined with a theodolite or a sextant, and the instant of observation is obtained from a chronometer. From the data, altitude, time, latitude, and longitude of the place, the hour angle of the star at the instant of observation is computed. The R.A. and declination of the star are obtained from the *Nautical Almanac*.

The star will be in the best position for observation when for a given change in altitude the change in time is a minimum. The more rapid the change in altitude, the less will an error of observation affect the time computed from it.

The position of the sun or star at which the change of altitude is most rapid is obtained as follows :— In Fig. 272 we have $\cos z = \sin l \cdot \cos p + \cos l \cdot \sin p \cos H$.
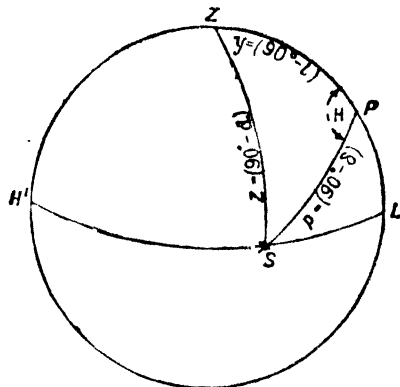
Fig. 272.

Differentiating, and remembering that z and H are the only variables, we get

$$\sin z \cdot \frac{dz}{dH} = \cos l \cdot \sin H \cdot \sin p,$$

or $\cdot$
$$\frac{dz}{dH} = \cos l \cdot \frac{\sin p \cdot \sin H}{\sin z}$$

$$= \cos l \cdot \sin P\,Z\,S \text{ (Fig. 272)}.$$

This has a maximum value when $P\,Z\,S$ is a right angle, hence the best position for observation is on, or near, the prime vertical.

Stars which cross the prime vertical—*i.e.*, those which rise and set (see Fig. 258)—should, therefore, be used, in this method, for the purpose of determining time. The altitude of the star at observation should, however, be not less than 10° or the advantage gained by observing the star on the prime vertical may be lost owing to the uncertainty in the value of the refraction.

For precision, several altitudes should be observed at intervals sufficiently short that the mean of the altitudes may be taken to correspond to the mean of the times and, to eliminate instrumental and other errors, observations should be taken in pairs on both sides of the meridian by observing a rising and a setting star, or, in the case of the sun, by morning and afternoon observations.

In taking observations on the sun with a theodolite, sight the lower (or the upper limb) and make contact with the horizontal centre wire. Note the time and the reading on the vertical circle, and also the readings at the two ends of the spirit level. Now, change the face of the instrument and make contact with the other limb, noting the readings as before. These operations are repeated as often as required, for accuracy, the mean of the altitudes (corrected for bubble reading) is taken as the mean observed altitude, and the mean of the times that at observation.

The procedure is the same when observing a star, except that the image of the star is bisected by the cross wires.

It is sometimes convenient to know beforehand the altitude of a star when crossing the prime vertical. This is easily determined, since the triangle P Z S (Fig. 272) is right-angled at Z, hence, using the formula

$$\cos b = \frac{\cos c}{\cos a} \text{ (Eq. 3, p. 535)},$$

we have, $\cos (90 - a) = \dfrac{\cos (90 - \delta)}{\cos (90 - l)}$,

or $\sin a = \dfrac{\sin \delta}{\sin l}$.   .   .   .   .   .   *(a)*

To compute the hour angle, since we know the three sides of the triangle Z P S (Fig. 272), we use the formula,

$$\tan^2 \tfrac{1}{2} A = \frac{\sin (s - c) \cdot \sin (s - b)}{\sin s \cdot \sin (s - a)} \text{ (Eq. VII., p. 532)}.$$

Here, $s = 90 - \dfrac{(a + l - p)}{2}$ (*vide* p. 584)

$a = p = 90 - \delta$; $b = 90 - a$; $(c) = 90 - l$;

hence, if $z = \dfrac{a + l + p}{2}$, $s = 90 - \overline{z - p}$

$$s - a = z - l,$$
$$s - b = 90 - z,$$
$$s - c = z - a,$$

and,   H = A.

Substituting these values in the above equation, we get

$$\tan^2 \tfrac{1}{2} H = \frac{\sin (z - a) \cdot \cos z}{\cos (z - p) \cdot \sin (z - l)},$$

or, $\tan \tfrac{1}{2} H = \sqrt{\cos z \cdot \sin (z - a) \cdot \sec (z - p) \cdot \operatorname{cosec} (z - l)}.$

In using this formula, we note that p is measured from the elevated pole, and the latitude l is always considered to be positive.

The hour angle H having been calculated, the sidereal time of observation is equal to the R.A. of the star $\pm \dfrac{H}{15}$, the upper or lower sign being used according as the star is west or east of the meridian. If the sun be observed, the apparent time will be $\dfrac{H}{15} + 12$ h. if the sun be west of the meridian, or 12 hours $- \dfrac{H}{15}$ if on the east.

Example 13.—In longitude 2 h. 30 m. E., latitude 54° 30′ N., on Nov. 8th, 1936, the stars β Herculis and β Tauri were observed with a theodolite for time. Watch supposed to be 3 m. 20 s. fast at noon, gaining 0·7 s. per day. Bar. 29·4 inches, therm. 40° F.

β Herculis (setting), mean of altitudes, 20° 12′ 24″,
mean of times, 7 h. 1 m. 45 s. p.m.

β Tauri (rising)   mean of altitudes, 25° 26′ 18″,
mean of times, 8 h. 32 m. 58 s. p.m.

Find the chronometer correction.

	H.	M.	S.
S.T. at G.M.M., Nov. 8/36	3	8	0·26
Correction for long. 2·5 × 9·856 s., —			24·64
L.S.T. of L.M.M.,	**3**	**7**	**35·62** Nov. 8/36.

β Herculis.

By Appendix A—

Refraction due to alt.,	$+2'\,36.9''$
Correct. for bar., .	$-\quad 3.2''$
Correct. for therm., .	$+\quad 3\,2''$
Corr. refraction, •	$2'\,36.9''$

From N.A., 1936,
    Declination (δ), .$+21°\,37'\,39''$
                  $90°\,00'\,00''$

Polar distance (p), . $68°\,22'\,21''$

Mean obs. alt., •	$20°\,12'\,24.0''$
Refraction, •	$-0°\,\,\,2'\,36.9''$
Corr. alt. (a), •	$20°\,\,\,9'\,47.1''$
Polar dist. (p),	$68°\,22'\,21''$
Latitude (l), •	$54°\,30'\,00.0''$

$$2\,)\,143°\,02'\,\,\,8.1''$$
$$z = 71°\,31'\,\,\,4.0''$$

Log cos z
  = log cos $71°\,31'\,4.0''$, 9.5010735

Log sin ($z-a$)
  = log sin $51°\,21'\,16.9''$, 9.8926659

Log sec ($z-p$)
  = log sec $3°\,08'\,43.0''$, 10.0006547

Log cosec ($z-l$)
  =log cosec $17°\,01'\,4.0''$, 10.5336242

$$2\,)\,19.9280183$$

Log tan $\frac{1}{2}$ H, • •	9.9640092
$\frac{1}{2}$ H, • •	$42°\,37'\,42.98''$
	2

$$5\,)\,85°\,15'\,25.96''$$
$$3\,)\,17°\,\,\,3'\,\,5.192''$$
$$5\text{h. }41\text{m. }1.73\text{s.}$$

β Tauri.

By Appendix A—

Refraction due to alt.,	$+\,2'\,1.8''$
Correct. for bar., .	$-\quad 2.45''$
Correct. for therm., .	$+\quad 2.47''$
Corr. refraction, •	$2'\,1.8''$

From N.A., 1936.
    Declination (δ), .$+\,28°\,33'\,21.8''$
                  $90°\,00'\,00''$

Polar distance (p), . $61°\,26'\,38.2''$

Mean obs. alt., •	$25°\,26'\,18.0''$
Refraction, •	$-0°\,\,\,2'\,\,1.8''$
Corr. alt. (a), . •	$25°\,24'\,16.2''$
Polar dist. (p),	$61°\,26'\,38.2''$
Latitude (l), •	$54°\,30'\,00.0''$

$$2\,)\,141°\,20'\,54.4''$$
$$z = \,\,\,70°\,40'\,27.2''$$

Log cos $70°\,40'\,27.2''$, 9.5197482

Log sin $45°\,16'\,11.0''$, 9.8515199

Log sec $9°\,13'\,49.0''$, 10.0056601

Log cosec $16°\,10'\,27.2''$, 10.5550828

$$2\,)\,19.9320110$$

Log tan $\frac{1}{2}$ H, . •	9.9660055
$\frac{1}{2}$ H, • •	$42°\,45'\,35.52$
	2

$$5\,)\,85°\,31'\,11.04''$$
$$3\,)\,17°\,\,\,6'\,14.21''$$
$$5\text{h. }42\text{m. }04.7\text{s.}$$

	H.	M.	S.
	5	42	04·7
	24	00	00·0

		H.	M.	S.					
Westerly hour angles,	.	5	41	1·7			18	17	55·3
From N.A.,									
R.A. β Herculis, .	.	16	27	29·6	R.A. β Tauri,		5	22	20·3
L.S.T. of observation,	.	22	08	31·3			23	40	15·6
From p. 611.									
L.S.T. of L.M.M., .	.	3	07	35·6			3	07	35·6
S.T. interval from L.M.M.		19	00	55·7			20	32	40·0

From N.A., M.T. equivalents—									
For 19 h., .	.	:	18	56	53·24	20 h., .	19	56	43·41
For 00 m., .	.	.	0	0	0	32 m., .	0	31	54·76
For 55 s., .	.	.	0	0	54·85	40 s., .	0	0	39·89
For 0·7 s., .	.	.	0	0	0·70	0·0 s., .	0	0	0·00
L.M.T. of observation,	.		18	57	48·79		20	29	18·06
Chron. time of „			19	1	45		20	32	58·00
Chron. fast, .	.	.	0	3	56·21		0	3	39·94
			0	3	39·94				
Sum, .	.	.2)0	7	36·15					
Mean correction at obs.,	.		0	3	48·07				
Chron. gain in 7½ hours,	.		—0	0	0·2				
Chron. correction at noon,		—0	3	47·87	(chron. fast).				
		0	3	20·0					
Error in assumed chron. time at noon,	.	.	0	0	27·87				

It should be noted that if the assumed time is found to be much in error, the computation should be repeated with new quantities obtained from the corrected value of the time.

(3) **By Circum-Meridian Altitudes of a Star or the Sun.**—In this method two altitudes of a star or the sun are taken when near the meridian, as far apart in time as possible, and the times of observation are noted, the procedure being the same as described (on p. 597) for determining latitude by circum-meridian altitudes.

The method of determining the chronometer correction is as follows :—In equation (α), p. 599, since H is small, we may write for $\sin^2 \frac{1}{2}$ H its approximate equivalent $(\frac{1}{2}$ H$)^2 \times \sin^2 1''$,

and to reduce the time to arc we multiply by 15^2. On making these substitutions, equation (α) becomes

$$z_0 = z - \frac{\cos l \cdot \cos \delta}{\sin z} \times \frac{225 \cdot H^2 \cdot \sin 1''}{2}$$

$$= z - \frac{C \cdot 225 \cdot \sin 1''}{2} H^2$$

$$= z - C \cdot d \cdot H^2, \text{ where } d = \frac{225 \cdot \sin 1''}{2}.$$

Similarly, $z_0 = z' - C \cdot d \cdot H'^2$,

and, by subtraction, we get:

$$z' - z = C \cdot d \cdot H'^2 - C \cdot d \cdot H^2$$
$$= C \cdot d \cdot (H' - H)(H' + H).$$

Now, if both observations of the star are made on the same side of the meridian $(H' - H)$ is known, and if on opposite sides of the meridian $(H' + H)$ is known, also the zenith distances z' and z are known in both cases, hence, either the sum or difference of the times may be computed, and, therefore, the chronometer correction at either observation.

Example 14.—At X in Bolivia, latitude 18° 0′ 00″ S., longitude 63° W., the star γ Toucanæ was observed near the meridian, on Nov. 11th, 1936, and the following readings (on opposite sides of the meridian) were obtained (Bar. 30″, therm. 50° F.):—

	Observed Altitude.	Mean of Times of Observation.
		H. M. S.
	49° 13′ 10″	19 42 24
	49° 14′ 47″	19 55 20

Diff. = 0° 1′ 37″
$z' - z = $ 97″

Determine the chronometer correction.

Note.—Refraction is the same for both observations and disappears on taking the difference.

To compute the time of transit:—

	H. M. S.
R.A. γ Toucanæ, Nov. 11th, 1936,	23 13 47·6

	H. M. S.
S.T. at G.M.M., Nov. 11/36,	3 19 49·9
Acceleration for 4·2 h. W.,	0 0 41·4

	H. M. S.	H. M. S.
L.S.T. of L.M.M.,	3 20 31·3	3 20 31·3

	H. M. S.
S.T. interval from L.M.M.,	19 53 16·3

	H.	M.	S.
M.T. equivalent for 19 h., . . .	18	56	53·24
,, ,, 53 m., . . .	0	52	51·32
,, ,, 16·3 s., . . .	0	0	16·26
L.M.T. at transit, ...	19	50	00·82
Time of observation, 1st reading, . .	19	42	24·00
$H_2 =$	0	7	36·82
	19	50	00·82
Time of observation, 2nd reading, . .	19	55	20·00
$H_3 =$	0	5	19·18

$$H_2 + H_3 = H' + H = 12 \text{ m. } 56 \text{ s.}$$
$$= 776 \text{ seconds. M.T. units.}$$
$$= 778·1 \text{ ,, } \quad \text{S.T. units.}$$

Greatest observed altitude, . . .	49° 14′ 47″
Refraction, ...	0° 0′ 50·1″
True altitude, ...	49° 13′ 56·9″
	90° 0′ 0″
Zenith distance z, . . .	40° 46′ 3·1″

$$\text{Log cos } l = \text{log cos } 18° \ 0′ \ 0″ = 9·97821$$
$$\text{Log cos } \delta = \text{log cos } 58° 35′ 01·4″ = 9·71705$$
$$\text{Log cosec } z = \text{log cosec } 40° 46′ 3·1″ = 10·18509$$

$$\text{Log C} = 9·88035$$
$$\text{Log sin } 1″ = \log\left\{\frac{\sin 1′}{60}\right\} = 4·68557$$
$$\text{Log } 225 = 2·35218$$
$$\text{Log (H}' + \text{H)} = \log 778·1 = 2·89104$$

$$\text{Sum} = 19·80914$$
$$\text{Log } 2 = 0·30103$$

$$\text{Log } \{C\, d\, (H' + H)\} = \text{Diff.} = 19·50811$$

$$\text{Co-log} = 0·49189$$
$$\text{Log } (z' - z) = \log 97 = 1·98677$$

$$\text{Log (H}' - \text{H)} = 2·47866$$

$$(H' - H) = 301 \cdot 06 \qquad H' = 8 \text{ m. } 59 \cdot 6 \text{ s. in S.T. units.}$$
$$(H' + H) = 778 \cdot 10 \qquad = 8 \text{ m. } 58 \cdot 13 \text{ s. in M.T. units.}$$

$$2 H' = 1079 \cdot 16$$

	H.	M.	S.
$H' = 538 \cdot 13 =$	0	8	58·13
Observed time =	19	42	24·00
Chron. time of transit =	19	51	22·13
L.M.T. of transit =	19	50	00·82
Chron. correction =	—0	1	21·31 (Chron. fast).

It should be noted that, if both observations of the star are made on the same side of the meridian, the difference of the observed times is equal to $H' - H$.

Graphical Construction for Time.—A graphical construction for time similar to that given (on p. 588) for azimuth is readily constructed as follows :—Draw a circle 8 or 10 inches diameter, centre O (Fig. 273). Through O draw the vertical and horizontal diameters Z N' and H L. Z and N' represent the zenith and nadir respectively, and H L represents the horizon. From O lay off the angle L O N equal to the latitude (l) of the place, prolong the line N O to S, and through O draw the line E Q at right angles to the line N S. The lines N S and E Q represent the axis and the equator of the stellar sphere respectively. At O lay off the angles Q O a, Q O b equal to the declinations of the stars observed, and from O, with O H as reference line, lay off the angles H O p, H O r equal to the observed altitudes. Through the points a and b, draw the lines $a c$ and $b d$ parallel to E Q. These lines show the paths of the two stars in elevation. Through the points p and r draw the lines $p q$ and $r s$; these lines represent the elevations of circles of constant altitude (α and α'). The circles of constant altitude cut the circles of constant declination at the points e and f, thus giving the positions of the stars at observation as seen in elevation. The figure, thus far, shows the condition of affairs as seen in elevation when projected on the plane of the meridian ; to determine the required hour angles, it is necessary to project the figure on the plane of the equator, and to see it in plan. To do this, imagine the figure to be rotated about the line E Q until the point N

occupies the position of the point O. During rabatment the points a and b will describe the lines $a\,n$ and $b\,m$ perpendicular to E Q, and after rabatment will occupy the positions of the points n and m; also the line N S now represents the prime vertical, and the line E Q the meridian through Z. With centre O, radii O m and O n, describe the circles m R, n W, which represent the plans of the circles of constant declination. To determine the positions of the stars at observation, let e and f be the elevations of the rising and setting stars respectively. Through
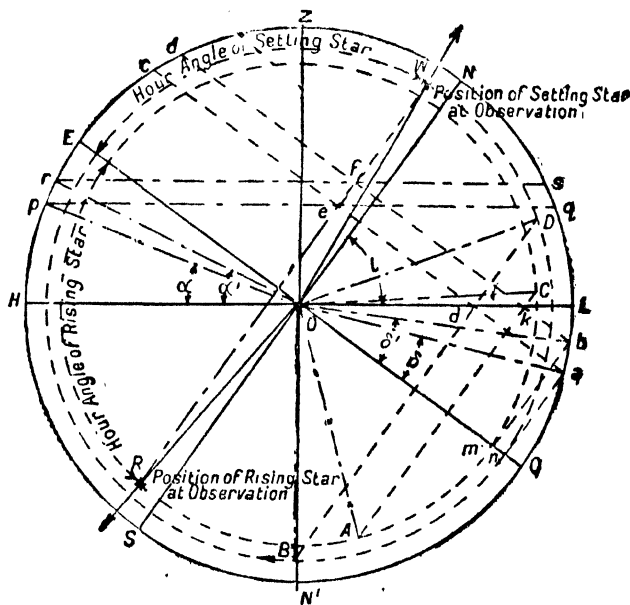
Fig. 273.

e and f draw lines parallel to N S to cut the corresponding circles of declination at the points R and W. Join R O and W O, then the required hour angles are the angles R O E and E O W in arc. The hour angles in arc divided by 15 give the corresponding angles in time which may be converted into sidereal or mean time by the methods already dealt with.

Fig. 273 has been drawn from the data given in Example 13. In the original figure the meridian circle is 5 inches diameter,

the angles were set out with a common 6-inch rectangular pro-
tractor; the hour angle (R O E) of β Tauri on the figure is
85·1°, and of β Herculis (E O W) 85·5°; in computation these
angles are 85·52° and 85·26° respectively.

Time of Rising and Setting.—Obviously the same figure may be
used to determine the hour angles of a star when rising or setting,
for in these positions the angular elevation of the star is zero.
These points are shown in elevation at the points d and k, and
on projecting these points on the plan we obtain the points
A, B, C, and D, the points of rising and setting of the stars
R and W respectively. Clearly, from the figure, the hour angle
of rising (A O E) is equal to the hour angle of setting (E O C),
and this is true in all cases.

The problem of finding the time of rising or setting of a known
body is really a particular case of finding the time by a single
altitude—in this case zero. To solve the problem, it is necessary
to find the angle Z P S (Fig. 272). The triangle P L S is right-
angled at L, since H′ S L is the horizon and H′ Z L the meridian
through the place of observation Z. Hence, we use the formula

$$\cos B = \tan a \cdot \cot c \text{ (Eq. 12, p. 535),}$$
or, $$\cos S P L = \tan P L \cdot \cot S P,$$
and $$\cos Z P S = \cos (\pi - S P L)$$
$$= - \tan l \cdot \cot (90 - \delta)$$
$$= - \tan l \cdot \tan \delta.$$

This gives the hour angle Z P S, from which the mean or sidereal
time of rising or setting may be determined.

The time thus obtained is the time of *true rising or setting*,
but this is altered by the effects of atmospheric haze, refraction,
and dip, so that the time of visible or apparent rising or setting
will differ appreciably from that of true rising or setting.

Determination of Longitude.

The position of a point on the earth's surface is clearly defined
if its latitude and longitude are known.

The difference of longitude of two places is known when the
difference of local (mean or sidereal) times at the places is known,
at any given instant, since to convert the time interval into
arc we have only to multiply severally the hours, minutes, and
seconds in the interval by 15 to obtain the required angle in
degrees, minutes, and seconds.

The problem of determining longitude has been attempted in

a variety of ways, but they all resolve themselves ultimately into finding the time at some initial meridian, such as Greenwich, at an instant corresponding to the local time at the place of observation. The local time is easily obtained; the difficulty has been to determine the corresponding instant at the other meridian. This may be done in the following ways :—

    (1) By transportation of chronometers.
    (2) By the electric telegraph, or by a luminous signal.
    (3) By moon-culminating stars.
    (4) By celestial signals, such as the eclipses, occultations, or transits of one of Jupiter's Satellites.
    (5) By lunar distances.
    (6) By wireless time signals.

The methods (1) to (5) are now mainly of historical interest as they have been superseded by (6), the advent of wireless signals having rendered the carrying of the time of the reference meridian comparatively easy.

Time signals are now sent out from various wireless stations at stated intervals, and the surveyor, by their aid, may check his chronometer in almost any part of the World. A list of wireless stations, their times and durations of emission, together with their wavelengths and type of signal, is given in the *Admiralty List of Wireless Signals,* which is published annually ; any changes and corrections are notified in the weekly *Notices to Mariners.* Greenwich mean time signals are sent and usually continue for a period of five minutes. The signals are rhythmic and consist of a series of 61 Morse dots to the minute, the beginning and end of each minute being denoted by a dash, which is counted as zero of the series which follows.

Equipment.—It is necessary that the surveyor be equipped with (1) a reliable (mean or sidereal time) chronometer, beating ½ seconds, and (2) a specially constructed longwave wireless receiving set with a frame aerial.

Checking the Chronometer.—Assuming in the first instance that the surveyor is provided with a mean time chronometer. As the time for receiving the signals approaches, the surveyor adjusts his wireless set to the wavelength he wishes to receive and, when the signals come through he notes that the wireless beats overtake the beats of his chronometer until they appear to coincide, the coincidence appearing to continue for several beats, and afterwards the signals get in front of the chronometer beats. This will occur at each minute, and the observer counting from

the zero (as 0) of each series will note the number of the wireless
beat at which coincidence appears to begin and that at which it
ends ; the mean beat between this is taken as that of coincidence.
Thus, if the former takes place at 41 and the latter at 45, 43 is
noted as the coincident beat. At each coincidence the surveyor
also notes the chronometer time. The chronometer correction
is now obtained as follows :—

We will suppose the surveyor is using the Paris (Eiffel) time
signal and obtains the following results :—

Paris (Eiffel) Time Signal.				Mean Time Chronometer.		
	H.	M.	BEATS.		H.	M.	S.
	9	31	42		9	32	05
	9	32	42		9	33	06
	9	33	42		9	34	06
	9	34	42		9	35	06
	9	35	41		9	36	06

5) (5) 9 165 209   5) (5) 9 170 29

$$9 \quad 33 \quad \frac{41 \cdot 8 \times 60}{61 \text{ s.}} \qquad\qquad 9 \quad 34 \quad 5 \cdot 8$$

$$= 9 \quad 33 \quad 41 \cdot 12 \text{ s.} \qquad\qquad 9 \quad 33 \quad 41 \cdot 12$$

Chron. fast on G.M.T.,   0   0   24·68

As shown in the table the average wireless beats are reduced
to seconds. There being 61 beats to each second, the average
Greenwich mean time given by the readings is 9 h. 33 m. 41·8
× 60/61 seconds. The average time given by the chronometer
being 9 h. 34 m. 5·8 s., it evidently is 24·68 seconds fast on
Greenwich mean time.

With the sidereal time chronometer the procedure is much
the same. Coincidence will take place at intervals of approxi-
mately 72 sidereal seconds. It will be necessary to convert the
observed Greenwich mean time to its equivalent sidereal time.
To this is added the G.S.T. of 0 h. for the given date, and from
this sum the error of the chronometer in sidereal time is at once
determined by difference. For example, the following coinci-
dences were obtained in an observation on July 17, 1936. Find
the chronometer correction.

Rugby Time Signal.				Sidereal Time Chronometer.		
H.	M.	BEATS.		H.	M.	S.
17	55	10		13	34	50
17	56	23		13	36	02
17	57	37		13	37	14
17	58	50		13	38	26

4) (4) 17 226 120   4) (4) 13 146 32

$$17 \ 56 \cdot 5 \quad \frac{30 \times 60}{61} \qquad 13 \ 36 \ 38$$

17 56 59·51, M.T. $=$ 17 59 56·43 S.T.

S.T. at 0 h. G.M.T., July 17/36 $=$ 19 38 33·08

G.S.T. of mean Rugby signal $=$ 13 38 29·51

Corresponding chronometer time $=$ 13 36 38·00

Chronometer slow on G.S.T.   00 01 51·51

Rating the Chronometer.—If the chronometer error be obtained daily at the same station over a period, its average stationary rate over this period is determined by dividing the total error by the number of days in the period. Knowing the rate of the chronometer the Greenwich (mean or sidereal) time may be found at any instant by multiplying the rate by the time interval which has elapsed since the chronometer was last checked and applying the result to the time indicated by the chronometer.

To determine the longitude of any station, the surveyor determines the local mean or sidereal time at the station by methods already described. The chronometer reading (corrected for rate) gives him the corresponding Greenwich mean or sidereal time, and the difference in these times gives the longitude of the station east or west of Greenwich : the direction is indicated by the excess or defect of the local (mean or sidereal) time over or from the corresponding instant at Greenwich. If the local time is greater than the corresponding Greenwich time, the place is east of Greenwich, and *vice versa* if less than Greenwich time it is west of Greenwich.

As the travelling rate of a chronometer is seldom the same as its stationary rate, the chronometer error on Greenwich time should be obtained as close as possible to the time of the observations for local mean time, thus reducing the uncertainty due to this factor.

The following stations send out standard rhythmic signals at the times stated ; these signals are clearly audible in all parts of the British Isles :—

Station.		Times of Emission.										
		H.	M.		H.	M.		H.	M.		H.	M.
Bordeaux,	.	8	01	to	8	06	and	20	01	to	20	06
Paris (Eiffel),		9	31	,,	9	36	,,	22	31	,,	22	36
Rugby,	. .	9	55	,,	10	00	,,	17	55	,,	18	00
Nauen,	. .	12	01	,,	12	06	,,	00	01	,,	00	06

For other stations the surveyor should consult the *Admiralty List of Wireless Signals* already referred to.

Example 15.—From the following observations determine the longitude of the observer's station :

Nauen Time Signals.				Mean Time Chronometer.		
H.	M.	BEATS.		H.	M.	S.
12	01	53		12	08	10
12	02	52		12	09	9
12	03	54		12	10	10
12	04	54		12	11	10
12	05	54		12	12	9

From star observations made on the same day the L.M.T. at a given instant was found to be 22 h. 25 m. 18 s. the chronometer at the same instant reading 22 h. 45 m. 5 s., its rate being 1·5 s. per day, losing.

From above timing observations we have :

	H.	M.	S.
G.M.T. =	12	3	52·53
Mean time chron. at same instant	12	10	9·6
Chron. fast on G.M.T.	00	6	17·07

	H.	M.	s.
Chron. time at observation	22	45	5
Chron. time when last checked	12	10	9·6

Time elapsed since checking	10	34	55·4

$$= 0·441 \text{ d.}$$

Chron. lost in this time $\quad 0·441 \times 1·5$

$$= 0·66 \text{ s.}$$

At 22 h. 45 m. 5 s. chron. is 6 m. 17·07 s. — 0·66 s. fast

$$= 6 \text{ m. } 16·41 \text{ s. fast.}$$

	H.	M.	S.	M.	S.
Hence, G.M.T. at observation $=$	22	45	5	—6	16·41
	$=$	22	38	48·59	
L.M.T. at given inst. $=$		22	25	18·00	

Required longitude $= \quad$ 0   13   30·59 W.

$$= 3° \quad 22' \quad 38·8'' \text{ W.}$$

Example 16.—The star i Pegasi was observed on Jan. 25th, 1936 for time and its W.H.A. was found to be 5 h. 20 m. 14·2 s. The mean time of observation given by a chronometer keeping G.M.T. was 18 h. 50 m. 17·55 s. The chronometer was checked on the same day at 9 h. 55 m. 20·4 s., and found to be 10 m. 15·8 s. slow, its rate being 2·35 s. per day, losing. Find the longitude of the place of observation.

	H.	M.	S.
Here, chron. time at observation $=$	18	50	17·55
Chron. time when last checked $=$	9	55	20·40

Interval since checking $=$	8	54	57·15

$$= 0·3715 \text{ d.}$$

Chron. lost in this time $\quad 0·3715 \times 2·35$ s.

$$= 0·87 \text{ s.}$$

	H.	M.	S.
Chron. slow at observation		10	15·8+0·87
	$=+$	10	16·67
Chron. time at observation $=$	18	50	17·55

G.M.T. at observation $=$ 19   00   34·22

	H.	M.	S.
From N.A., S.T. at 0 h. G.M.T., Jan. 25th =	8	12	32·449
S.T. equivalent for 19 h. 00 m. 34·22 s. =	19	03	41·587
G.S.T. at observation =	3	16	14·036

From N.A., R.A. of   H. M.   S.
    star at given inst. = 22 04 01·56
W.H.A. of star at
    given inst. =   5 20 14·20

L.S.T. at
    given inst. =   3 24 15·76   **3   24   15·760**

             Required longitude in time =   0   8   01·724 E.
                        =   2°   00'   25·86" E.

Example 17.—The easterly hour angle of the sun computed from observations on May 18th, 1936, was 3 h. 32 m. 18·5 s., the mean time of observation, read on a chronometer keeping G.M.T., was 8 h. 52 m. 18·5 s. The chronometer was checked on the preceding day at (chronometer time) 17 h. 58 m. 35·4 s., and found to be 2 m. 41·33 s. slow, its rate being 1·32 s. per day, gaining. Find the longitude of the place of observation.

To compute G.M.T. at observation :—

	H.	M.	S.
	24	0	0
Chronometer checked at	17	58	35·4
	6	01	24·6
Chronometer time at obs.	8	52	18·5
Time interval since checking	14	53	43·1

               =   0·6206 d.
      Change in error =   0·6206 × 1·32
               =   0·82 s.

	H.	M.	S.
Chron. error at observation $=+$		2	$41 \cdot 33 - 0 \cdot 82$
$=+$		2	$40 \cdot 51$ slow.
Chron. time at observation $=$	8	52	$18 \cdot 50$

G.M.T. at observation   8   54 $59 \cdot 01$

$= \quad 0 \cdot 3715$ d.

From N.A., 1936 $n = \quad 0 \cdot 3715$

E. at 0 h. $B'' = - \quad 0 \cdot 0584$

DATE	G.M.T.	VAR.
	M. S.	S.
May 17		$-1 \cdot 70$
,, 18	$+3 \; 43 \cdot 05$	$-2 \cdot 28$
,, 19	$+3 \; 40 \cdot 77$	$-2 \cdot 84$
, 20		

$0 \cdot 3715 \times -2 \cdot 28 = -0 \cdot 847$

$- 0 \cdot 0584 \times -1 \cdot 14 = +0 \cdot 066$

Corrn. to E. $= -0 \cdot 781$

                  M.   S.

E. at 0 h. G.M.T., May 18th $=+$ 3 $43 \cdot 05$

Correction $=-$ $0 \cdot 78$

E. at observation $=+$ 3 $42 \cdot 27$

	H.	M.	S.
	24	0	0
Easterly hour angle	3	32	$18 \cdot 5$
Westerly hour angle,	20	27	$41 \cdot 5$
Add 12 h.	12	00	$00 \cdot 0$
L.A.T. at observation,	8	27	$41 \cdot 5$
E. at observation,	$-$	3	$42 \cdot 27$
L.M.T. at observation,	8	23	$59 \cdot 23$
G.M.T. at observation,	8	54	$59 \cdot 01$

Required longitude $= 0$ 30 $59 \cdot 78$ W.

Required longitude $= 7° \; 44' \; 56 \cdot 7''$ W.

Identification of Stars.

Obviously, before an assistant may be trusted to make observations on stars for azimuth, latitude, or time, it is necessary that he be able, not only to identify the different constellations, but to identify the principal stars forming them. The identification of a star in a given constellation is most readily accomplished by reference to a star chart, such as that given in Appendix B, which shows the N.A. stars. All the principal stars are designated by their names and constellations, thus β (Beta) Cygni— *i.e.*, the star β in the constellation Cygnus.

To certain bright stars in the various constellations proper names have been given ; for example, the star a Pegasi is known as " Markab." The brightest star in the heavens is Sirius (a Canis Major). From the identification of these bright stars that of the constellations follows, and by reference to the star chart the identification of the principal stars forming them. The angular distance between two stars can be estimated with a fair degree of accuracy by comparison with half the angular distance from the zenith to the horizon—*i.e.*, 45°— or with that between the stars a and β in Ursa Major (the Great Bear), which is 5°.

The following short description of the positions of some of the principal stars will enable the reader to identify them :—

(*a*) The prolongation of the line formed by joining β and a in Ursa Major passes very near **Polaris** the Pole Star (a Ursa Minoris). This star forms the end of the tail of Ursa Minoris or Little Bear. The stars a and β are generally known as the " pointers."

(*b*) The line drawn through Polaris at right angles to the line from the pointers passes through **Capella** (a Aurigæ), which is about 50° from Polaris.

(*c*) If the curve formed by the tail of the Great Bear be continued southwards, the star **Arcturus** (a Boötes) is found, about 30° from the end of the tail.

(*d*) About 30° from Arcturus and in the line formed by joining Polaris and the last star but one (ϱ) in the tail of the Great Bear the star **Spica** (a Virginis) will be found.

(*e*) If the line of the pointers be continued beyond Polaris for about 60°, the " Great Square of Pegasus " will be seen. Three stars only (a, β, γ) of the square belong to the constellation Pegasus, the fourth star, that nearest the pole, is the star a in Andromedæ. The star furthest from the pole is **Markab** (a Pegasi).

(*f*) The constellation " **Cassiopeia** " will be found about mid-way between the " Great Square " and the pole star. The five stars forming this constellation are arranged in the form of the letter W.

(*q*) The diagonal of the " Great Square " from the S.E. to the N.W. corners produced about 40° passes near **Deneb** (*a* Cygni).

(*h*) If the diagonal be produced 25° further, 10° to the right of it, the star **Vega** (*a* Lyræ) will be seen.

(*i*) If we imagine an isosceles triangle having Deneb and Vega at the ends of its base with its apex downwards, the star **Altair** (*a* Aquilæ) will occupy the southern apex.

(*j*) A line drawn from the Pole Star through Capella will touch **Rigel** (*β* Orionis) about 65° from Capella. This line also passes between the bright stars **Aldebaran** (*a* Tauri) 10° to the west, at about 30° from Capella ; and **Betelguese** (*a* Orionis) 10° to the east, at about 40° from Capella.

(*k*) The star **Sirius** (*a* Canis Majoris) lies on the line joining Aldebaran to " Orion's Belt." Aldebaran and Sirius are about equidistant from the belt. Aldebaran, Rigel, Sirius, and Betel-guese form a trapezium having the belt at its centre.

(*l*) About 50° to the east of Betelguese the star **Procyon** (*a* Canis Minoris) will be seen. This star, with the stars Betelguese and Sirius, form a triangle, which is nearly equilateral.

(*m*) The bright star about 30° north of Procyon is **Castor** (*a* Geminorum), and 5° south of Castor is **Pollux** (*β* Geminorum).

(*n*) At the apex of an isosceles triangle having Castor and Procyon at the ends of its base, and 45° east, is **Regulus** (*a* Leonis).

(*o*) About 30° east of Regulus, and on the line joining Procyon and Regulus produced, is **Denebola** (*β* Leonis). This star forms an equilateral triangle with Arcturus and Spica.

(*p*) At about 30° from the South Pole is the **Southern Cross,** formed by four stars (*a, β, γ, δ* Crucis). When the line joining *a* and *γ* is in a vertical plane, the stars are very nearly on the meridian.

(*q*) South of Sirius, and about 40° from it, is **Canopus** (*a* Argus).

(*r*) A line drawn through Canopus at right angles to the line joining Canopus and the Southern Cross passes through **Achernar** (*a* Eridani).

(*s*) Canopus, Achernar, and **Fomalhaut** (*a* Pisces Australis) are nearly equidistant, being about 40° apart, and are in the same straight line.

Selecting Stars for Observation.—The star chart is also useful in enabling the observer to determine readily which star, or stars,

will be suitably placed for the purpose of his observations. Although the right ascensions and declinations of the stars can only be read very roughly from the star chart, yet the data so obtained will be accurate enough for the purpose. For example, suppose we wish to make an observation on a star when near the meridian on Nov. 10th, at about 11 h. 30 m. p.m., what star will be suitably placed for our purpose ? On referring to the chart (Appendix B), we see that the R.A. of the sun on Nov. 10th is about 15 h. 10 m., hence

S.T. = 15 h. 10 m. + 12 h., and the R.A. of the star
   = 23 h. 30 m. + 3 h. 10 m. − 24 h. = 2 h. 40 m.

We now refer to the chart again, and find that (among others) the stars γ Persei, α Ceti, and β Fornacis have about this R.A., and pass the meridian within a few minutes of each other, and from these we select that star which is most suitably placed for our purpose.

The converse problem of finding the approximate time of transit of a given star is solved as follows :—From the chart take the R.A. of the star and the R.A. of the sun (+12 h.) and subtract the latter from the former, the result (increased by 24 hours if necessary) giving the approximate time of transit. Thus, find the time at which the star Deneb (α Cygni) will transit on Dec. 20th. From the chart—

Approximate R.A. of α Cygni is . 20 h. 40 m.
Approx. S.T. at 0 h., Dec. 20th, . 5 h. 50 m.
                                   _____
Approx. time of transit,   14 h. 50 m., Dec. 20.

The Solar Attachment.—If, by any means, we set the vertical axis of a theodolite exactly parallel to the earth's axis at any place of observation, and directed the telescope to the sun, the sun's apparent path as viewed in the telescope would, for a short interval of time, appear to be horizontal, the departure of the apparent path from strict horizontality being caused by the sun's change in declination during the time of observation ; also, if after bisecting the sun's image his hour angle be set-off on the divided circle towards the west or east, according as the observation be made in the morning or afternoon, the telescope would be in the plane of the meridian ; the inclination of the telescope measured from a reference plane at right angles to the axis of the instrument would be the declination of the sun

at the instant of observation, and further, if the inclination of the axis of the instrument be determined, we should at once know the latitude of the place. It is evident that a theodolite as ordinarily constructed, cannot be used in this way, but a
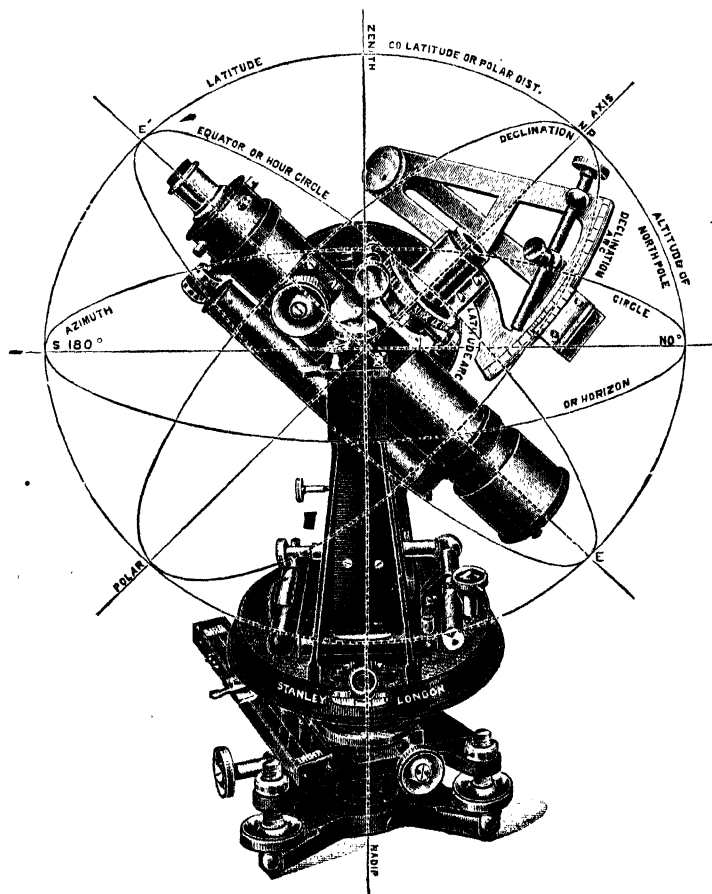
Fig. 274.

special apparatus, known as the Solar Attachment, has been devised which may be attached to the telescope of an ordinary

theodolite, so that the requisite settings and movements may
be made for the purpose of determining azimuth, latitude, and
time.   The use of the instrument does not, however, supersede
the methods given on the preceding pages, as the results obtain-
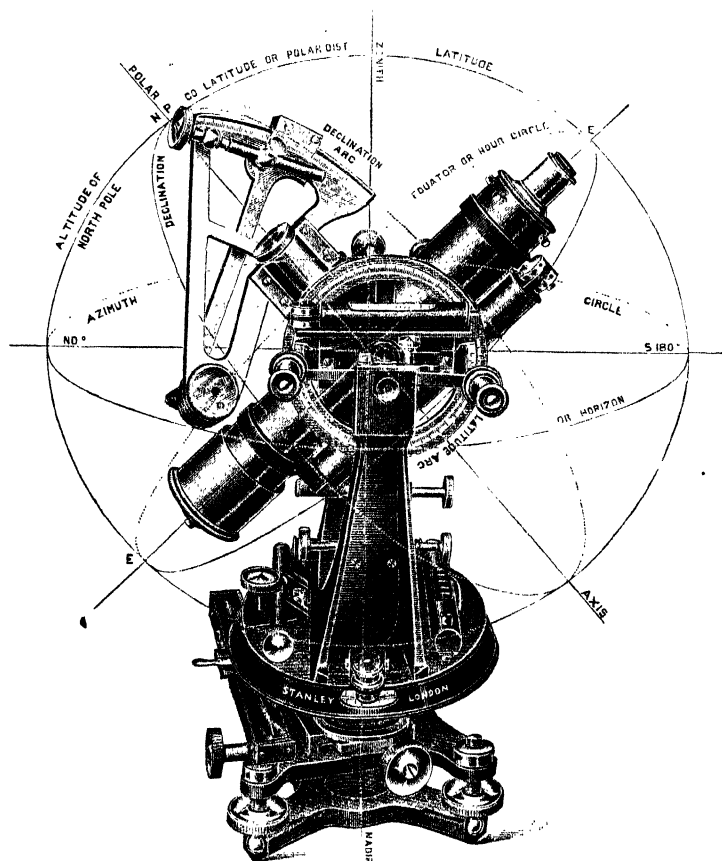
Fig. 275.

able are approximate only, being about one minute of the truth,
hence the chief field of usefulness of the Solar Attachment lies

in those branches of surveying in which the magnetic needle is used. With this instrument the directions of survey lines are referred to the direction of the true meridian, which does not change.

An improved form of this instrument, as manufactured by W. F. Stanley & Co., Ltd., London, is shown attached to a theodolite in Figs. 274 and 275. The instrument consists of a " polar axis," which is fixed in the centre of the transverse axis of the telescope perpendicular to its optical axis. Revolving upon and around the polar axis is the declination arc, which is read by a vernier carried at the end of a radial arm fitted with a clamp and tangent screw. The radial arm also carries two bracket pieces, one at either end ; each bracket piece carries a lens and a small silver plate (Fig. 276), either lens throws an image of the sun on the silver plate of the opposite bracket. Four lines are drawn upon each silver plate, two horizontal and two vertical; the lines are so spaced that the square formed by their intersection just encloses the image of the sun thrown
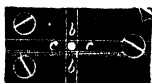
Fig. 276.

by the opposite lens. The horizontal lines (*c, c,* Fig. 276) are called equatorial lines, as the image of the sun is kept between them only when the polar axis of the instrument is placed in the plane of the meridian, and parallel to the earth's axis. The vertical lines *b, b* are called hour lines, as the sun's image is kept or brought between them for ascertaining or setting time.

Attached to the declination arc and turning with it around the polar axis is the hour circle, which is fitted with a clamp and tangent screw for fine adjustment. This circle reads to a fixed index.

Use of Solar Attachment.

To Determine the Direction of the Meridian.—The instrument is first set up in the usual way, the principal spirit level being used for final levelling ; the latitude of the place of observation is then ,set-off on the vertical arc of the theodolite and the declination of the sun, for the given day and hour (*vide*, Ex. 15, p. 560), is set-off on the declination arc. Next clamp the hori-

zontal limb at zero and point the telescope approximately north by the compass. Leave the plates clamped at zero, unclamp the vertical axis and point the solar lens on the arm of the declination arc, towards the sun, then slowly turning the declination arc with one hand and the limb of the theodolite with the other, observe the course of the sun's image on the silver plate or solar screen. The horizontal limb must be rotated until the position is found in which the declination arc will revolve about the polar axis, keeping the sun's image between the equatorial lines on the solar screen the whole of their length. The final adjustment of the limb in azimuth is made with the clamp and tangent screw to the vertical axis, and when the position is found at which the sun's image will remain between the equatorial lines the telescope is in the meridian, with the plates still clamped at zero. On unclamping the limbs and vertical circle and directing the telescope on any object, the true azimuth of the line through the object and the observer's station will be given by the reading of the horizontal limb.

It is obvious that it will be impossible to keep the sun's image between the equatorial lines on the solar screen unless the polar axis of the instrument is parallel to the earth's axis ; any error due to dislevelment, or in setting-off the latitude on the vertical arc of the instrument, or in setting-off the declination of the sun, will cause the image to deviate above or below the lines and thus discover the error. If a magnifier is used to observe the sun's image, an error of thirty seconds in the direction of the meridian can be easily detected.

When the sun's declination is south, as is the case from the 22nd of September to the 20th of March in each year, the declination arc is turned downwards or towards the plates of the theodolite, and is turned upwards for the rest of the year.

To Determine the Latitude.—The instrument having been set up and carefully levelled, the vertical arc is clamped at zero and brought to exact line by the clip screws until the bubble remains central in all positions ; then, allowing for refraction in declination, set the declination arc to read the declination of the sun at 12 o'clock, for the given day, note the equation of time and, fifteen or twenty minutes before noon, unclamp the telescope and depress its object end. Now, by moving the telescope and the declination arc from side to side, as already explained, bring the image of the sun between the equatorial lines on the solar screen and clamp the vertical axis. Next bring the declination arc exactly in line with the telescope,

using the clamp and tangent screw to the hour circle; now clamp the vertical circle, and with the tangent screws of the hour and vertical circles bring the sun's image exactly between the equatorial and hour lines, and keep it there by turning the tangent screws as may be necessary. When the sun reaches the meridian the image will remain stationary in altitude for an instant, and will then begin to rise on the solar screen. The moment the image ceases to run below the equatorial lines is apparent noon, when the index of the hour circle should indicate XII., and the required latitude is given by the complement of the angle read on the vertical circle of the theodolite. We take the complement of the angle read on the vertical arc, because the reading at the vernier is the angle moved through by the polar axis of the instrument as measured from the zenith; hence the complement of this is the inclination of the polar axis, or is the latitude of the place.

To Obtain the Mean Time.—Whenever the telescope is set in the plane of the meridian as already described, and the image of the sun is brought exactly in the square of the solar screen, the hour circle reads by its index apparent time. The corresponding mean time is then obtained as explained on p. 557, and illustrated by Example 14 on the same page. The time obtained with the instrument is only approximate.

If the instant at which the observation is made be noted from a chronometer keeping Greenwich mean time, the difference of the mean times gives the approximate longitude of the place.

All the foregoing remarks concerning the manipulation of the instrument apply when making observations north of the equator, when working south of the equator the methods described are exactly reversed.

For the convenience of surveyors, a table of mean refractions in declination is supplied by Messrs. Stanley with each of their instruments, and is published in their pamphlet on "The Solar Attachment: its Principles and Use," to which the author is indebted for much of the matter dealing with this instrument.

Projection of Maps.

When a survey has for its object the construction of engineering works, such as a line of communication, a dock or a harbour, the map or plan must be so drawn that the engineer may apply a scale to any part of it and read off actual distances. A plan of this kind is constructed without reference to the earth's cur-

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vature, or of reduction to sea level. When, however, the area covered by the survey is large, some method of representation must be adopted which will allow for the spheroidal form of the earth, and for the reduction of the whole survey to a common datum plane—usually that of the mean sea level.

With the exception of the geographical globes, all maps are drawn on plane surfaces, and as a spherical surface is not a developable surface—*i.e.*, it cannot be unrolled into a flat sheet —all systems of map projection produce distortion in a greater or less degree, the amount of distortion produced depending on the method of projection adopted and the extent of the area represented.

The first step towards the production of a map is the graticulation of the sheet on which the map is to be drawn. Each graticule is quadrilateral in outline, the sides of the graticule representing short arcs of the meridians and of the parallels of latitude respectively, the magnitudes of the arcs represented will depend on the scale of representation, and the sides of the graticules will be straight or curved according to the system of projection made use of. The sides of the graticules are laid down from the calculated lengths, as given in geodetic tables constructed for this purpose. Within the graticules the principal stations and main lines are laid down from their known latitudes and longitudes, from the distances and bearings of the sides of the main triangles, or from the " latitudes " and " departures " of the stations. The secondary and tertiary figures are plotted in like manner, and from these the plotting of the detail proceeds in the usual way.

As in every system of projection, the bearings, relative lengths, areas, latitudes, and longitudes will be more or less inaccurate, the system of projection adopted must be decided by, (1) the purpose for which the map is intended ; (2) the extent of the area to be represented ; (3) the degree of accuracy required ; and (4) the scale of representation.

Systems of Projection.—The various systems of projection used in the preparation of maps may be divided into two main classes : (1) those in which the portion of the globe represented is drawn as seen when viewed from a fixed point, called the " point of sight," and (2) those which are developments of certain curved surfaces on which the areas to be represented are projected.

To class (1) belong orthographic, gnomonic, stereographic, globular, and Sir H. James's projections. These systems are used for maps of the world in hemispheres, and for star maps.

The systems of class (2) are used for the representation of comparatively small areas in which a minimum amount of distortion is required ; they include the rectangular, trapezoidal, Colonel Blacker's, conical, De L'Isle's, polyconic, and Mercator's projections.

Orthographic Projection.—In orthographic projection the point of sight is assumed to be at an infinite distance from the plane of projection, the projectors are parallel to each other, and are perpendicular to the plane of projection. If the plane of projection contains the earth's axis, or is parallel to it, the circles of latitude
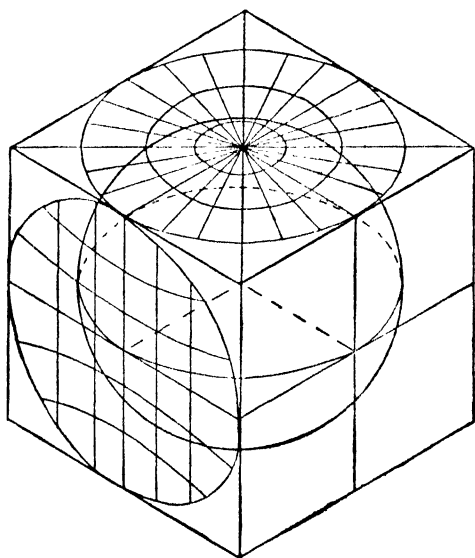
Fig. 277.

project into straight lines at right angles to the axis, but the meridian circles project into ellipses. Further, it is only in the neighbourhood of the central meridian that the area represented is free from distortion, near the profile of the sphere the distortion is very great, and causes the detail to be crowded together.

If the plane of projection is parallel to the equator, circles of latitude project into concentric circles having the pole as centre, and the meridian circles project into straight lines passing

through the pole ; hence, this system is convenient for maps of the polar regions, it is not of much use for ordinary maps.

Several examples of orthographic projection are given in Chap. XVI.

Gnomonic Projection.—In this system the point of sight is supposed to be at the centre of the earth, which may be looked upon as a transparent globe, and the portions of the earth's surface are drawn as seen projected on the faces of the circumscribing cube (Fig. 277). As the point of sight is placed at the centre of the earth, on the vertical faces of the cube, the meridians project into parallel straight lines, but the circles of latitude project into hyperbolas. On the horizontal faces of the cube, the meridians are represented by straight lines radiating from the pole, and the parallels are concentric circles having the pole as centre. At the edges of the map the detail is distorted, the distance between the meridians becoming rapidly greater as the distance from the central meridian increases (Fig. 277).

This projection is not used for terrestrial maps, but is used for star maps of small portions of the heavens.

Stereographic Projection.—The disadvantages of the gnomonic system are much reduced by placing the point of sight in the surface of the sphere, and taking the plane of projection at right angles to the diameter in which the point of sight is situated, either tangent to the surface of the sphere or passing through its centre. In either case the distortion near the edges is not so great as in the previous system.

This projection, which is known as stereographic projection, possesses the following advantages :—(1) Figures on the sphere project into *similar figures—i.e.*, circles project into circles, triangles project into similar triangles, etc., except when the planes of the figures pass through the point of sight, when they project into straight lines—under this condition *equal* arcs project into unequal *straight* lines ; (2) the angles between lines on the sphere are equal to the angles between their projections.

This projection is largely used for maps of the world in hemispheres and for star maps.

Globular Projection.—In this projection the point of sight P (Fig. 278) is outside the spherical surface and at a distance therefrom equal to one-half the quadrantal chord ; thus, in Fig. 278, $P Q = \frac{1}{2} Q S = \frac{1}{2} r/\sin 45° =$ (say) $\frac{3}{4} r.$ The plane of projection R T is tangent to the spherical surface at R. In this system the meridians and parallels are so nearly circular that they are made so on maps, the circular arcs representing the

meridians and parallels, have their centres on the equatorial
and polar diameters produced; the former pass through the
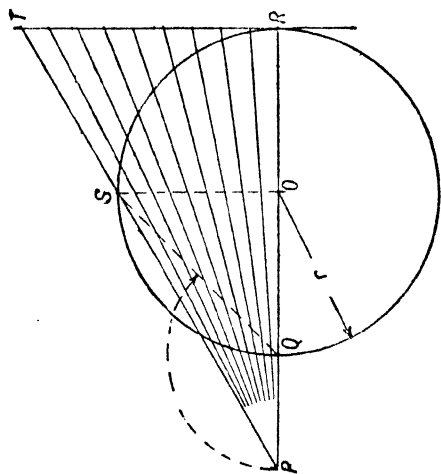extremities of the central meridian and divide the equatorial radii
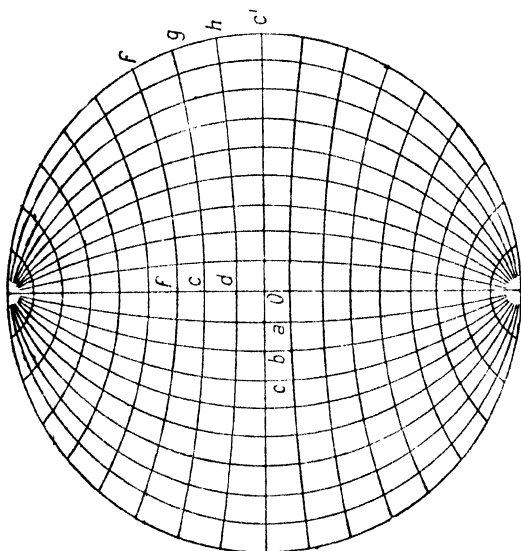

Fig. 279.—Globular Projection.

Fig. 278.—Globular Projection.

into equal parts ; the latter divide the polar radii and the quad-
rantal arcs into equal parts. Thus, in Fig. 279, the distances
$O\,a$, $a\,b$, $b\,c$, $O\,d$, $d\,e$, and $e\,f$, etc., are all equal ; also the arcs
$c'\,h$, $h\,g$, and $g\,f$, etc., are equal. The centres of the circular arcs
passing through the points of division are found geometrically
or by trial.

This projection is also largely used for the representation of
maps of the world in hemispheres.

Sir H. James's Projection is similar to the preceding, the point
of sight is placed at a distance of half the radius of the sphere
below its surface, the plane of projection is made to coincide
with that of a small circle situated between the equator and the
point of sight and 23° 30′ from the equatorial plane.

Rectangular Projection.—In this projection the meridians and
parallels of latitude are represented by straight lines drawn at
right angles to each other. To construct the graticules, draw
a central meridian and parallel of latitude, on the former lay-off
the degrees and minutes of latitude according to their values
as given in tables for this purpose ; similarly, lay-off the degrees
and minutes of longitude along the central parallel according
to their values for the given latitude, and through the points of
division thus found draw horizontal and vertical lines.

The greatest error in this system arises from taking the meri-
dians to be parallel ; in latitude 40° two meridians 1 mile apart
converge at the rate of about 1 foot per mile, and, in an area of
10 miles square, the distortion at the corners, in longitude, is
about 20 feet, with reference to the centre of the map.

This system is suitable for use on surveys in which the whole
survey has been referred to a single meridian, and, if the plotting
is done from " latitudes " and " departures " referred to the
central meridian, it will give correct distances and bearings
with reference to that meridian ; but the latitudes and longitudes
will only be approximate, the degree of error depending on the
distance from the equator—at the equator this projection is
correct.

This method is not suitable for the representation of extensive
areas, and should not be used for more than 100 square miles.

Trapezoidal Projection.—For an area greater than 100 square
miles, and when the survey has been referred to a central meridian,
trapezoidal projection is often used. In this both the meridians
and parallels are represented by straight lines, but the former
are drawn to converge. The graticules are constructed by first
drawing the central meridian along which the degrees and

minutes of latitude are laid down from the values given in tables,
and through the points of division the parallels are drawn at
right angles to the central meridian. On the two parallels
situated at one-quarter of the length of the central meridian
from the top and bottom of the sheet the degrees and minutes
of longitude are plotted from the corresponding values of the
latitudes of these parallels as given in tables, and points of equal
longitude are joined by straight lines, thus dividing the sheet
into trapezoidal spaces.

The principal error in this projection arises from drawing
the parallels of latitude as straight lines, the error caused thereby
being about 30 feet in 10 miles, in latitude 40°, and is nearly
proportional to the square of the distance east or west of the
central meridian.

The plotting is carried out as in the previous system, or from
the computed latitudes and longitudes of the principal stations.
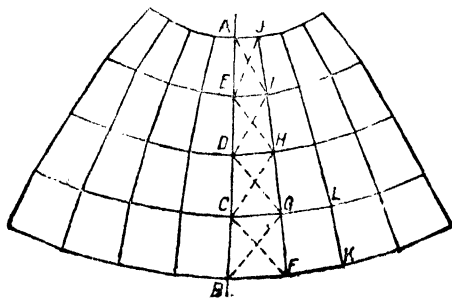
Fig. 280.

This system should not be used for an area greater than
25 miles square.

Colonel Blacker's Projection.—This projection is an improve-
ment on the trapezoidal system when the object of the map is
the representation of small areas to large scales, for which the
trapezoidal system is not suitable. It was invented by Colonel
Blacker, and has been largely used in the delineation of the
Survey of India.

A central meridian (A B, Fig. 280) is first drawn on the sheet,
and along this line the points C, D, E, $\frac{1}{4}$°, $\frac{1}{2}$°, etc. (say 1°), apart
are laid down from tabular data, thus giving the points at which
corresponding parallels would cut A B. With centre B, radius
equal to $\frac{1}{2}$° of longitude at the latitude of B, strike an arc; with

centre C, radius equal to $\frac{1}{2}$° of longitude at latitude C, strike an arc. Again, with centre C, radius equal to the diagonal C F, strike an arc cutting the arc centre B at F; similarly with centre B and the same radius, strike an arc cutting the arc centre C at G. Join G F, B F, and C G. In the same way the quadrilaterals C G H D, D H I E, and E I J A are built up from the lengths corresponding to their respective latitudes.

It is obvious that for the same change in latitude, lengths measured on the meridian are all equal (*i.e.*, G F = C B, G H = C D, etc.); also, for a given change of longitude, lengths measured on the same parallel are equal (B F = F K, C G = G L, etc.), consequently the remaining graticules may be drawn by copying those already in position between their respective parallels. In this way the graticulation of the sheet would be in 1° of latitude and $\frac{1}{2}$° of longitude; by bisecting the meridians and joining the points of section, $\frac{1}{2}$° of latitude is obtained.

The length of the diagonal of any quadrilateral may be obtained as follows :—

Let a = the length measured on the lower parallel,

         b =   ,,   ,,   upper   ,,

         c = the corresponding length measured on the meridian,

and C = the angle between the sides a and c :

then $\cos C = \dfrac{a-b}{2\,c}$, if d is the length of the required diagonal,

we have $d^2 = a^2 + c^2 - 2 \cdot a \cdot c \cos C$

$$= a^2 + c^2 - 2 \cdot a \cdot c \cdot \frac{a-b}{2\,c},$$

or, $d = c \sqrt{1 + \dfrac{a \cdot b}{c^2}}.$

The lengths a, b, and c are obtained from tables specially constructed for this system.*

Conical Projection.—In this projection the point of sight is supposed to be placed at the centre of the earth, and points on the earth's surface are projected on the surface of a tangent cone having its vertex in the earth's axis produced. The circle of contact is a parallel of latitude, and by suitably arranging the length of the slant side of the cone it may be made to touch the earth's surface at any parallel of latitude we please. On

* See *The Manual of Surveying for India* in which this projection is fully described.

developing the conical surface a map is produced in which the parallels of latitude are concentric circles having the apex of the cone as centre, and the meridians are represented by straight lines converging on the apex of the cone.

This projection is well suited for the projection of maps not extending more than $10°$ on each side of the central parallel of latitude (which is made the circle of contact of the assumed tangent cone and the earth's surface); latitudes are practically correct, but the longitudes are exaggerated on all the parallels except that at the centre.

The graticules are drawn in this system, as follows :—The central meridian is first laid down and is divided into degrees and minutes from data given in geodetic tables, and through these points the parallels are drawn as concentric circles having the apex of the cone as centre. The distance of the apex of the cone from the central parallel (i.e., the length of the slant side of the cone) is also given in tables constructed for this purpose. The central parallel is next divided into degrees and minutes corresponding to its latitude, and through the points of division straight lines are drawn passing through the apex of the cone.

As the meridians are radii of the concentric circles representing the parallels of latitude, the meridians and parallels are everywhere at right angles.

De L'Isle's Projection.—This is a modification of the preceding system, the conical surface on which the map is projected is assumed to cut the earth's surface in two parallels of latitude, one of which is placed at one-quarter and the other at three-quarters the height of the graticulated sheet. The parallels are all drawn as arcs of circles having the apex of the cone as centre, the parallels of intersection only being correctly divided. Areas between the graduated parallels are too small and the remainder too large. The area of the whole map is more nearly correct than in the preceding system.

Bonne's Projection.—In this the parallels are drawn as concentric circles ; all the parallels are properly divided, and the meridians are drawn through the divisions of equal longitude. The meridians are perpendicular to the parallels at the centre of the map only. The distortion at the corners of the map in this system is very small, and is due to drawing the parallels of latitude as concentric circles.

Polyconic Projection.—It is evident that a closer approximation to the true relative positions of the sides of the graticules in the three previous systems would be arrived at by using

several tangent cones, the cones being made tangent to the
earth's surface at the successive parallels of latitude. By this
means the radii of the successive parallels would be reduced,
not only by the increase in latitude, but also by the movement
of the apex of the cone towards the earth's surface. Conse-
quently, the circles of latitude would be no longer concentric,
but would separate as they recede from the centre of the map.
A degree of latitude at the central meridian does not, therefore,
correspond with a degree at the edges. This difference is, however,
very small, and is only perceptible on maps of very large areas.

This system is employed for maps of the United States Coast
and Geodetic Surveys.

The graticules are constructed by drawing the central meridian
A B (Fig. 281), which is divided into degrees of latitude at the
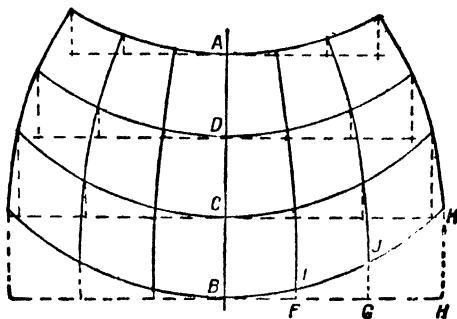
Fig. 281.

points C, D, from tabular data. Through the points of division
lines are drawn at right angles to A B, and on these lines the
meridional distances B F, F G, G H, etc., are plotted. At the
points F, G, H the divergences of the parallels of latitude F I,
G J, H K are drawn parallel to A B. Points corresponding to
I, J, and K having been found on each parallel, the meridians
and parallels are drawn through the points, either as straight
lines or as curves.

The detail of the map may be plotted from computed latitudes
and longitudes, or by polar or rectangular co-ordinates.

Mercator's Projection.—In Mercator's projection the point of
sight is assumed to be at the centre of the earth, and the curved
surface on which the map is projected is that of a cylinder tangent
to the earth at the equator. On unrolling the cylinder, the meri-
dians and parallels of latitude are represented by straight lines

at right angles to each other. At the equator this projection is correct, the meridians being drawn parallel to each other at their equatorial distance apart. At higher latitudes the distance apart of the meridians is greatly exaggerated. The distance apart of the parallels of latitude is not constant, but is augmented in the same ratio as the distance along the parallels is increased at each parallel. Thus, at the equator the distance between two meridians 1° apart is 69·00 miles, and at latitude 30°, 59·75 miles, the distance apart of two parallels at latitude 30° would be $\dfrac{6,900}{5,975}$ of its distance at the equator ; similarly, in latitude 60° (distance 34·5 miles) the length would be $\dfrac{690}{345}$, or twice the length at the equator. In latitude 60° a line of given length would represent only one-half the distance it would represent if drawn at the equator. This great variation in scale renders maps drawn on this system very deceptive to the eye ; Alaska appears nearly as large as the United States, and Greenland looks larger than Africa. In spite of this defect this projection is universally used for charts on account of its general convenience, as on this system the course of a ship sailing on a constant bearing is plotted as a straight line, thus greatly simpli‐fying the work of the navigator.

EXAMPLES.

1. The boundary between two provinces following the parallel of latitude at 45° 30′ N. is to be set-out. If the length of the boundary is 30 miles, find the convergence at its extremities. If the boundary is set-out as a straight line, by how much must the pegs at 10, 15, and 20 miles be moved south to place them in the given parallel of latitude ? Take R = 20,889,000 feet.

(*Ans.* 26′ 31·62″ ; 126·5 feet ; 156·6 feet ; 126·5 feet.)

2. Give two approximate methods of determining the azimuth of a line. Explain why the method of shadows gives a correct result at the solstices only.

3. Determine the azimuth of the line A B from the given observations made on a star when at equal altitudes :—

Star east of meridian.

Angle between R.O. at B, and star.	Circle	left,	74° 18′ 20″
,,	,,	,, right,	74° 16′ 10″
,,	,,	,, left,	73° 15′ 25″
,,	,,	,, right,	73° 13′ 10″

Star west of meridian.

Angle between R.O. at B and star. Circle left, 50° 25′ 18″
,,   ,,   ,,   ,,   right, 50° 23′ 10″
,,   ,,   ,,   ,,   left,   49° 21′ 10″
,,   ,,   ,,   ,,   right, 49° 20′ 45″

Both positions of the star are on the same side of the line and north of the observer.   (*Ans.* N. 61° 49′ 11″ W.)

4. If in Example 3 the first and last pairs of readings were obtained on opposite sides of the line, find its true bearing.
(*Ans.* N. 12° 28′ 8·7″ W.)

5. In lat. 53° 32′ 00″ N., on May 31st, 1936, the following readings on the sun were obtained with a theodolite, reading to minutes, for the purpose of determining the azimuth of a line, the sun being at equal altitudes :—

From N.A., 1936.

DATE.   VAR. IN DEC.

		1st angle, .	0° 23′, sun east of line.	
May 30	+532·8″	2nd ,,	. 0° 42′, ,, west ,,	
,, 31	+510·0″	3rd ,,	. 24° 42′, ,, ,, ,,	
June 1		4th ,,	. 25° 47′, ,, ,, ,,	

Mean time interval between the 1st and 4th angles, 57 minutes.
,,   ,,   ,,   2nd and 3rd ,,   51   ,,

Find the true bearing of the line.   (*Ans.* S. 12° 39′ 40·4″ E.)

6. Describe how you would determine the declination of the magnetic needle by aid of a 5″ theodolite fitted with a trough compass. To what degree of accuracy would you expect your result ? If the magnetic bearing of the line in Example 5 is 183° 41′, find the declination for the given date and place.
(*Ans.* N. 16° 21′ $\pm$ 3′ W.)

7. The star *β Ursæ Majoris* was observed at eastern elongation in lat. 53° 32′ N. on December 6th, 1936, and the mean angle between a line and the star was found to be 75° 18′ 20″, the star and line being on opposite sides of the meridian. Find the azimuth of the line. Declination of *β Ursæ Majoris* on the given date, 56° 42′ 53·2″ N.   (*Ans.* N. 7° 52′ 56·3″ W.)

8. In Example 7, find the altitude of the star at observation (neglecting refraction), and, if the longitude of the place is 5 h. 40 m. 18 s. W., the R.A. of the star at the given date 10 h.

58 m. 3·9 s., and S.T. at G.M.M. 4 h. 58 m. 23·84 s., find the
L.M.T. of observation. (*Ans.* 74° 9′ 32·9″ ; 4 h. 8 m. 41·8″ s.)

9. Determine the azimuth of A B from the following observations :—Mean altitude of φ *Pegasi* (rising), 30° 20′ 18″ ; latitude
of place, 50° 17′ 30″ N. ; bar., 29·8″ ; therm., 39° F. ; angle
between the star and line, 75° 18′ 45″ ; star south of the
observer ; line between the star and elevated pole ; declination
of star at date of observation, N. 18° 39′ 46·31″.
$$\text{(}Ans.\text{ N. 21° 47′ 47·2″ E.)}$$

10. On July 27th, 1936, in latitude 59° 51′ 39″ N., longitude
1 h. 10 m. 39 s. E. ; mean observed altitude of the sun's centre,
22° 30′ 18″ ; mean of times, 4 h. 10 m. 5 s. p.m. ; angle between
the line and the sun at observation, 40° 17′ 33″, the line being
between the sun and the depressed pole. Watch 3 seconds fast
at noon, gaining 0·8 second per day. Bar., 29·5″ ; therm.,
50° F.

<div align="center">From N.A., 1936.</div>

Date, July 26.	0 h., G.M.T.,		Var. − 796·6″
„ „ 27.	Sun's App. Dec. + 19° 18′ 56·4″,		Var. − 815·7″
„ „ 28.			Var. − 834·5″
„ „ 29.			

$$n = 0·62547 \text{ d.} \quad B'' = -\ 0·0586.$$

Find the azimuth of the line.   (*Ans.* S. 49° 27′ 10·2″ W.)

11. Solve Examples 9 and 10 graphically.

12. Determine the latitude of the observer from the following
data :—Meridian altitude of α *Ursæ Majoris*, after correction
for the instrument, 52° 30′ 10″, the star being between the
horizon and the pole. Declination of star at date of observation,
+ 62° 14′ 34″. Bar., 29·4″ ; therm., 55° F.
$$\text{(}Ans.\text{ 80° 14′ 52·7″ N.)}$$

13. From the data given in Example 12, find the latitude of the
place on the assumption that the star was at upper culmination
at the time of observation.   (*Ans.* 24° 44′ 00·7″ N.)

14. Find the latitude of the place of observation from the
following data :—Meridian altitude of α *Pavonis*, 34° 15′ 0″.
Therm., 44° F. ; bar., 28·56″. Star south of the observer.
Declination of α *Pavonis* at the date of observation, 57° 4′ 30″ S.
$$\text{(}Ans.\text{ 67° 9′ 7·9″ S.)}$$

15. In long. 4 h. 34 m. E., on May 25th, 1936, the meridian
altitude of the sun's upper limb, 45° 30′ 15″, sun south of the
observer. Bar., 28·6″ ; therm., 40° F. Semi-diameter, 15′ 48·73″ ;
level error, − 7″. Find the latitude of the place.

Declination at G.A.N., 1936.

DATE.	DECLINATION.	VARIATION.
May 23	+ 20° 36′ 00·2″	
,, 24	+ 20° 47′ 16·4″	+ 676·2″
,, 25	+ 20° 58′ 11·2″	+ 654·8″
,, 26	+ 21° 08′ 44·2″	+ 633·0″

(*Ans.* 65° 42′ 38·4″ N.)

16. From the following data obtain the latitude of the place :—
Mean altitude of *Polaris* (out of the meridian), 60° 17′ 20″ ;
longitude of place, 5 h. 15 m. 10 s. W. ; L.M.T. at observation,
11 h. 15 m. 40 s. p.m., December 9th, 1916. Bar., 29·5″ ; therm.,
35° F.   Watch 5 seconds fast at noon, gaining 0·6 second per
day.

Apparent Altitude.	Refraction Barometer 30″ Thermometer 50° F.	Add difference for + 1″ bar.	Subtract difference for + 1° F.
60°	0′ 33·6″	0′ 1·1″	0′ 0·07″
61°	0′ 32·3″	0′ 1·1″	0′ 0·07″

Declination of *Polaris*, N. 88° 52′ 06·05″ on December 9th.
       ,,   ,,   N. 88° 52′ 06·33″   ,,   10th.
R.A. of *Polaris*, 1 h. 30 m. 49·92 s.
S.T., at G.M.N., December 9th, 1916, 17 h. 11 m. 34·79 s.

(*Ans.* 59° 29′ 8·38″ N.)

17. From the data in the preceding example, and using the
tables given in the *Nautical Almanac,* find the latitude of the
place.

From Table I.—Argument, L.S.T. of observation.
        1st correction   4 h. 30 m.,   − 48′ 5″
                      4 h. 28 m.,   − 48′ 30″

From Table II.—Arguments, L.S.T. and altitude.

		Altitude.	
	L.S.T.	60°	65°
2nd correction—	4 h. 0 m.,	+0′ 26″	+0′ 32″
	4 h. 30 m.,	+0′ 35″	+0′ 43″

From Table III.—Arguments, L.S.T. and date (December 9th,
1916).

		Date.	
	L.S.T.	Dec. 1.	Dec. 31.
3rd correction—	4 h.	+0′ 53″	+1′ 3″
	6 h.	+0′ 46″	+0′ 56″

(*Ans.* 59° 29′ 8·84″ N.)

18. In long. 2 h. 1 m. 19 s. E., the star δ *Aquarii* was observed when near the meridian on December 9th, 1936, and the following readings were obtained :—

Observed Altitude.			Mean Times of Observation (p.m., corrected).		
°	′	″	H.	M.	S.
13	46	52	5	32	4
13	47	15	5	34	6
13	47	25	5	35	54
13	47	35	5	36	40
13	47	54	5	37	56
13	47	30	5	39	50
13	47	10	5	41	3
13	46	40	5	44	1
13	46	30	5	45	6

Star south of observer.

Determine the latitude of the place. Assume refraction = 58″ cot mean altitude.

From N.A.

Declination, δ *Aquarii.*   R.A. δ *Aquarii.*

DATE, 1936.	DEC.	VAR.	DATE, 1936.	H.	M.	S.	VAR.
Nov. 25·8			Nov. 25·8				
Dec. 5·7	−16° 09′ 20·83″	67	Dec. 5·7	22	51	19·281	126
„ 15·7	−16° 09′ 21·37″	54	„ 15·7	22	51	19·161	120
„ 25·7		41	„ 25·7				108

(*Ans.* 60° 07′ 06·5″ N.)

19. In long. 4 h. 32 m. 10 s. W., lat. 52° 37′ N., on May 30th, 1936, the sun was observed at equal altitudes for the purpose of determining time ; the mean chronometer readings obtained were 10 h. 40 m. 15 s. a.m., and 1 h. 12 m. 43 s. p.m. Determine the chronometer correction at noon.

Declination at 0 h. G.M.T., 1936.   G.M.T. of G.A.N., 1936.

DATE.	DEC.	VAR.	DATE.	G.M.T.			VAR.
				H.	M.	S.	S.
May 29			May 29				
„ 30	+ 21° 42′ 47·0″	+ 555·4″	„ 30	11	57	22·60	+7·96″
„ 31	+ 21° 51′ 39·8″	+ 532·8″	„ 31	11	57	30·98	+8·38″
June 1		+ 510·0″	June 1				+8·78″

(*Ans.* + 1 m. 00·5 s. Chronometer slow.)

20. In long. 4 h. 30 m. W., latitude 53° 27′ N., on December 13th, 1936, the stars β *Pegasi* and α *Tauri* were observed for

time. Watch supposed to be 42·4 seconds slow at noon, losing at the rate of 1·2 seconds per day.

β *Pegasi* (setting) mean observed altitude, 23° 15′ 18″.
    Mean of times,   .   .   .   11 h. 20 m. 40 s. p.m.
α *Tauri* (rising) mean observed altitude, 18° 50′ 14″.
    Mean of times,   .   .   .   5 h. 40 m. 42 s. p.m.
Bar., 30·2″; therm., 29° F. Find chronometer correction at noon.

From N.A., R.A., β *Pegasi*, Dec. 13th, 1936, 23 h. 00 m. 43·6 s.
                    Declination, +27° 44′ 43·1″ N.
    ,,   ,,   α *Tauri*,   4 h. 32 m. 20·5 s.
                    Declination,   16° 23′ 6·7″ N.
S.T. at G.M.M. on given date, 5 h. 25 m. 59·7 s.
                    (*Ans.* 34·7 seconds slow at noon.)

21. At X in Australia, approx. latitude 33° 40′ S., longitude 10 h. 4 m. 50 s. E., the star ζ *Aquilœ* was observed when near the meridian for time, on June 17th, 1936, and the following observations (on the same side of the meridian) were made :—

Mean Observed Altitude.	Mean of Times.
o ′ ″	H. M. S.
42 54 18	1 15 12 (a.m.)
42 56 4	1 19 5

Correction for refraction, 1′ 4″.
From N.A., R.A. ζ *Aquilœ* 19 h. 2 m. 31·6 s., declination 13° 46′ 3·2″ N.
S.T. at G.M.M., June 17th, 1936, 17 h. 40 m. 16·3 s.
    Find the chronometer correction.
                    (*Ans.* + 16·7 s. (Chron. slow))

22. Determine graphically the hour angles of the stars at observation in Example 20.

23. Determine the approximate L.M.T. of true rising and setting of the sun in latitude 40° 30′ N., longitude 60° 15′ W., on July 18th, 1936. Verify your answers by a graphical construction.

DATE	DECLINATION AT G.A.N.	VAR.	DATE.	E. AT 0 H. G.M.T.	VAR.
July 17			July 17	M. S.	
,, 18	+21° 01′ 09·1″	− 623·7″	,, 18	− 6 00·76	−5·19
,, 19	+20° 50′ 24·1″	− 645·0″	,, 19	− 6 05·42	−4·66
,, 20		− 661·0″	,, 20		−4·12

                    (*Ans.* 4 h. 50 m. ; 19 h. 23 m.)

24. From the following data determine the longitude of the observer's station :- -

Rugby Time Signals.			Mean Time Chron.		
H.	M.	BEATS.	H.	M.	S.
9	55	45	9	57	20
9	56	46	9	58	21
9	57	45	9	59	20
9	58	45	10	00	21
9	59	46	10	1	21

A star observation on the same day gave the L.M.T. as 20 h. 15 m. 25 s., the reading given by the chronometer at the same instant being 19 h. 40 m. 35 s., chronometer rate 1·3 seconds per day, gaining. (*Ans.* 9° 06′ 36·75″ E.)

25. The computation of the results of an observation on the star *μ Ursæ Majoris* on Feb. 1st, 1936, gave its easterly hour angle as 6 h. 45 m. 26·35 s. The mean time of observation read on a chronometer keeping G.M.T. was 18 h. 35 m. 20·26 s. The chronometer was checked at 17 h. 55 m. 18·52 s. on the same day and found to be 1 m. 18·63 s. slow, its rate being 3·65 s. per day, losing. Find the longitude of the place of observation.

From N.A., 1936.

S.T. at 0 h. G.M.T., Feb. 1st, 8 h. 40 m. 08·323 s.
R.A. of *μ Ursæ Majoris*.

		R.A.		VAR.
Jan. 21·1 d.	H.	M.	S.	259
,, 31·1 ,,	10	18	34·673	199
Feb. 10·0 ,,	10	18	34·872	134
,, 20·0 ,,				

(*Ans* 3° 19′ 24·1″ E.)

26.—From the following data determine the longitude of the place of observation :—Date, July 8th, 1936, W.H.A. of true sun, 7 h. 16 m. 40·5 s., chronometer time of observation, 19 h. 20 m. 16·55 s., chronometer error on G.M.T. at noon (12 h.) of the preceding day 4 m. 25·63 s. slow, gaining 2·32 s. per day.

From N.A., 1936.

DATE	G.M.T. AT G.A.N.			VAR.
July 7	H.	M.	S.	+9·40
,, 8	12	04	52·42	+9·03
,, 9	12	05	01·45	+8·64
,, 10				

(*Ans.* 0° 45′ 51·7″ W.)

CHAIN SURVEY

AT

WIGGLESWORTH HALL.

LONG PRESTON, YORKSHIRE.

———————

To be plotted to a scale of 3 chains to 1 inch,

Wigglesworth Beck
Line ② F to H

F
30
22
23
000
Plank
58
98

F
②
6531
22
23
30
4935
Ditch
4893
4886   2
4878
14
22
4686
25
5
E
Rail
Post and
3521
3
3241
4
D   Rubble
DTY   Wall
Post & Rail
1939
15a
1933   g
16
1707   x
11
5
1552   C   10
13
914   B
12
152   a
8
A
000   9

Line ① A to F

Mag Bearing E to A N 7°30 W
Bearing of Base Line E to A N 10° 5 W
Base Line E to A 4683
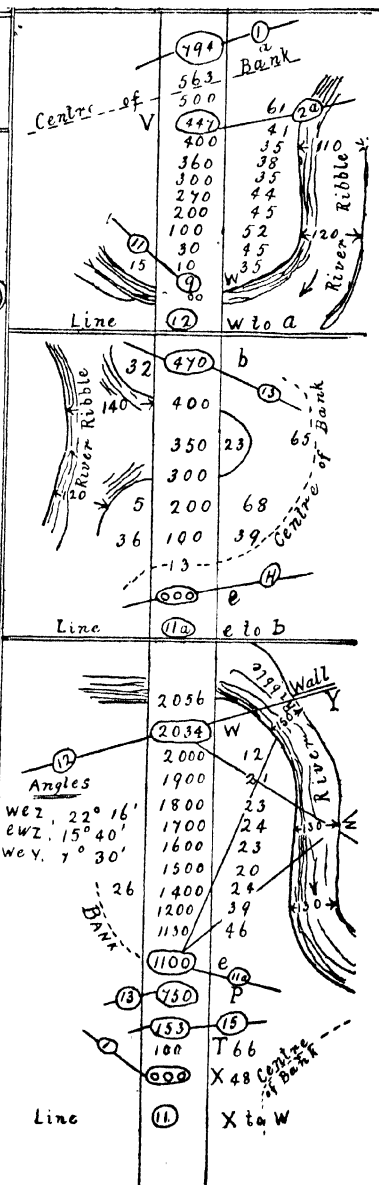Base Line A to E 4686

27,8   1300   35
2
7,   1230
140   24
38   1200
1197   1
1163   25
47   1100   33
64   1000   21
N45   50   900   18
35   800
26   700   38
41   600
120   47   500
River
49   400   64
11   318   U
300   64
38   230
200   50
of Bank
42
Centre
18   000
D   4
Line ③ D to H.

Ribble
Centre of Bank
Fence Following Centre Line of Bank

3   2406   H
2335   12
Long Plantation
Bank Begins
5   1498   G
Slip   Ditch
Pasture
1267
Ditch
1231
Straight

ERRATUM.—Line 3, D to H.   Centre of Bank should read **142** (not 42).

ERRATUM.—Line 4, D to I. Fence is *post and rail throughout,* not as on plan—part post and rail and part wall.
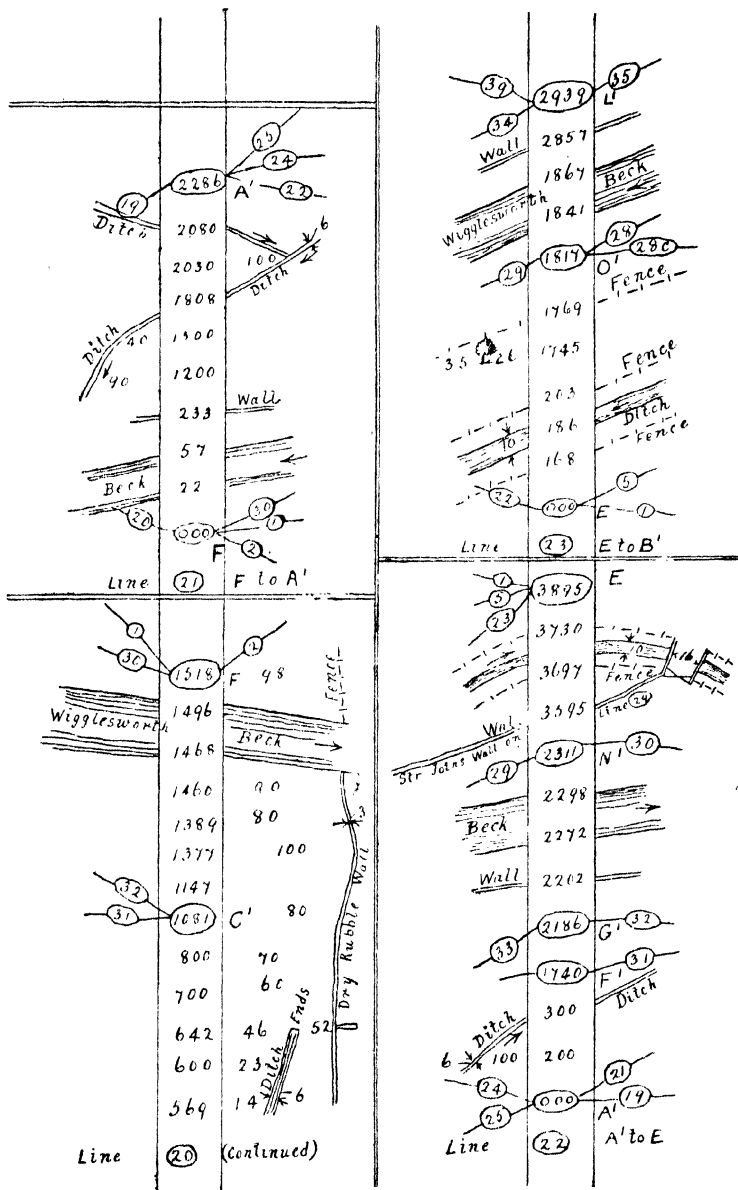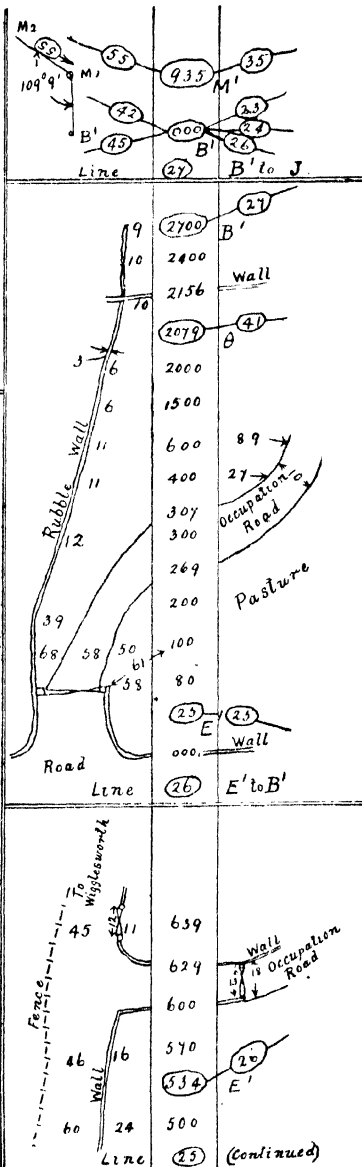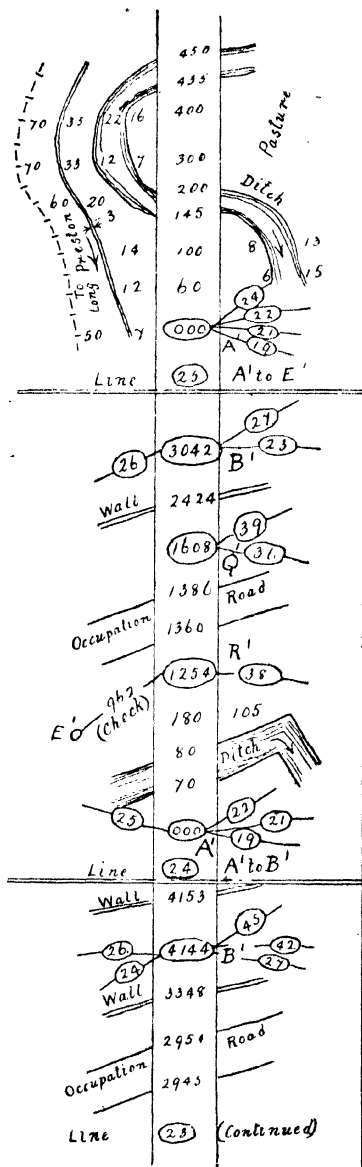
Line   N to C

Wall   4240   Gate
L   22
3263   15
2929   15
2850   15
2845
2582
2232
1953   13
1937   19
155   19
A   15
139   17
Bottom of   108   Bank
95   30
Centre of   88   Bank
Bottom of   64   Bank
55   28
(Top of Bank   26   O
8 links wide)   25   28
14   28

River   Ribble
Line   9   O to L

A   3777
Pasture   910   N
909   25   WALL
100   7   Meadow
27   J
Line   8   J to A

Centre of   V
Bank   794   Bank
563
500
447   61
400   41   110
360   35
300   38
270   35
200   44
100   45   120
52
15   45   River Ribble
10   35
W   60
Line   12   W to a

River Ribble   32   470   b
140   400   73
350   23   65
300
5   200   68   Centre of Bank
36   100   39
13
e   H
Line   11a   e to b

2056   Wall
2034   W   Y
Angles   2000   12
wez. 22° 16'   1900   2
ewz. 15° 40'   1800   23
wey. 7° 30'   1700   24
1600   23
1500   20
1400   24
26   1200   39
1130   46
BANK   1100   e
13   750   P
153   15
100   T 66
X 48   Centre of Bank
Line   11   X to W

Left column (top to bottom):

140
3
9
120 10
Y
Line ⑰ Y to U

550 a⸱
500 58
400 61
200 64
⑱
⑯
000

Ribble River
30 7
15
22
22
33
120
19
19
34
58
125 ·3

680
⑰⑱
672
600 46
500 35
400
330
300 36
272 Fence
250 40 |
200 36 |
150 24 |
104
100
000
⑪ X
①
Line ⑯ X to Y.

Fence
Wall
432
428 ①
9
370
300 55
231
200 18
River Ribble 170 16
117
100 73
000
⑮ f
Line ⑮a f to g

Right column:

Ditch
523 9
506 8
478 6
313 29 Well
⑲ 000 D' 4
Line ⑳ D' to F.

960 2
⑳ 932 D'4 3
800 27
600 26 Dry Ribble Wall
500 23 To Long Preston
380 20
300 17
200 16
184
177 Ditch
㉑㉒ To Wiglesworth Road 33
㉔ 000 A' 6
㉕ 33
Line ⑲ A' to D'

936 ④
③ D' ①
Centre 270 of Bank
⑰ 000 Y
⑯
Line ⑱ Y to D

39 1131 64
19 1100 U ③
9 1046 Fence
21 1000 105
Ribble 75 950
135 43 900 72 Centre of Bank
54 800 42
34 700 43
6 600 52
River
Line ⑰ (Continued)

Left upper column (Line 21, F to A'):

㉕
㉔
⑲ ②②⑥ A' ㉒
Ditch
2080
2030   →
1808   100 Ditch ⑥
Ditch 40 1500
      90 1200
         233   Wall
          57
Beck   22
⑳ ⓪⓪⓪ ㉚
       F ②
Line ㉑ F to A'

Left lower column (Line 20, continued):

①
㉚ ⑤①⑧ ① F 48   Fence
Wigglesworth 1496
         1468   Beck →
         1460   90
         1389   80   3
         1377   100
㉝   1147
㉛ ⑩⑧① C'   80
         800   70
         700   6c
         642   46   52
         600   23
         569   14 6
Line ⑳ (continued)

Right upper column (Line 23, E to B'):

㊴ ②⑨③⑨ L' ㊶
㉞
      2857   Wall
      1867   Beck
Wigglesworth 1841
㉙ ⑧①⑨ O' ②⑧ ㉘c
      1769   Fence
35 Lat 1745   Fence
      203   Ditch
  70 186   Fence
      168
㉒ ⓪⓪⓪ ⑤ E ①
Line ㉓ E to B'

Right lower column (Line 22, A' to E):

①
⑤ ③⑧⑨⑤ E
㉓
      3730
      3697   Fence Line㉔
      3595
Str Joins Wall on Wall ㉙ ②③①① N' ㉚
      2298   →
Beck 2272
Wall 2202
㉝ ②①⑧⑥ G' ㉜
   ⑰④⓪ F' ㉛ Ditch
      300
   Ditch
6 100 200
㉔ ⓪⓪⓪ A' ⑲ ㉑
㉕
Line ㉒ A' to E

Jans Mell on (24) at 3595

Wall 28

Fence Stn

Centre of Embankment 7 Links Wide at top

(1306)	30
N¹	13 22
1000	39
698	20
540	26
380	
8 356	19
300	
60	29
Bank Begins	23
48 42	
(28b) (000)	O¹ 28
Line (29)	O¹ to N¹

Wigglesworth Beck

23
19
27
35

Gate
43
46 14 19
21
Wall

(522)	29
O	23 23
246	at
100	22
	224 to V.
(000)	28a
Line (28b)	Ps to O¹

Beck

Wall 3

661	
(657)	28b P₃
23 500	39
Line (28a)	(Continued)

31	(640)	34a	
		H¹	
	577	34	
	500	85	
	100	44	
32	(000)	G 22	
Line (33)	G¹ to H¹		

Wall

	852	
		3
22 (802)	G¹ 14	
	400	43
31		
20 (000)	C¹ 58	
Line (32)	C¹ to G¹	

Wall

	36
(672)	F¹ 22
32	
(000)	C¹ 20
Line (31)	C¹ to F¹

Long Plantation

	793
	752
2 (693)	122
1	26
600	32
370	20 27
200	21 36
(000)	N¹ 22 29
Line (30)	N¹ to F¹

Beck

Bank

Pasture

Centre

Column 1 (left):

34b — Check to J' 319.

462   I'

3
46   449
35   400

Wall

26   300
18   148
34   000   K'
61

Pasture

Line   34a   K' to I'

2085   of road
Centre
2055   61
39   23   L'
1679   64   3
1661   58   Wall
1639   53
1529   34a   K' Check to I' 319
830   J'
449   34b   33   H'
435   70   Wall
141   114   Teenley Spring
100   76   133
46   100

Pasture

000   22   F'
31   Line   34   F' to L'

Column 2 (right):

38   100   S' 5 6   Wall   Quarry
80
60   18
40   33
20   60
000   142
Line   36   S' to Q'

617   27   M'
500   93   16   113
448   46
Spring   446   67
415   43
413   54
400   76   93
245   8   Gate
240   44   62

Pasture   Wall

41   222   φ
200   45   61
42   Road
Occupation 18   24
38   000   23   L'
Line.   35   L' to M'

70   677
Wall Str.
33   673   38   34   H'
62   100
60   10   Check to J' 319
000   I'   34a
Line   34b   I' to H'

Quarry Wall

54	392	8
	379	
	367	Teenley Hill
	(000)	(24) R'

Line (35) R' to H'

(38)

(1070) T'

800	70
700	36
600	6
500	4
400	17
300	43
200	72

Quarry Wall

Pasture

(39) (000) U'

Line (37) U' to T'

(24)

(877) (39)

Q

780	Road
767	Occupation
600	154
550	99 3
500	70
435	41
400	29
300	15
260	8
200	5

Pasture

Quarry Wall

Line (36) Continued

Quarry Wall

22		941
53		910
41	32	900
72	63	850
	(37)	(780) U'
83	74	775
52	45	560
49	43	300
52	45	230
51	44	170
43	34	130
		85
		74

Occupation Road

Pasture

Pasture

(29)

(35)

(39) (66) L'

(34)

Wall ooc

Gate

Line (39) L' to Q'

Wall

20 1625

(34b)

(34) (1556) (33)

(37) (1000) H'

(36) (624) T'

460 S,

Pasture

Quarry Wall

84

416 15

Line (38) (Continued)

ERRATUM.—Line 40, I to A₂. Wall ends at **622** (not 303).

28 50

Wall

2 (000) (42)

(43)

Line (47) B₂ to P¹

561

(549) (44)

Wall B₅

Yard 32 533

Stable 35 481

3 64 260

(59) (000) (43)

(45)

B₄

Line (46) B₄ to B₅

585 Wall

15 572

8

15 (490) (61)

Wall (46)

15 400 B₄ (43)

(42) (27)

Pasture 10 (000) (23)

B₁ (26) (24)

Line (45) B₁ to B₄

116

(106) B₅

(46) 46 31 3

Pasture 27 56

90° (50)

(000) (43)

Line (44) B₃ to B₅

(49) (251) B₇

High Barn 233 Manure Tip 48

158 at 40

Crate 118 Joins Wall at

785 in (55)

Manure Tip

37 4 54 68

5

4 Wall

(43) (000) (47)

(42) B₂

Line (48) B₂ to B₇

(27) (966) P¹ 45

(56) 900 44 25

850 38

778

735

687

Beck Wall 666

(650) D₃

600

(450) D₂

(531) F₂

Wall

320

90 251

Line (47) (Continued)

Traverse round High Barn

Wigglesworth Hall

54   72   (63)   M₂

Check to   57   25
Y₂3½8   22
46

cuts   11
32   000   14 16 11
Dairy
50

Line   (55)   M₂ to M₁

(53)   (292)   (52)
12   264   Z₂
252   Lawn
Lawn   10 18   130
37
8
(55)   (000)   (51)
X₂
Line   (54)   X₂ to Z₂

(55)   (296)   M₂ Wigglesworth Hall
14   260   2
194
193   Cuts
Lawn   Arch   175   Farm House   Wall
4 13   135   6
120   13   Cuts
119
2   91   15   34
89   57   19   107
50   10
(54)
(52)   (000)   Z₂
Line   (53)   Z₂ to M₂

(27)   (1267)   M₁   Wall
19   900
11   837
Gate   15   832
827
50   802
Gate   75   785   56   Join's Wall on (43)
100   10k   783   53
775
760
740   67   Barn
694   46   82
653   60 78   73
629   8
621   93
606   74
603   61   Gate
600   cuts
Wigglesworth Beck   578
548
Footpath   Stile   Higher   Bridge
(535)
515   C₂ 50   (57)
503
9   47   450   47   Road
9   428
383
29   47
Garden   150   2   363
33   241   18   Plantation
(57)   (235)   X₂   12
(54)   200   6
Lawn   15   158
19   100   11
Line   (55)   (Continued)

Bridge
17
26
25
Line

100
54
000
57

13
55
C_2
C_2 to C_3

57
10   860
3   800
740
Wall   672
10   600
570
520
500
400
at   373
350
79   300
275
258
256

C_2
Wall   Stile
34
27
3
16
35
Low
Barn
26
27

Wigglesworth Beck
16
36
25

Footpath
Wall

67
72
17
20

Bridge
57
73
80
89
86
27
70
47
53
Line

242
240
200
150
100
50
000
56

25
Wall
3
55
27
P_1 to C_2

284
274
59   000
Line   60
61
C_6 16
C_5 35
3
C_5 to C_6

60   301
345
290
190
115
58   000
Line   59
Wall
C_6
38
32
67
C_4
C_4 to C_5

Tofts Plantation

Wall   587
54   444
Pasture   200
100
90
80
50
8
57   000
Line   58
50
C_4 71   100
32   70   25
2
20
3
Wall   29
C_3
C_3 to C_4
Beck
Cart track
Str   11

58   547
8
83
20
400
200
C_3 23
15
4
Str   44
Road
25
12

Line	Bearings Fore	Back
X'a'	54° 30'	234° 20'
a' b'	354° 30'	174° 15'
b' c'	100° 30'	280° 30'
c' d'	136° 00'	315° 5 0
d' e'	292° 30'	112° 00'
e' a'	272° 10'	92° 30'

Sketch Plan
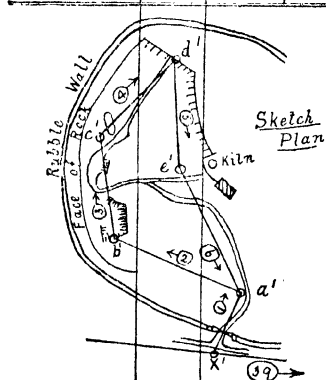
Compass Traverse in (Limestone) Quarry

Compass Traverse from C₂ to B₄.

Line.	Bearings. Fore.	Back.
C₂ to C₃	256° 30'	76° 30'
C₃ to C₄	234° 15'	54° 15'
C₄ to C₅	173° 30'	353° 30'
C₅ to C₆	121° 45'	301° 30'
C₆ to B₄	145° 00'	325° 00

Wall

322

15   (286)   ①
          ②

12   152

121   38

76   117   12

55 23   99

88

60   56   76   12

15   68   38
Kiln   20

46   15

⑤ (000)   e'

Line   ⑥   e' to a'

Centre line of Cart track

Retaining Wall

68

Kiln → (15)   ⑥   (333)

82   50   ④
       (000)
          d'

Line   ⑤   d' to e'

76   236   27

(232)   ⑤
      d'

137

Line   ④   Continued

River Ribble

Bearings

Line	Fore	Back	Distance
A p	86°50'	267°0'	87
p q	126°20'	306°15	208
q r	136°10'	316°5	127
r s	164°30'	344°30	194
s t	198°30'	18°30	351
t u	173°30'	353 00	383
u v	140°20'	320°00'	237
v w	179°50'	359°40	227
w x'	233°20'	53°20	226
x' y	270°30'	90°30	402
y x	255°20'	75°30	135
X A	7°30'	187°30	1707

Traverse on Centre line of Embankmt.
N B *Stations* are on Centre line
   Average Width at Bottom of Bank
   35   and at top   8 links

End of Survey

Details of Farm Buildings

APPENDIX A.

ASTRONOMICAL REFRACTION.

Note.—If the Barometer is over 30 inches the Barometer correction is *additive* to the Mean Refraction ; if the Thermometer is less than 50° the Thermometer correction is *additive* to the Mean Refraction ; and *vice versâ*.

The Mean Refraction is for Barometer 30″ and Temperature 50° Fahr.				
Apparent Altitude.	Mean Refraction.	Difference for 1′ Altitude.	Difference for + 1 in. Bar.	Difference for − 1° Temp.
0° 0′	33′ 51″	11·7″	74″	8·1″
5′	32′ 53″	11·3″	71″	7·6″
10′	31′ 58″	10·9″	69″	7·3″
15′	31′ 5″	10·5″	67″	7·0″
20′	30′ 13″	10·1″	65″	6·7″
25′	29′ 24″	9·7″	63″	6·4″
30′	28′ 37″	9·4″	61″	6·1″
35′	27′ 51″	9·0″	59″	5·9″
40′	27′ 6″	8·7″	58″	5·6″
45′	26′ 24″	8·4″	56″	5·4″
50′	25′ 43″	8·0″	55″	5·1″
55′	25′ 3″	7·7″	53″	4·9″
1° 0′	24′ 25″	7·4″	52″	4·7″
5′	23′ 48″	7·1″	50″	4·6″
10′	23′ 13″	6·9″	49″	4·5″
15′	22′ 40″	6·6″	48″	4·4″
20′	22′ 8″	6·3″	46″	4·2″
25′	21′ 37″	6·1″	45″	4·0″
30′	21′ 7″	5·9″	44″	3·9″
35′	20′ 38″	5·7″	43″	3·8″
40′	20′ 10″	5·5″	42″	3·6″
45′	19′ 43″	5·3″	40″	3·5″
50′	19′ 17″	5·1″	39″	3·4″
55′	18′ 52″	4·9″	39″	3·3″

ASTRONOMICAL REFRACTION—*Continued.*

Apparent Altitude.	Mean Refraction.	Difference for 1′ Altitude.	Difference for + 1 in. Bar.	Difference for – 1° Temp.
2° 0′	18′ 29″	4·8″	38″	3·2″
5′	18′ 5″	4·6″	37″	3·1″
10′	17′ 43″	4·4″	36″	3·0″
15′	17′ 21″	4·3″	36″	2·9″
20′	17′ 0″	4·1″	35″	2·8″
25′	16′ 40″	4·0″	34″	2·8″
30′	16′ 21″	3·9″	33″	2·7″
35′	16′ 2″	3·7″	33″	2·7″
40′	15′ 43″	3·6″	32″	2·6″
45′	15′ 25″	3·5″	32″	2·5″
50′	15′ 8″	3·4″	31″	2·4″
55′	14′ 51″	3·3″	30″	2·3″
3° 0′	14′ 35″	3·2″	30″	2·3″
5′	14′ 19″	3·1″	29″	2·2″
10′	14′ 4″	3·0″	29″	2·2″
15′	13′ 50″	2·9″	28″	2·1″
20′	13′ 35″	2·8″	28″	2·1″
25′	13′ 21″	2·7″	27″	2·0″
30′	13′ 7″	2·7″	27″	2·0″
35′	12′ 53″	2·6″	26″	2·0″
40′	12′ 41″	2·5″	26″	1·9″
45′	12′ 28″	2·4″	25″	1·9″
50′	12′ 16″	2·4″	25″	1·9″
55′	12′ 3″	2·3″	25″	1·8″
4° 0′	11′ 52″	2·2″	24·1″	1·70″
10′	11′ 30″	2·1″	23·4″	1·64″
20′	11′ 10″	2·0″	22·7″	1·58″
30′	10′ 50″	1·9″	22·0″	1·53″
40′	10′ 32″	1·8″	21·3″	1·48″
50′	10′ 15″	1·7″	20·7″	1·43″
5° 0′	9′ 58″	1·6″	20·1″	1·38″
10′	9′ 42″	1·5″	19·6″	1·34″
20′	9′ 27″	1·5″	19·1″	1·30″
30′	9′ 11″	1·4″	18·6″	1·26″
40′	8′ 58″	1·3″	18·1″	1·22″
50′	8′ 45″	1·3″	17·6″	1·19″

ASTRONOMICAL REFRACTION—*Continued*.

Apparent Altitude.	Mean Refraction.	Difference for 1′ Altitude.	Difference for + 1 in. Bar.	Difference for − 1″ Temp.
6° 0′	8′ 32″	1·2″	17·2″	1·15″
10′	8′ 20″	1·2″	16·8″	1·11″
20′	8′ 9″	1·1″	16·4″	1·09″
30′	7′ 58″	1·1″	16·0″	1·06″
40′	7′ 47″	1·0″	15·7″	1·03″
50′	7′ 37″	1·0″	15·3″	1·00″
7° 0′	7′ 27″	1·0″	15·0″	0·98″
10′	7′ 17″	0·9″	14·6″	·95″
20′	7′ 8″	·9″	14·3″	·93″
30′	6′ 59″	·8″	14·1″	·91″
40′	6′ 51″	·8″	13·8″	·89″
50′	6′ 43″	·8″	13·5″	·87″
8° 0′	6′ 35″	0·7″	13·3″	0·85″
10′	6′ 28″	·7″	13·1″	·83″
20′	6′ 21″	·7″	12·8″	·82″
30′	6′ 14″	·7″	12·6″	·80″
40′	6′ 7″	·7″	12·3″	·79″
50′	6′ 0″	·6″	12·1″	·77″
9° 0′	5′ 54″	0·6″	11·9″	0·76″
10′	5′ 47″	·6″	11 7″	·74″
20′	5′ 41″	·6″	11·5″	·73″
30′	5′ 36″	·6″	11·3″	·71″
40′	5′ 30″	·5″	11·1″	·71″
50′	5′ 25″	·5″	11·0″	·70″
10° 0′	5′ 20″	0·5″	10·8″	0·69″
10′	5′ 15″	·5″	10·6″	·67″
20′	5′ 10″	·5″	10·4″	·65″
30′	5′ 5″	·5″	10·2″	·64″
40′	5′ 0″	·5″	10·1″	·63″
50′	4′ 56″	·4″	9·9″	·62″
11° 0′	4′ 51″	0·4″	9·8″	0·60″
10′	4′ 47″	·4″	9·6″	·59″
20′	4′ 43″	·4″	9·5″	·58″
30′	4′ 39″	·4″	9·4″	·57″
40′	4′ 35″	·4″	9·2″	·56″
50′	4′ 31″	·4″	9·1″	·55″

ASTRONOMICAL REFRACTION—*Continued.*

Apparent Altitude.	Mean Refraction.	Difference for 1' Altitude.	Difference for + 1 in. Bar.	Difference for − 1° Temp.
12° 0′	4′ 28·1″	0·38″	9·00″	0·556″
10′	4′ 24·4″	·37″	8·86″	·548″
20′	4′ 20·8″	·36″	8·74″	·541″
30′	4′ 17·3″	·35″	8·63″	·533″
40′	4′ 13·9″	·33″	8·51″	·524″
50′	4′ 10·7″	·32″	8·41″	·517″
13° 0′	4′ 7·5″	0·31″	8·30″	0·509″
10′	4′ 4·4″	·31″	8·20″	·503″
20′	4′ 1·4″	·30″	8·10″	·496″
30′	3′ 58·4″	·30″	8·00″	·490″
40′	3′ 55·5″	·29″	7·89″	·482″
50′	3′ 52·6″	·29″	7·79″	·476″
14° 0′	3′ 49·9″	0·28″	7·70″	0·469″
10′	3′ 47·1″	·28″	7·61″	·464″
20′	3′ 44·4″	·27″	7·52″	·458″
30′	3′ 41·8″	·26″	7·43″	·453″
40′	3′ 39·2″	·26″	7·34″	·448″
50′	3′ 36·7″	·25″	7·26″	·444″
15° 0′	3′ 34·3″	0·24″	7·18″	0·439″
30′	3′ 27·3″	·22″	6·95″	·424″
16° 0′	3′ 20·6″	·21″	6·73″	·411″
30′	3′ 14·4″	·20″	6·51″	·399″
17° 0′	3′ 8·5″	·19″	6·31″	·386″
30′	3′ 2·9″	·18″	6·12″	·374″
18° 0′	2′ 57·6″	0·17″	5·98″	0·362″
19° 0′	2′ 47·7″	·16″	5·61″	·340″
20° 0′	2′ 38·7″	·15″	5·31″	·322″
21° 0′	2′ 30·5″	·13″	5·04″	·305″
22° 0′	2′ 23·2″	·12″	4·79″	·290″
23° 0′	2′ 16·5″	·11″	4·57″	·276″
24° 0′	2′ 10·1″	0·10″	4·35″	0·264″
25° 0′	2′ 4·2″	·09″	4·16″	·252″
26° 0′	1′ 58·8″	·09″	3·97″	·241″
27° 0′	1′ 53·8″	·08″	3·81″	·230″
28° 0′	1′ 49·1″	·08″	3·65″	·219″
29° 0′	1′ 44·7″	·07″	3·50″	·209″

ASTRONOMICAL REFRACTION—*Continued.*

Apparent Altitude.	Mean Refraction.	Difference for 1′ Altitude.	Difference for + 1 in. Bar.	Difference for − 1° Temp.
30° 0′	1′ 40·5″	0·07″	3·36″	0·201″
31° 0′	1′ 36·6″	·06″	3·23″	·193″
32° 0′	1′ 33·0″	·06″	3·11″	·186″
33° 0′	1′ 29·5″	·06″	2·99″	·179″
34° 0′	1′ 26·1″	·05″	2·88″	·173″
35° 0′	1′ 23·0″	·05″	2·78″	·167″
36° 0′	1′ 20·0″	0·05″	2·68″	0·161″
37° 0′	1′ 17·1″	·05″	2·58″	·155″
38° 0′	1′ 14·4″	·05″	2·49″	·149″
39° 0′	1′ 11·8″	·04″	2·40″	·144″
40° 0′	1′ 9·3″	·04″	2·32″	·139″
41° 0′	1′ 6·9″	·04″	2·24″	·134″
42° 0′	1′ 4·6″	0·038″	2·16″	0·130″
43° 0′	1′ 2·4″	·036″	2·09″	·125″
44° 0′	1′ 0·3″	·034″	2·02″	·120″
45° 0′	0′ 58·1″	·034″	1·94″	·117″
46° 0′	56·1″	·033″	1·88″	·112″
47° 0′	54·2″	·032″	1·81″	·108″
48° 0′	0′ 52·3″	0·031″	1·75″	0·104″
49° 0′	50·5″	·030″	1·69″	·101″
50° 0′	48·8″	·029″	1 63″	·097″
51° 0′	47·1″	·028″	1·58″	·094″
52° 0′	45·4″	·027″	1·52″	·090″
53° 0′	43·8″	·026″	1·47″	·088″
54° 0′	0′ 42·2″	0·026″	1·41″	0·085″
55° 0′	40·8″	·025″	1·36″	·082″
56° 0′	39·3″	·025″	1·31″	·079″
57° 0′	37·8″	·025″	1·26″	·076″
58° 0′	36·4″	·024″	1·22″	·073″
59° 0′	35·0″	·024″	1·17″	·070″
60° 0′	0′ 33·6″	0·023″	1·12″	0·067″
61° 0′	32·3″	·022″	1·08″	·065″
62° 0′	31·0″	·022″	1·04″	·062″
63° 0′	29·7″	·021″	0·99″	·060″
64° 0′	28·4″	·021″	·95″	·057″
65° 0′	27·2″	·020″	·91″	·055″

ASTRONOMICAL REFRACTION—*Continued.*

Apparent Altitude.	Mean Refraction.	Difference for 1' Altitude.	Difference for + 1 in. Bar.	Difference for − 1° Temp.
66° 0′	0′ 25·9″	0·020″	0·87″	0·052″
67° 0′	24·7″	·020″	·83″	·050″
68° 0′	23·5″	·020″	·79″	·047″
69° 0′	22·4″	·020″	·75″	·045″
70° 0′	21·2″	·020″	·71″	·043″
71° 0′	19·9″	·020″	·67″	·040″
72° 0′	0′ 18·8′	0·019″	0·63″	0·038″
73° 0′	17·7″	·018″	·59″	·036″
74° 0′	16·6″	·018″	·56″	·033″
75° 0′	15·5″	·018″	·52″	·031″
76° 0′	14·4″	·018″	·48″	·029″
77° 0′	13·4″	·017″	·45″	·027″
78° 0′	0′ 12·3″	0·017″	0·41″	0·025″
79° 0′	11·2″	·017″	·38″	·023″
80° 0′	10·2″	·017″	·34″	·021″
81° 0′	9·2″	·017″	·31″	·018″
82° 0′	8·2″	·017″	·27″	·016″
83° 0′	7·1″	·017″	·24″	·014″
84° 0′	0′ 6·1″	0·017″	0·20″	0·012″
85° 0′	5·1″	·017″	·17″	·010″
86° 0′	4·1″	·017″	·14″	·008″
87° 0′	3·1″	·017″	·10″	·006″
88° 0′	2·0″	·017″	·07″	·004″
89° 0′	1·0″	·017″	·03″	·002″

APPENDIX C.

$$h_2 = \frac{2 \sin^2 (\tfrac{1}{2} H \times 15)}{\sin 1''}$$

H.	0 m.	1 m.	2 m.	3 m.	4 m.	5 m.	6 m.	7 m.	8 m.	9 m.
s.	″	″	″	″	″	″	″	″	″	″
0	0·0	2·0	7·8	17·7	31·4	49·1	70·7	96·2	125·7	159·0
1	0·0	2·0	8·0	17·9	31·7	49·4	71·1	96·7	126·2	159·6
2	0·0	2·1	8·1	18·1	31·9	49·7	71·5	97·1	126·7	160·2
3	0·0	2·2	8·2	18·3	32·2	50·1	71·9	97·6	127·2	160·8
4	0·0	2·2	8·4	18·5	32·5	50·4	72·3	98·0	127·8	161·4
5	0·0	2·3	8·5	18·7	32·7	50·7	72·7	98·5	128·3	162·0
6	0·0	2·4	8·7	18·9	33·0	51·1	73·1	99·0	128·8	162·6
7	0·0	2·4	8·8	19·1	33·3	51·4	73·5	99·4	129·3	163·2
8	0·0	2·5	8·9	19·3	33·5	51·7	73·9	99·9	129·9	163·8
9	0·0	2·6	9·1	19·5	33·8	52·1	74·3	100·4	130·4	164·4
10	0·1	2·7	9·2	19·7	34·1	52·4	74·7	100·8	130·9	165·0
11	0·1	2·7	9·4	19·9	34·4	52·7	75·1	101·3	131·5	165·6
12	0·1	2·8	9·5	20·1	34·6	53·1	75··	101·8	132·0	166·2
13	0·1	2·9	9·6	20·3	34·9	53·4	75·9	102·3	132·6	166·8
14	0·1	3·0	9·8	20·5	35·2	53·8	76·3	102·7	133·1	167·4
15	0·1	3·1	9·9	20·7	35·5	54·1	76·7	103·2	133·6	168·0
16	0·1	3·1	10·1	20·9	35·7	54·5	77·1	103·7	134·2	168·6
17	0·2	3·2	10·2	21·2	36·0	54·8	77··	104·2	134·7	169·2
18	0·2	3·3	10·4	21·4	36·3	55·1	77·9	104·6	135·3	169·8
19	0·2	3·4	10·5	21·6	36·6	55·5	78·3	105·1	135·8	170·4
20	0·2	3·5	10·7	21·8	36·9	55·8	78·8	105·6	136·3	171·0
21	0·2	3·6	10·8	22·0	37·2	56·2	79·2	106·1	136·9	171·6
22	0·3	3·7	11·0	22·3	37·4	56·5	79·6	106·6	137·4	172·2
23	0·3	3·8	11·2	22·5	37·7	56·9	80·0	107·0	138·0	172·9
24	0·3	3·8	11·3	22·7	38·0	57·3	80·4	107·5	138·5	173·5
25	0·3	3·9	11·5	22·9	38·3	57·6	80·8	108·0	139·1	174·1
26	0·4	4·0	11·6	23·1	38·6	58·0	81·3	108·5	139·6	174·7
27	0·4	4·1	11·8	23·4	38·9	58·3	81·7	109·0	140·2	175·3
28	0·4	4·2	11·9	23·6	39·2	58·7	82·1	109·5	140·7	175·9
29	0·5	4·3	12·1.	23·8	39·5	59·0	82·5	110·0	141·3	176·6

$$h_2 = \frac{2 \sin^2 (\tfrac{1}{2} H \times 15)}{\sin 1''}$$

H.	0 m.	1 m.	2 m.	3 m.	4 m.	5 m.	6 m.	7 m.	8 m.	9 m.
s.	″	″	″	″	″	″	″	″	″	″
30	0·5	4·4	12·3	24·0	39·8	59·4	83·0	110·4	141·8	177·2
31	0·5	4·5	12·4	24·3	40·1	59·8	83·4	110·9	142·4	177·8
32	0·6	4·6	12·6	24·5	40·3	60·1	83·8	111·4	143·0	178·4
33	0·6	4·7	12·8	24·7	40·6	60·5	84·2	111·9	143·5	179·0
34	0·6	4·8	12·9	25·0	40·9	60·8	84·7	112·4	144·1	179·7
35	0·7	4·9	13·1	25·2	41·2	61·2	85·1	112·9	144·6	180·3
36	0·7	5·0	13·3	25·4	41·5	61·6	85·5	113·4	145·2	180·9
37	0·7	5·1	13·4	25·7	41·8	61·9	86·0	113·9	145·8	181·6
38	0·8	5·2	13·6	25·9	42·1	62·3	86·4	114·4	146·3	182·2
39	0·8	5·3	13·8	26·2	42·5	62·7	86·8	114·9	146·9	182·8
40	0·9	5·4	14·0	26·4	42·8	63·0	87·3	115·4	147·5	183·5
41	0·9	5·6	14·1	26·6	43·1	63·4	87·7	115·9	148·0	184·1
42	1·0	5·7	14·3	26·9	43·4	63·8	88·1	116·4	148·6	184·7
43	1·0	5·8	14·5	27·1	43·7	64·2	88·6	116·9	149·2	185·4
44	1·1	5·9	14·7	27·4	44·0	64·5	89·0	117·4	149·7	186·0
45	1·1	6·0	14·8	27·6	44·3	64·9	89·5	117·9	150·3	186·6
46	1·2	6·1	15·0	27·9	44·6	65·3	89·9	118·4	150·9	187·3
47	1·2	6·2	15·2	28·1	44·9	65·7	90·3	118·9	151·5	187·9
48	1·3	6·4	15·4	28·3	45·2	66·0	90·8	119·5	152·0	188·5
49	1·3	6·5	15·6	28·6	45·5	66·4	91·2	120·0	152·6	189·2
50	1·4	6·6	15·8	28·8	45·9	66·8	91·7	120·5	153·2	189·8
51	1·4	6·7	15·9	29·1	46·2	67·2	92·1	121·0	153·8	190·5
52	1·5	6·8	16·1	29·4	46·5	67·6	92·6	121·5	154·3	191·1
53	1·5	7·0	16·3	29·6	46·8	68·0	93·0	122·0	154·9	191·8
54	1·6	7·1	16·5	29·9	47·1	68·3	93·5	122·5	155·5	192·4
55	1·6	7·2	16·7	30·1	47·5	68·7	93·9	123·1	156·1	193·1
56	1·7	7·3	16·9	30·4	47·8	69·1	94·4	123·6	156·7	193·7
57	1·8	7·5	17·1	30·6	48·1	69·5	94·8	124·1	157·3	194·4
58	1·8	7·6	17·3	30·9	48·4	69·9	95·3	124·6	157·8	195·0
59	1·9	7·7	17·5	31·2	48·8	70·3	95·7	125·1	158·4	195·7

$$h_2 = \frac{2 \sin^2 (\frac{1}{2} H \times 15)}{\sin 1''}$$

H.	10 m.	11 m.	12 m.	13 m.	14 m.	15 m.	16 m.	17 m.	18 m.	19 m.
S.	″	″	″	″	″	″	″	″	″	″
0	196·3	237·5	282·7	331·7	384·7	441·6	502·5	567·2	635·9	708·4
1	197·0	238·5	283·5	332·6	385·6	442·6	503·5	568·3	637·0	709·7
2	197·6	239·0	284·3	333·4	386·6	443·6	504·6	569·4	638·2	710·9
3	198·3	239·7	285·1	334·3	387·5	444·6	505·6	570·5	639·4	712·1
4	198·9	240·4	285·8	335·2	388·4	445·6	506·7	571·6	640·6	713·4
5	199·6	241·1	286·6	336·0	389·3	446·6	507·7	572·8	641·7	714·6
6	200·3	241·9	287·4	336·8	390·2	447·5	508·8	573·9	642·9	715·9
7	200·9	242·6	288·2	337·7	391·2	448·5	509·8	575·0	644·1	717·1
8	201·6	243·3	289·0	338·6	392·1	449·5	510·9	576·1	645·3	718·4
9	202·2	244·1	289·9	339·4	393·0	450·5	511·9	577 2	646·5	719·6
10	202·9	244·8	290·6	340·3	393·9	451·5	513·0	578·4	647·7	720·9
11	203·6	245·5	291·4	341·2	394·9	452·5	514·0	579·5	648·9	722·1
12	204·2	246·3	292·2	342·0	395·8	453·5	515·1	580·6	650·0	723·4
13	204·9	247·0	293·0	342·9	396·7	454·5	516·1	581·7	651·2	724·6
14	205·6	247·7	293·8	343·7	397·6	455·5	517·2	582·9	652·4	725·9
15	206·3	248·5	294·6	344·6	398·6	456·5	518·3	584·0	653·6	727·2
16	206·9	249·2	295·4	345·5	399·5	457·5	519·3	585·1	654·8	728·4
17	207·6	249·9	296·2	346·4	400·5	458·5	520·4	586·2	656·0	729·7
18	208·3	250·7	297·0	347·2	401·4	459·5	521·5	587·4	657·2	730·9
19	208·9	251·4	297·8	348·1	402·3	460·5	522·5	588·5	658·4	732·2
20	209·6	252·2	298·6	349·0	403·3	461·5	523·6	589·6	659·6	733·5
21	210·3	252·9	299·4	349·8	404·2	462·5	524·7	590·8	660·8	734·7
22	211·0	253·6	300·2	350·7	405·1	463·5	525·7	591·9	662·0	736·0
23	211·7	254·4	301·0	351·6	406·1	464·5	526·8	593·0	663·2	737·3
24	212·3	255·1	301·8	352·5	407·0	465·5	527·9	594·2	664·4	738·5
25	213·0	255·9	302·6	353·3	408·0	466·5	529·0	595·3	665·6	739·8
26	213·7	256·6	303·5	354·2	408·9	467·5	530·0	596·5	666·8	741·1
27	214·4	257·4	304·3	355·1	409·8	468·5	531·1	597·6	668·0	742·3
28	215·1	258·1	305·1	356·0	410·8	469·5	532·2	598·7	669·2	743·6
29	215·8	258·9	305·9	356·9	411·7	470·5	533·3	599·0	670·4	744·9
30	216·4	259·6	306·7	357·7	412·7	471·5	534·3	601·0	671·6	746·2
31	217·1	260·4	307·5	358·6	413·6	472·6	535·4	602·2	672·8	747·4
32	217·8	261·1	308·4	359·5	414·6	473·6	536·5	603·3	674·1	748·7
33	218·5	261·9	309·2	360·4	415·5	474·6	537·6	604·5	675·3	750·0
34	219·2	262·6	310·0	361·3	416·5	475·6	538·7	605·6	676·5	751·3

$$h_2 = \frac{2 \sin^2(\frac{1}{2}\,\mathrm{II} \times 15)}{\sin 1''}$$

H.	10 m.	11 m.	12 m.	13 m.	14 m.	15 m.	16 m.	17 m.	18 m.	19 m.
s.	″	″	″	″	″	″	″	″	″	″
35	219·9	263·4	310·8	362·2	417·4	476·6	539·7	606·8	677·7	752·6
36	220·6	264·1	311·6	363·1	418·4	477·6	540·8	607·9	678·9	753·8
37	221·3	264·9	312·5	364·0	419·4	478·7	541·9	609·1	680·1	755·1
38	222·0	265·7	313·3	364·8	420·3	479·7	543·0	610·2	681·3	756·4
39	222·7	266·4	314·1	365·7	421·3	480·7	544·1	611·4	682·6	757·7
40	223·4	267·2	315·0	366·6	422·2	481·7	545·2	612·5	683·8	759·0
41	224·1	268·0	315·8	367·5	423·2	482·8	546·3	613·7	685·0	760·2
42	224·8	268·7	316·6	368·4	424·2	483·8	547·4	614·8	686·2	761·5
43	225·5	269·5	317·4	369·3	425·1	484·8	548·4	616·0	687·4	762·8
44	226·2	270·3	318·3	370·2	426·1	485·8	549·5	617·2	688·7	764·1
45	226·9	271·0	319·1	371·1	427·0	486·9	550·6	618·3	689·9	765·4
46	227·6	271·8	319·9	372·0	428·0	487·9	551·7	619·5	691·1	766·7
47	228·3	272·6	320·8	372·9	429·0	488·9	552·8	620·6	692·4	768·0
48	229·0	273·3	321·6	373·8	429·9	490·0	553·9	621·8	693·6	769·3
49	229·7	274·1	322·4	374·7	430·9	491·0	555·0	623·0	694·8	770·6
50	230·4	274·9	323·3	375·6	431·9	492·0	556·1	624·1	696·0	771·9
51	231·1	275·6	324·1	376·5	432·8	493·1	557·2	625·3	697·3	773·1
52	231·8	276·4	325·0	377·4	433·8	494·1	558·3	626·5	698·5	774·5
53	232·5	277·2	325·8	378·3	434·8	495·2	559·4	627·6	699·7	775·7
54	233·2	278·0	326·7	379·3	435·8	496·2	560·5	628·8	701·0	777·1
55	234·0	278·8	327·5	380·2	436·7	497·2	561·6	630·0	702·2	778·4
56	234·7	279·5	328·4	381·1	437·7	498·3	562·8	631·2	703·5	779·7
57	235·4	280·3	329·2	382·0	438·7	499·3	563·9	632·3	704·7	781·0
58	236·1	281·1	330·0	382·9	439·7	500·4	565·0	633·5	705·9	782·3
59	236·8	281·9	330·9	383·8	440·6	501·4	566·1	634·7	707·1	783·6

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